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APPLICATION OF DERIVATIVES

Contents

Syllabus

Geometrical interpretation of the derivative, tangents and normals, increasing and decreasing functions, maximum and minimum values of a function, Rolle's Theorem and Lagrange's Mean Value theorem.

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APPLICATION OF DERIVATIVES

Rate of change , Tangent and Normal, Error and Approximation, Rolle's Theory, LMVT, Monotonicity, Maxima Minima.

Derivative as rate of change

In various fields of applied mathematics one has the quest to know the rate at which one variable is changing, with respect to other. The rate of change naturally refers to time. But we can have rate of change with respect to other variables also.

An economist may want to study how the investment changes with respect to variations in interest rates.

A physician may want to know, how small changes in dosage can affect the body's response to a drug.

A physicist may want to know the rate of charge of distance with respect to time.

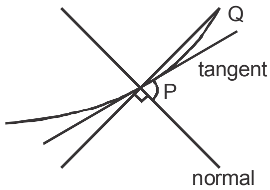
All questions of the above type can be interpreted and represented using derivatives.

Definition: The average rate of change of a function f(x) with respect to x over an interval [a, a + h] is defined as $\frac{f(a+h)-f(a)}{h}$.

Definition: The instantaneous rate of change of f(x) with respect to x is defined as $f'(x) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ provided the limit exists.

Note: To use the word 'instantaneous', x may not be representing time. We usually use the word 'rate of change' to mean instantaneous rate of change'.

Tangent and Normal



Figure

Let y = f(x) be function with graph as shown in figure. Consider secant PQ. If Q tends to P along the curve passing through the points Q_1, Q_2, ...

I.e. $Q \to P$, secant PQ will become tangent at P. A line through P perpendicular to tangent is called normal at P.

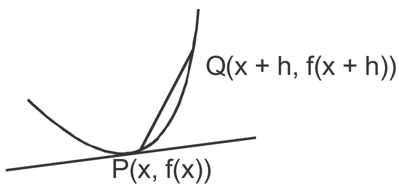
Geometrical Meaning of $\frac{dy}{dx}$

As $Q \to P$, $h \to 0$ and slope of chord PQ tends to slope of tangent at P (see figure).

Slope of chord PQ = $\frac{f(x+h)-f(x)}{h}$

$\lim_{Q\to P}$ slope of chord PQ = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow$ slope of tangent at P = $f'(x) = \frac{dy}{dx}$



Equation of tangent and normal

$\left.\frac{dy}{dx}\right]_{(x_1, y_1)} = f'(x_1)$ denotes the slope of tangent at point (x_1, y_1) on the curve y = f(x). Hence the equation of tangent at (x_1, y_1) is given by

$(y - y_1) = f'(x_1)(x - x_1)$; when, $f'(x_1)$ is real.

Also, since normal is a line perpendicular to tangent at (x_1, y_1) so its equation is given by

$(y - y_1) = -\frac{1}{f'(x_1)}(x - x_1)$ when $f'(x_1)$ is nonzero real.

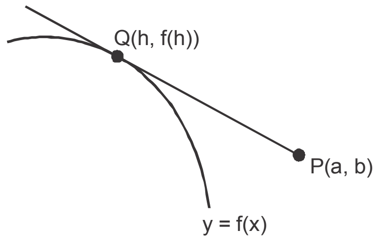
If $f'(x_1) = 0$, then tangent is the line $y = y_1$ and normal is the line $x = x_1$

If $\lim\limits_{h \to 0} \dfrac{f(x_1 + h) - f(x_1)}{h} = \infty$ or $-\infty$, then $x = x_1$ is tangent **(VERTICAL TANGENT)** and $y = y_1$ is normal.

Tangent from an external point

Given a point P(a, b) which does not lie on the curve y = f(x), then the equation of possible tangents to the curve y = f(x) , passing through (a, b) can be found by solving for the point of contact Q.

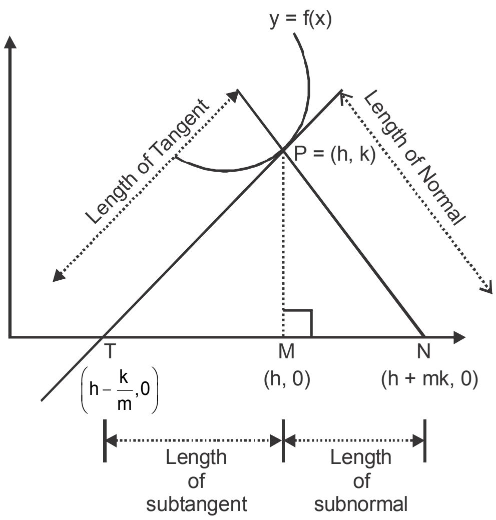
$$f'(h) = \frac{f(h) - b}{h - a}$$



And equation of tangent is $y - b = \dfrac{f(h) - b}{h - a}(x - a)$

Lengths of tangent, normal, subtangent and subnormal:

Let P (h, k) be any point on curve y =f(x). Let tangent drawn at point P meets x-axis at T and normal at point P meets x-axis at N. Then the length PT is called the length of tangent and PN is called length of normal. (as shown in figure)



Projection of segment PT on x-axis, TM, is called the subtangent and similarly projection of line segment. PN on x axis, MN is called subnormal.

Let $m = \left.\dfrac{dy}{dx}\right]_{(h,k)}$ = slope of tangent.

Hence equation of tangent is $m(x - h) = (y - k)$.

Putting y = 0, we get x-intercept of tangent is $x = h - \dfrac{k}{m}$

Similarly, the x-intercept of normal is $= h + km$

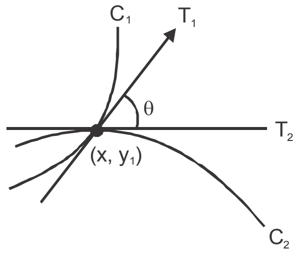
Now, length PT, PN, TM, MN can be easily evaluated using distance formula

(i) $PT = |k| \sqrt{1+\frac{1}{m^2}}$ = Length of Tangent (ii) $PN = |k| \sqrt{1+m^2}$ = Length of Normal

(iii) $TM = \left|\frac{k}{m}\right|$ = Length of subtangent (iv) $MN = |km|$ = Length of subnormal.

Angle between the curves

Angle between two intersecting curves is defined as the acute angle between their tangents (or normal) at the point of intersection of two curves (as shown in figure).



$$\tan q = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

where m_1 & m_2 are the slopes of tangents at the intersection point (x_1, y_1).

Notes: (i) The angle is defined between two curves if the curves are intersecting. This can be ensured by finding their point of intersection or graphically.

(ii) If the curves intersect at more than one point then angle between curves is found out with respect to the point of intersection.

(iii) Two curves are said to be orthogonal if angle between them at each point of intersection is right angle. i.e. $m_1\, m_2 = -1$.

Shortest distance between two curves

Shortest distance between two non-intersecting differentiable curves is always along their common normal. (Wherever defined)

Error and Approximation.

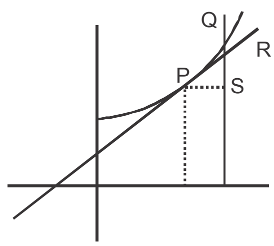
Let y = f(x) be a function. If these is an error dx in x then corresponding error in y is dy = f(x + dx) – f(x).

We have $\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x_1)}{\delta x} = \frac{dy}{dx} = f'(x)$

We define the differential of y, at point x, corresponding to the increment dx as f'(x) dx and denote it by dy.

i.e. dy = f'(x) dx.

Let P(x, f(x)), Q((x + dx), f(x + dx)) (as shown in figure)



dy = QS,

dx = PS,

dy = RS

In many practical situations, it is easier to evaluate dy but nor dy.

Rolle's Theorem:

If a function f defined on [a, b] is

(i) continuous on [a, b]

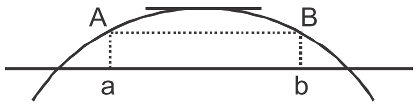
(ii) derivable on (a, b) and

(iii) f(a) = f(b),

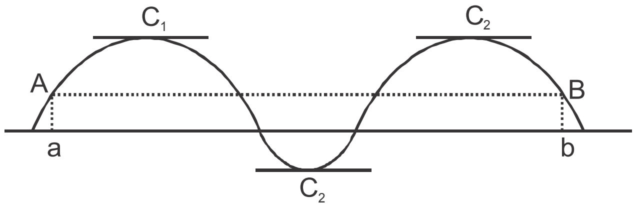
Then there exists at least one real number c between a and b (a < c < b) such that f'(c) = 0

Geometrical explanation of Rolle's Theorem:

Let the curve y = f(x), which is continuous on [a, b] and derivable on (a, b) be drawn (as shown in figure).



A(a, f(a)) , B(b f(b)), f(a) = f(b) , C(c, f(c)) , f' (c) = 0.



$C_1 (c_1 , f(c_1))$, $f'(c_1) = 0$

$C_2 (c_2 , f(c_2))$, $f'(c_2) = 0$

$C_3 (c_3 , f(c_3))$, $f'(c_3) = 0$

The theorem simply states that between two points with equal ordinates on the graph of f(x), there exists at least one point where the tangent is parallel to x-axis.

Algebraic Interpretation of Rolle's Theorem:

Between two zeros a and b of f(x) (i, e. between two roots a and b of f(x) = 0) there exists at least one zero of f'(x)

Lagrange's Mean value Theorem (LMVT):

If a function f defined on [a , b] is

(i) continuous on [a, b] and

(ii) derivable on (a, b)

then there exists at least one real numbers between a and b (a < c < b) such that $\frac{f(b)-f(a)}{b-a} = f'(c)$

Proof: Let us consider a function g(x) = f(x) + λx, x ∈ [a, b]

where λ is a constant to be determined such that g(a) = g(b).

$$\therefore \quad \lambda = -\frac{f(b)-f(a)}{b-a}$$

Now the function g(x), being the sum of two continuous and derivable functions it self

(i) continuous on [a, b]

(ii) derivable on (a, b) and

(iii) g(a) = g(b).

Therefore, by Rolle's theorem there exists a real number c ∈ (a, b) such that g'(c) = 0

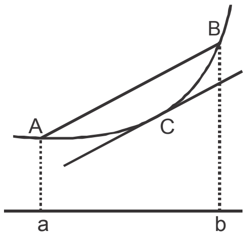
But g'(x) = f'(x) + λ

$\therefore \quad 0 = g'(c) = f'(c) + \lambda$

$$f'(c) = -\lambda = \frac{f(b)-f(a)}{b-a}$$

Geometrical Interpretation of LMVT:

The theorem simply states that between two points A and B of the graph of f(x) there exists at lease one point where tangent is parallel to chord AB.



C(c, f(c)), f'(c) = slope of AB.

Alternative Statement : If in the statement of LMVT, b is replaced by a + h, then number c between a and b may be written as a + θh , where 0 < θ < 1. Thus

$$\frac{f(a+h)-f(a)}{h} = f'(a+\theta h)$$

or

$f(a + h) = f(a) + hf'(a + \theta h),\ 0 < \theta < 1$

Monotonicity of A function:

Let f be a real valued function having domain D(D $\subset$ R) and S be a subset of D. f is said to be monotonically increasing (non decreasing) (increasing) in S if for every $x_1, x_2 \in S$, $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$. f is said to be monotonically decreasing (non increasing) (decreasing) in S if for every $x_1, x_2 \in S$, $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$

f is said to be strictly increasing in S if for $x_1, x_2 \in S$, $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$. Similarly, f is said to be strictly decreasing in S if for $x_1, x_2 \in S$, $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

Notes: (i) f is strictly increasing $\Rightarrow$ f is monotonically decreasing (non increasing). But converse need not be true.

(ii) f is strictly decreasing $\Rightarrow$ f is monotonically decreasing (non increasing). Again, converse need not be true

(iii) If f(x) = constant in S, then f is increasing as well as decreasing in S

(iv) A function f is said to be an increasing function if it is increasing in the domain. Similarly, if f is decreasing in the domain, we say that f is monotonically decreasing

(v) f is said to be a monotonic function if either it is monotonically increasing or monotonically decreasing

(vi) If f is increasing in a subset of S and decreasing in another subset of S, then f is non monotonic in S.

Application of Differentiation for detecting monotonicity:

Let I be an interval (open or closed or semi open and semi closed)

(i) If $f'(x) > 0\ \forall\ x \in I$, then f is strictly increasing in I

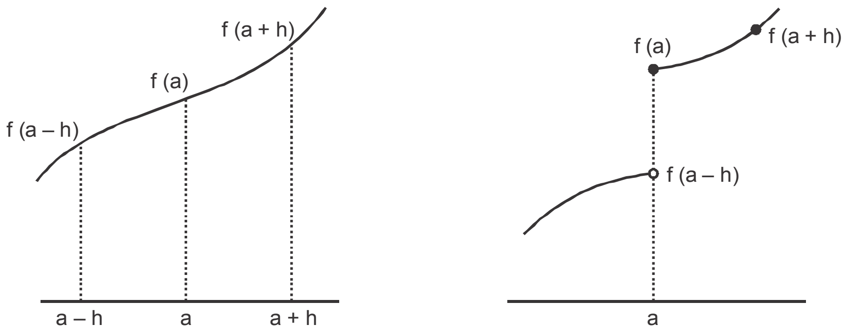
(ii) If $f'(x) > 0\ \forall\ x \in I$, then f is strictly increasing in I

Note: Let I be an interval (or ray) which is a subset of domain of f. If $f'(x) > 0 < \forall\ x \in I$, except for countably many points where f'(x) =0, then f(x) is strictly increasing in I.

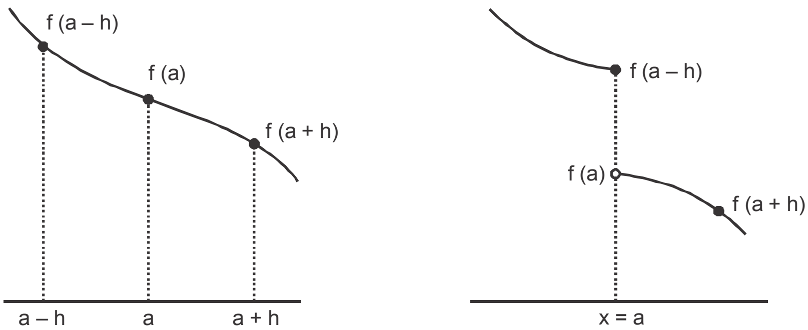
{f'(x) = 0 at countably many points $\Rightarrow$ f'(x) = 0 does not occur on an interval which is a subset of I}

Monotonicity of function about a point:

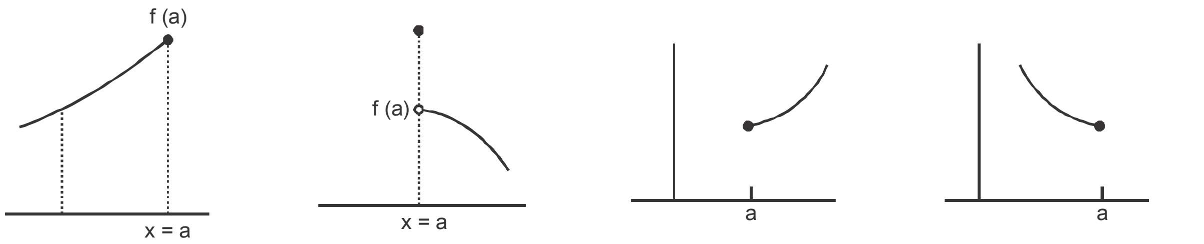
1. A function f(x) is called as a strictly increasing function about a point (or at a point) $a \in D_f$ if it is strictly increasing in an open interval containing a (as shown in figure).



2. A function f(x) is called a strictly decreasing function about a point x = a, if it is strictly decreasing in an open interval containing a (as shown in figure).

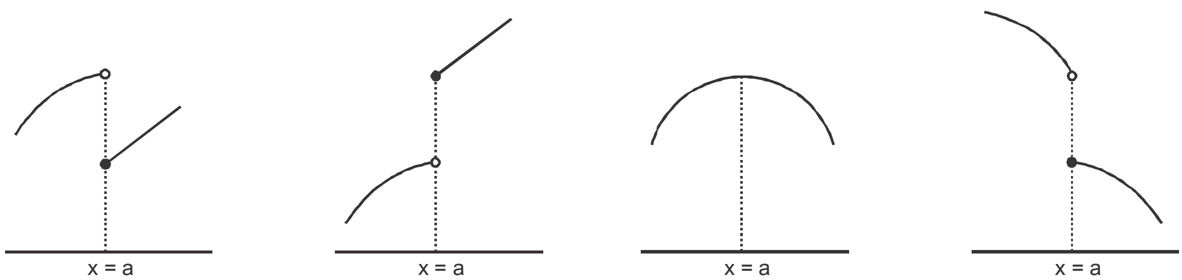


Note: If x = a is a boundary point then use the appropriate one sided inequality to test monotonicity of f(x).



f is increasing at x = a   f is decreasing at x = a   f is increasing at x = a   f is decreasing at x = a

e.g.: Which of the following functions (as shown in figure) is increasing, decreasing or neither increasing nor decreasing at x = a.



neither increasing nor decreasing at x = a   increasing at x= a   neither increasing nor decreasing at x= a   decreasing at x= a

Test for increasing and decreasing functions about a point

Let f(x) be differentiable.

(1) If f'(a) > 0 then f(x) is increasing at x = a.

(2) If f'(a) < 0 then f(x) is decreasing at x = a.

(3) If f'(a) = 0 then examine the sign of f'(x) on the left neighborhood and the right neighborhood of a.

(i) If f''(x) is positive on both the neighbourhoods, then f is increasing at x = a.

(ii) If f'(x) is negative on both the neighbourhoods, then f is decreasing at x = a.

(iii) If f' (x) have opposite signs on these neighbourhoods, then f is non-monotonic at x = a.

Use of monotonicity for proving inequalities

Comparison of two functions f(x) and g(x) can be done by analysing the monotonic behaviour of h(x) = f(x) – g(x)

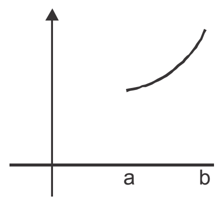
Concavity, convexity, point of inflection

A function f(x) is concave in (a, b) if-tangent drawn at every point $(x_0, (f(x_0))$, for $x_0 \in (a, b)$ lie below the curve. f(x) is convex in (a, b) if tangent drawn at each point $(x_0, f(x_0))$, $x_0 \in (a, b)$ lie above the curve.

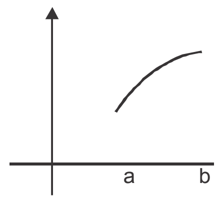
A point (c, f(c)) of the graph y = f(x) is said to be a point of inflection of the graph, if f(x) is concave in $(c - \delta, c)$ and convex in $(c, c + \delta)$ (or vice verse), for some $\delta \in R^+$.

Results:

1. If f''(x) > 0 x , $\in$ (a, b), then the curve y = f(x) is concave in (a, b)



2. If f''(x) < 0 $\forall x \in (a, b)$ then the curve y = f(x) is convex in (a, b)



3. If f is continuous at x = c and f''(x) has opposite signs on either sides of c, then the point (c, f(c)) is a point of inflection of the curve

4. If f''(c) = 0 and f''' (c) $\neq$ 0, then the point (c, f(c)) is a point of inflection

Proving inequalities using curvature:

Generally these inequalities involve comparison between values of two functions at some particular points.

Global Maximum:

A function f(x) is said to have global maximum on a set E if there exists at least one $c \in E$ such that $f(x) \le f(c)$ for all $x \in E$.

We say global maximum occurs at x = c and global maximum (og global maximum value) is f(c).

Local Maxima:

A function f(x) is said to have a local maximum at x = c if f(c) is the greatest value of the function in a small neighbourhood (c – h, c + h), h > 0 of c.

i.e. for all $x \in (c - h, c + h)$, $x \neq c$, we have $f(x) \leq f(c)$.

i.e. $f(c - \delta) \leq f(c) \geq f(c + \delta)$, $0 < \delta < h$

Note: If x = c is a boundary point then consider (c – h, c) or (c, c + h) (h > 0) appropriately.

Global Minimum:

A function f(x) is said to have a global minimum on a set E if there exists at least one $c \in E$ such that $f(x) \geq f(c)$ for all $x \in E$.

Local Minima:

A function f(x) is said to have a local minimum at x = c if f(c) is the least value of the function in a small neighbourhood (c – h, h + h), h> 0 of c.

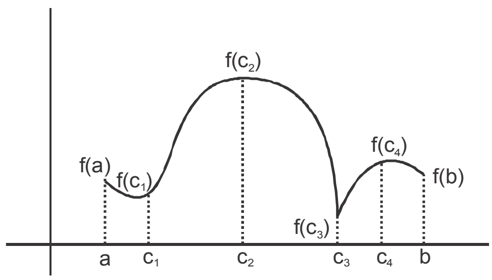
i.e. for all $x \in (c - h, c + h)$, $x \neq c$, we have $f(x) \geq f(c)$.

i.e. $f(c - \delta) \geq f(c) \leq f(c + \delta)$, $0 < \delta < h$

Extrema:

A maxima or a minima is called an extrema.

Explanation:



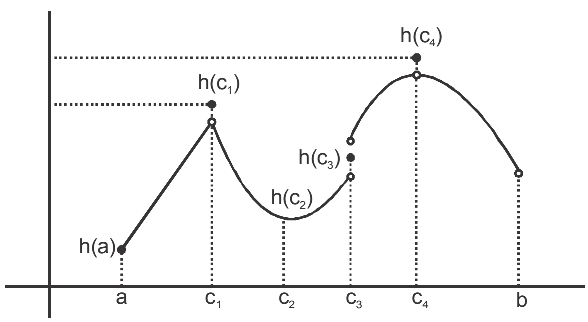
$x = a_1$ $x = c_2$, $x = c_4$ are points of local maxima, with maximum values f(a), $f(c_1)$, $f(c_4)$ respectively.

$x = c_1$, $x = c_{3,}$ x= b are points of local minima, with minimum values $f(c_1)$, $f(c_3)$, f(b) respectively

$x = c_2$ is a point of global minimum

$x = c_3$ is a point of global minimum

Consider the graph of y = h (x), $x \in [a, b)$



$x = c_1$, $x = c_4$ are points of local maxima, with maximum values $h(c_1)$, h (c_4) respectively.

x = a, $x = c_2$ are points of local minima, with minimum values h(a), $h(c_2)$ respectively.

$x = c_3$ is neither a point of maxima nor a minima.

Global maximum is $h(c_4)$

Global minimum is h(a)

Maxima, Minima for differentiable functions:

Mere definition of maxima, minima becomes tedious in solving problems. We use derivative as a tool overcome this difficulty.

A necessary condition for an extrema:

Let f(x) be differentiable at x = c.

Theorem: A necessary condition for f(c) to be an extremum of f(x) is that f'(c) = 0.

i.e. f(c) is extremum $\Rightarrow$ f'(c) = 0

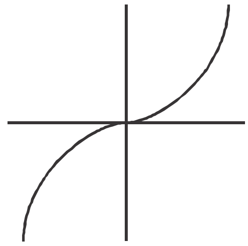
Note: f'(c) = 0 is only a necessary condition but not sufficient

i.e. f'(c) = 0 $\not\Rightarrow$ f(c) is extremum.

consider f(x) = x^3

f'(0) = 0

but f(0) is not an extremum (see figure).

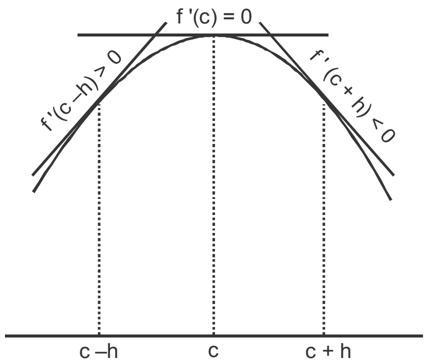


Sufficient condition for an extrema:

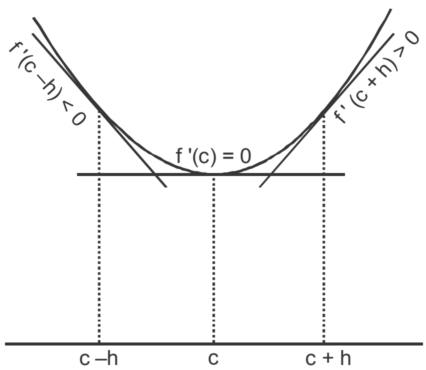
Let f(x) be a differentiable function.

Theorem: A sufficient condition for f(c) to be an extremum of f(x) is that f'(x) changes sign as x passes through c

i.e. f(c) is an extrema (see figure) $\Leftrightarrow$ f'(x) changes sign as x passes through c.



x = c is a point of maxima. f'(x) changes sign from positive to negative.



x = c is a point of local minima (see figure), f'(x) changes sign from negative to positive.

Stationary points:

The points on graph of function f(x) where f'(x) = 0 are called stationary points.

Rate of change of f(x) is zero at a stationary point.

First Derivative Test:

Let f(x) be continuous and differentiable function.

Step-I. find f'(x)

Step-II. Solve f'(x) = 0, let x = c be a solution. (i.e. Find stationary points)

Step-III. Observe change of sign.

(i) If f'(x) changes sign from negative to positive as x crosses c from left to right then x = c is a point of local minima

(ii) If f'(x) changes sign from positive to negative as x crosses c from left to right then x = c is a point of local maxima.

(iii) If f'(x) does not changes sign as x crosses c then x = c is neither a point of maxima nor minima.

Maxima, Minima for continuous functions:

Let f(x) be a continuous function

Critical points:

The points where f'(x) = 0 or f(x) is not differentiable are called critical points.

Stationary points $\subseteq$ Critical points.

Important note:

For f(x) defined on a subset of R, points of extrema (if exists) occur at critical points

Global extrema for continuous functions:

(i) Function defined on closed interval

Let f(x), $x \in [a , b]$ be a continuous function

Step-I Find critical points. Let it be c_1, c_2 ..... , c_n

Step-II Find f(a) , $f(c_1)$ ......, $f(c_n)$, f(b)

Let M = max . {f(a), $f(c_1)$, ..., $f(c_n)$, f(b)}

m = min . {f(a), $f(c_1)$, ...$f(c_n)$, f(b)}

Step-III M is global maximum.

m is global minimum.

(ii) Function defined on open interval

Let f(x), $x \in (a, b)$ be continuous function.

Step-I Find critical points. Let it be c_1, c_2, ...c_n

Step-II Find $f(c_1)$, $f(c_2)$, ..., $f(c_n)$

Let M = max . {$f(c_1)$, ... $f(c_n)$}

m = min. {$f(c_1)$, ...$f(c_n)$}

Step-III $\lim\limits_{x \to a^+} f(x) = \ell_1(\text{say})$, $\lim\limits_{x \to b^-} f(x) = \ell_2(\text{say})$.

Let l = min. $\{\ell_1, \ell_2\}$, L = max . $\{\ell_1, \ell_2\}$

Step-IV

(i) If $m \le \ell$ then m is global minimum

(ii) If $m > \ell$ then f(x) has no global minimum

(iii) If $M \ge L$ then M is global maximum

(iv) If $M < L$ then f(x) has no global minimum

Maxima, Minima by higher order derivatives:

Second derivative text :

Let f(x) have derivatives up to second order

Step-I Find $f'(x)$

Step-II Solve $f'(x) = 0$ Let $x = c$ be a solution

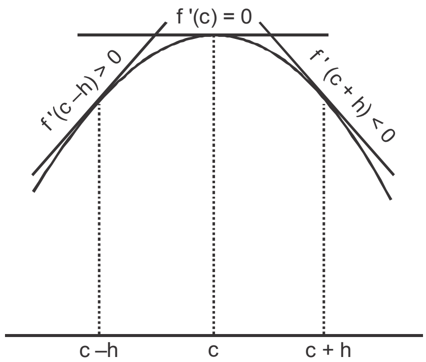
Step-III Find $f''(c)$

Step-IV.

(i) If $f''(c) = 0$ then further investigation is required

(ii) If $f''(c) > 0$ then $x = c$ is a point of minima

(iii) If $f''(c) < 0$ then $x = c$ is a point of maxima.



For maxima $f'(x)$ changes from positive to negative (as shown in figure).

$\Rightarrow$ $f'(x)$ is decreasing hence $f''(c) < 0$

n^{th} Derivative test:

Let f(x) have derivatives up to n^{th} order

If $f'(c) = f''(c) = ...= f^{n-1}(c) = 0$ and

$f^{n}(c) \ne 0$ then we have following possibilities

(i) n is even, $f^{(n)}(c) < 0 \Rightarrow x = c$ is point of maxima

(ii) n is even, $f^{(n)}(c) > 0 \Rightarrow x = c$ is point of minima

(iii) n is odd, $f^{(n)}(c) < 0 \Rightarrow$ f(x) is decreasing about $x = c$

(iv) n is odd, $f^{(n)} > 0 \Rightarrow$ f(x) is increasing about $x = c$.

Application of Maxima, Minima:

For a given problem, an objective function can be constructed in terms of one parameter and there extremum value can be evaluated by equating the differential to zero. As discussed in n^{th} derivative test maxima/minima can be identified.

Useful Formulae of Mensuration to Remember:

1. Volume of a cuboid = ℓbh.

2. Surface area of cuboid = $2(\ell b + bh + h\ell)$.

3. Volume of cube = a^3

4. Surface area of cube = $6a^2$

5. Volume of a cone = $\frac{1}{3}\pi r^2 h$

6. Curved surface area of cone = $\pi r\ell$ (ℓ = slant height)

7. Curved surface area of a cylinder = $2\pi rh$.

8. Total surface area of a cylinder = $2\pi rh + 2\pi r^2$

9. Volume of a sphere = $\frac{4}{3}\pi r^3$

10. Surface area of a sphere = $4\pi r^2$

11. Area of a circular sector = $\frac{1}{2}r^2\theta$, when θ is in radians.

12. Volume of a prism = (area of the base) × (height)

13. Lateral surface area of a prism = (perimeter of the base) × (height).

14. Total surface area of a prism = (lateral surface area) + 2 (area of the base)
    (Note that lateral surfaces of a prism are all rectangle).

15. Volume of a pyramid = $\frac{1}{3}$ (area of the base) × (height).

16. Curved surface area of a pyramid = $\frac{1}{2}$ (perimeter of the base) × (slant height).

    (Note that slant surfaces of a pyramid are triangles).

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

* ***Marked Questions are having more than one correct option.***

Section (A) : Rate of change, Approximation

A-1. Water is poured into an inverted conical vessel of which the radius of the base is 2m and height 4m, at the ratio of 77 litre/minute. The rate at which the water level is rising at the instant when the depth is 70 cm is (use $\pi = 22/7$)

(A) 10 cm/min (B) 20 cm/min (C) 40 cm/min (D) None of these

A-2. On the curve $x^3 = 12y$. The interval in which abscissa changes at a faster rate then its ordinate

(A) $(0, 2)$ (B) $(-\infty, -2) \cup (2, \infty)$ (C) $(-2, 2)$ (D) None of these

A-3. Using differentials, find the approximate value of $\sqrt{25.2}$

(A) 5.02 (B) 5.01 (C) 5.03 (D) 5.04

A-4. The approximate change in the volume of a cube of side x meters caused by increasing the side by 4% is

(A) $0.06x^3m^3$ (B) $0.09\,x^3m^3$ (C) $0.12\,x^3m^3$ (D) $0.15\,x^3m^3$

Section (B) : Tangent, Normal, Angle between curves, Orthogonality of Curves, Shortest distance, Length of Tangent & Normal

B-1. Equation of the normal to the curve $y = -\sqrt{x} + 2$ at the point of its intersection with the curve $y = \tan(\tan^{-1} x)$ is

(A) $2x - y - 1 = 0$ (B) $2x - y + 1 = 0$ (C) $2x + y - 3 = 0$ (D) None

B-2. The curve $y - e^{xy} + x = 0$ has a vertical tangent at

(A) (1, 1) (B) (0, 1) (C) (1, 0) (D) no point

B-3. If the tangent to the curve $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ at $\theta = \frac{\pi}{3}$ makes an angle α $(0 \le \alpha < \pi)$. with x-axis, then α =

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

B-4. Number of tangents drawn from the point (–1/2, 0) to the curve $y = e^{\{x\}}$. (Here {} denotes fractional part function)

(A) 2 (B) 1 (C) 3 (D) 4

B-5*. If tangent to curve $2y^3 = ax^2 + x^3$ at point (a, a) cuts off intercepts α, β on co-rodinate axes, where $\alpha^2 + \beta^2 = 61$, then the value of 'a' is equal to

(A) 20 (B) 25 (C) 30 (D) – 30

B-6*. The co-ordinates of point (s) on the graph of the function, $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 7x - 4$ where the tangent drawn cut off intercepts from the co-ordinate axes which are equal in magnitude but opposite in sign, is

(A) (2, 8/3) (B) (3, 7/2) (C) (1, 5/6) (D) none

B-7. If curve $y = 1 - ax^2$ and $y = x^2$ intersect orthogonally then the value of a is
(A) 1/2 (B) 1/3 (C) 2 (D) 3

B-8. The coordinates of the point of the parabola $y^2 = 8x$, which is at minimum distance from the circle $x^2 + (y + 6)^2 = 1$ are
(A) (2, – 4) (B) (18, – 12) (C) (2, 4) (D) None of these

B-9*. The co-rodinates of a point on the parabola $2y = x^2$ which is nearest to the point (0, 3) is
(A) (2, 2) (B) $\left(-\sqrt{2}, 1\right)$ (C) $\left(\sqrt{2}, 1\right)$ (D) (– 2, 2)

Section (C) Monotonicity an on interval, Monotonicity about a point

C-1. The function $\frac{|x-1|}{x^2}$ is monotonically decreasing at the point
(A) x = 3 (B) x = 1 (C) x = 2 (D) none of these

C-2. The values of p for which the function f(x) $\left(\frac{\sqrt{p+4}}{1-p}-1\right)x^5 - 3x + \ln 5$ decreases for all real x is

(A) $(-\infty, \infty)$ (B) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$

(C) $\left[-3, \frac{5-\sqrt{27}}{2}\right] \cup (2, \infty)$ (D) $[1, \infty]$

C-3*. Which of the following statements is/are correct?
(A) x + sin x is increasing function (B) sec x is neither increasing nor decreasing function
(C) x + sin x is decreasing function (D) sec x is an increasing function

C-4*. If $f(x) = 2x + \cot^{-1} x + \ln\left(\sqrt{1+x^2} - x\right)$, then f(x):
(A) increases in $[0, \infty)$ (B) decreases in $[0, \infty)$
(C) neither increases nor decreases in $[0, \infty)$ (D) increases in $(-\infty, \infty)$

C-5. Let $g(x) = 2f(x/2) + f(1 - x)$ and $f''(x) < 0$ in $0 \le x \le 1$ then g(x)
(A) decreases in $\left[0, \frac{2}{3}\right]$ (B) decreases $\left[\frac{2}{3}, 1\right]$ (C) increases in $\left[0, \frac{2}{3}\right]$ (D) increases in $\left[\frac{2}{3}, 1\right]$

Section (D) Local maxima, Local minima, Global maxima, Global minima, Application of Maxima and Minima

D-1. If $f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + ...+ 100\,x^{100}$ is a polynomial in a real variable x, then f(x) has:
(A) neither a maximum nor a minimum (B) only one maximum
(C) only one minimum (D) one maximum and one minimum

D-2. If $f(x) = \sin^3 x + \lambda \sin^2 x$; $-\pi/2 < x < \pi/2$, then the interval in which λ should lie in order that f(x) has exactly one minima and one maxima

(A) $(-3/2, 3/2) - \{0\}$ (B) $(-2/3, 2/3) - \{0\}$ (C) R (D) $\left[-\frac{3}{2}, 0\right)$

D-3. The greatest, the least values of the function , $f(x) = 2 - \sqrt{1+2x+x^2}$, $x \in [-2, 1]$ are respectively

(A) 2, 1 (B) 2, –1 (C) 2, 0 (D) none of these

D-4. If $f(x) = a \ln |x| + bx^2 + x$ has its extremum values at x = –1 and x = 2, then

(A) a = 2, b = –1 (B) a = 2, b = – 1/2 (C) a = – 2, b = 1/2 (D) none of these

D-5*. Let $f(x) = (x^2 - 1)^n (x^2 + x + 1)$. f(x) has local extremum at x = 1 if

(A) n = 2 (B) n = 3 (C) n = 4 (D) n = 6

D-6*. If $f(x) = \frac{x}{1 + x\tan x}, x \in \left(0, \frac{\pi}{2}\right)$, then

(A) f(x) has exactly one point of minima (B) f(x) has exactly one point of maxima

(C) f(x) is increasing in $\left(0, \frac{\pi}{2}\right)$ (D) maxima occurs at x_0 where $x_0 = \cos x_0$

D-7*. If $f(x) = \begin{cases} -\sqrt{1-x^2} & ,0 \le x \le 1 \\ -x & ,x > 1 \end{cases}$, then

(A) Maximum of f(x) exist at x = 1 (B) Maximum of f(x) doesn't exists

(C) Minimum of $f^{-1}(x)$ exist at x = –1 (D) Minimum of $f^{-1}(x)$ exist at x = 1

D-8*. If $f(x) = \tan^{-1} x - (1/2) \log x$. Then

(A) the greatest value of f(x) on $\left[1/\sqrt{3}, \sqrt{3}\right]$ is $\pi/6 + (1/4) \ln 3$

(B) the least value of f(x) on $\left[1/\sqrt{3}, \sqrt{3}\right]$ is $\pi/3 - (1/4) \ln 3$

(C) f(x) decreases on $(0, \infty)$

(D) f(x) increases on $(-\infty, 0)$

D-9. Let $f(x) = (1+ b^2) x^2 + 2bx + 1$ and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is

(A) [0, 1] (B) $\left(0, \frac{1}{2}\right]$ (C) $\left[\frac{1}{2}, 1\right]$ (D) (0, 1]

D-10. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone is

(A) one third that of the cone (B) $1/\sqrt{2}$ times that of the cone

(C) 2/3 that of the cone (D) 1/2 that of the cone

D-11. The dimensions of the rectangle of maximum area that can be inscribed in the ellipse $(x/4)^2 + (y/3)^2 = 1$ are

(A) $\sqrt{8}, \sqrt{2}$ (B) 4, 3 (C) $2\sqrt{8}, 3\sqrt{2}$ (D) $\sqrt{2}, \sqrt{6}$

D-12. The largest area of a rectangle which has one side on the x-axis and the two vertices on the curve $y = e^{-x^2}$ is

(A) $\sqrt{2}\ e^{-1/2}$ (B) $2\ e^{-1/2}$ (C) $e^{-1/2}$ (D) none

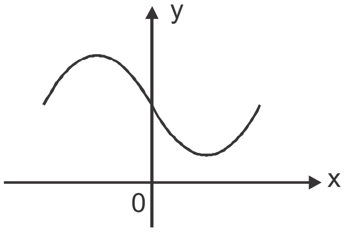
D-13. The maximum distance of the point (k, 0) from the curve $2x^2 + y^2 - 2x = 0$ is equal to

(A) $\sqrt{1+2k-k^2}$ (B) $\sqrt{1+2k-2k^2}$ (C) $\sqrt{1-2k-2k^2}$ (D) $\sqrt{1-2k+k^2}$

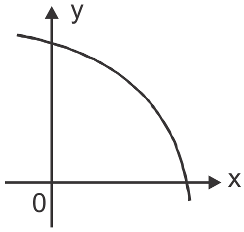
Section (E) Curvature, Points of inflection, Inequalities

E-1. The curve y = f(x) which satisfies the condition f '(x) > 0 and f " (x) < 0 for all real x, is:

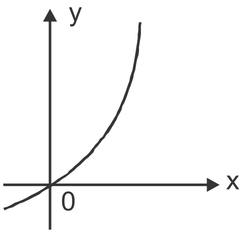
(A)



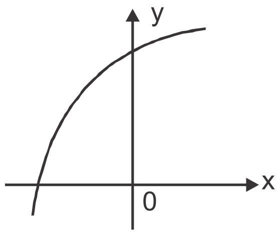
(B)



(C)



(D)



E-2. For which values of 'a' will the function $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$ will be concave upward along the entire real line

(A) $a \in [0, \infty)$ (B) $a \in (-2, \infty)$ (C) $a \in [-2, 2]$ (D) $a \in (0, \infty)$

E-3. If the point (1, 3) serves as the point of inflection of the curve $y = ax^3 + bx^2$ then the value of 'a' and 'b' are :

(A) a = 3/ 2& b = –9/2 (B) a = 3/2 & b = 9/2

(C) a = – 3/ 2& b = –9/2 (D) a = – 3/2 & b = 9/2

E-4*. If $f(x) = \ln(x-2) - \frac{1}{x}$, then

(A) f(x) is M.I. for $x \in (2, \infty)$ (B) f(x) is M.I. for $x \in [-1, 2]$

(C) f(x) is always concave downwards (D) $f^{-1}(x)$ is M.I. wherever defined

Section (F): Rolle's Theorem, LMVT

F-1. The function $f(x) = x^3 - 6x^2 + ax + b$ satisfy the conditions of Rolle's theorem on [1, 3]. Which of these are correct?

(A) a = 11 , $b \in R$ (B) a = 11, b = – 6 (C) a = – 11, b = 6 (D) a = – 11, $b \in R$

F-2. The function $f(x) = x(x+3)e^{-x/2}$ satisfies all the conditions of Rolle's theorem on [–3, 0]. The value of c which verifies Rolle's theorem, is

(A) 0 (B) – 1 (C) – 2 (D) 3

F-3. If f(x) satisfies the requirements of Lagrange's mean value theorem on [0, 2] and if f(0) = 0 and $f'(x) \le \frac{1}{2}\ \forall\ x \in [0, 2]$, then

(A) $|f(x)| \le 2$ (B) $f(x) \le 1$ (C) f(x) = 2x (D) f(x) = 3 for at least one x in [0, 2]

PART - II : MISCELLANEOUS QUESTIONS

SUBJECTIVE QUESTION

A-1. The length x of rectangle is decreasing at a rate of 3 cm/min and width y is increasing at a rate of 2 cm/min. When x = 10 cm and y = 6 cm, find the rate of change of (i) the perimeter, (ii) the area of rectangle.

A-2. x and y are the sides of two squares such that . Find the rate of change of the area of the second square with respect to the first square.

A-3. A man 1.5m tall walks away from a lamp post 4.5 m high at a rate of 4 km/hr.

(i) How fast is his shadow lengthening?

(ii) How fast is the farther end of shadow moving on the pavement?

A-4. If the radius of a sphere is measured as 8cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

Section (B) : Tangent, Normal, Angle between curves, Orthogonality of Curves, Shortest distance, Length of Tangent & Normal

B-1. Find the equation of normal to the curve $x^3 + y^3 = 8xy$ at point where it is met by the curve $y^2 = 4x$, other than origin.

B-2. If the tangent to the curve $xy + ax + by = 0$ at (1, 1) is inclined at an angle $\tan^{-1} 2$ with x-axis, then find a and b?

B-3. Prove that the length of segment of all tangents to curve $x^{2/3} + y^{2/3} = a^{2/3}$ intercepted between coordinate axes is same.

B-4. Find equation of tangents drawn to the curve $y^2 - 2x^2 - 4y + 8 = 0$ from the point (1, 2).

B-5. If the tangent at (1, 1) on $y^2 = x(2 - x)^2$ meets the curve again at P, then find coordinates of P

B-6. Find angle of intersection of the curves $y = 2\sin^2 x$ and $y = \cos 2x$.

B-7. Let f(x) and g(x) be two functions which cut each other orthogonally at their common point of intersection (x_1). Both f(x) and g(x) are equal to 0 at $x = x_1$. Also $|f'(x_1)| = |g'(x_1)|$, then find $\lim_{x \to x_1} [f(x) \cdot g(x)]$, where [.] denotes greatest integer functions.

B-8. Find the point on hyperbola $3x^2 - 4y^2 = 72$ which is nearest to the straight line $3x + 2y + 1 = 0$

B-9. Find the shortest distance between the curves $f(x) = -6x^6 - 3x^4 - 4x^2 - 6$ and $g(x) = e^x + e^{-x} + 2$

B-10. For parabola $y^2 = 4ax$, prove that the ratio of subtangent to abscissa is constant. Also find the ratio.

Section (C) Monotonicity an on interval, Monotonicity about a point

C-1. Show that $f(x) = \frac{x}{\sqrt{1+x}} - \ell n(1+x)$ is an increasing function for $x > -1$.

C-2. Find the intervals of monotonicity for the following functions.

(i) $\frac{x^4}{4} + \frac{x^3}{3} - 3x^2 + 5$ (ii) $\log_3^2 x + \log_3 x$

C-3. Check monotonocity at following points for

(i) $f(x) = x^3 - 3x + 1$ at $x = -1, 2$ (ii) $f(x) = |x-1| + 2|x-3| - |x+2|$ at $x = -2, 0, 3, 5$

C-4. Find the values of 'a' for which the function $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ decreases for all real values of x.

C-5. Let $f(x) = \begin{cases} x^2 & x \ge 0 \\ ax & x < 0 \end{cases}$. Find real values of 'a' such that f(x) is strictly monotonically increasing at x = 0.

C-6. If g(x) is monotonically increasing and f(x) is monotonically decreasing for $x \in R$ and if (gof) (x) is defined for $x \in R$, then prove that (gof) (x) will be monotonically decreasing function. Hence prove that $(gof)(x+1) \le (gof)(x-1)$.

C-7. Prove the inequality, $\frac{\tan x_2}{\tan x_1} > \frac{x_2}{x_1}$ for $0 < x_1 < x_2 < \frac{\pi}{2}$.

C-8. For $x \in \left(0, \frac{\pi}{2}\right)$ identify which is greater $(2\sin x + \tan x)$ or $(3x)$. Hence find $\lim_{x \to 0^+} \left[\frac{3x}{2\sin x + \tan x}\right]$ where [.] denote the GIF.

C-9. Let f and g be differentiable on R and suppose $f(0) = g(0)$ and $f'(x) \le g'(x)$ for all $x \ge 0$. Then show that $f(x) \le g(x)$ for all $x \ge 0$.

Section (D) Local maxima, Local minima, Global maxima, Global minima, Application of Maxima and Minima

D-1. Find the points of local maxima/minima of following functions

(i) $f(x) = 2x^3 - 21x^2 + 36x - 20$ (ii) $f(x) = -(x-1)^3 (x+1)^2$ (iii) $f(x) = x \log x$

D-2. Find the absolute maximum/minimum value of following functions

(i)	$f(x) = x^3$	;	$x \in [-2, 2]$
(ii)	$f(x) = \sin x + \cos x$	;	$x \in [0, \pi]$
(iii)	$f(x) = 4x - \frac{x^2}{2}$	;	$x \in \left[-2, \frac{9}{2}\right]$
(iv)	$f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$	;	$x \in [0, 3]$
(v)	$f(x) = \sin x + \frac{1}{2}\cos 2x$	;	$x \in \left[0, \frac{\pi}{2}\right]$

D-3. Draw graph of $f(x) = x|x-2|$ and, hence find points of local maxima/minima.

D-4. Let $f(x) = x^2$; $x \in [-1, 2)$. The show that f(x) has exactly one point of local maxima but global maximum is not defined.

D-5. Let $f(x) = \begin{cases} 3 - x & 0 \le x < 1 \\ x^2 + \ell nb & x \ge 1 \end{cases}$. Find the set of values of b such that f(x) has a local minima at x = 1.

D-6. John has 'x' children by his first wife and Anglina has 'x + 1' children by her first husband. They both marry and have their own children. The whole family has 24 children. It is given that the children of the same parents don't fight. Then find then maximum number of fights that can take place in the family.

D-7. If the sum of the lengths of the hypotenuse and another side of a right angled triangle is given, show that the area of the triangle is a maximum when the angle between these sides is $\pi/3$

D-8. Find the volume of the largest cylinder that can be inscribed in a sphere of radius ‘ r' cm.

D-9. Show that the semi vertical angle of a right circular cone of maximum volume. of a given slant height is $\tan^{-1}\sqrt{2}$

D-10. A running track of 440m. is to be laid out enclosing a football field, the shape of which is a rectangle with semi circle at each end. If the area of the rectangular portion is to be maximum, find the length of its sides.

D-11. Find the area of the largest rectangle with lower base on the x-axis and upper vertices on the curve $y = 12 - x^2$.

D-12. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible when revolved around one of its side.

Section (E) Curvature, Points of inflection

E-1. Find number of point of inflection for the following functions

(i) $f(x) = (x-1)^3(x-2)^2$ (ii) $f(x) = x + \sin x$ in $(0, 2\pi)$

E-2. Show that the locus of point of inflection of the curve $y = x \sin x$ is $y^2(4 + x^2) = 4x^2$

Section (F): Rolle's Theorem, LMVT

F-1. Verify Rolle's theorem for the function, $f(x) = \log_e\left(\frac{x^2+ab}{x(a+b)}\right) + p$ for [a, b] where $0 < a < b$.

F-2. Using Rolle's theorem prove that the equation $3x^2 + px - 1 = 0$ has at least on real root in the interval (–1, 1).

F-3. If a, b are two real numbers with $a < b$ show that a real number ‘c' can be found between a and b such that $3c^2 = b^2 + ab + a^2$

F-4. Let f(x) be differentiable function and g(x) be twice differentiable function. Zeros of f(x) , g'(x) be a, b respectively (a < b). Show that there exists at least one root of equation $f'(x)\,g'(x) + f(x)\,g''(x) = 0$ on (a ,b).

COMPREHENSIONS :

Read the following passage carefully and answer the questions.

Comprehension # 1

Let a(t) be a function of t such that $\frac{da}{dt} = 2$ for all values of t and a = 0 when t= 0. Further $y = m(t)\,x + c(t)$ is tangent to the curve $y = (x^2 - 2ax + a^2 + a)$ at the point whose abscissa is 0. Then

1. If the rate of change of distance of vertex of $y = x^2 - 2ax + a^2 + a$ from the origin with respect to is k, then k =

(A) 2 (B) $2\sqrt{2}$ (C) $\sqrt{2}$ (D) $4\sqrt{2}$

2. If the rate of change of c(t) with respect to t, when t= k, is ℓ, then

(A) $16\sqrt{2} - 2$ (B) $8\sqrt{2} + 2$ (C) $10\sqrt{2} + 2$ (D) $16\sqrt{2} + 2$

3. The rate of change of m(t), with respect to t, at t = ℓ is

(A) –2 (B) 2 (C) – 4 (D) 4

Comprehension # 2

Consider a function f defined by $f(x) = \sin^{-1}\sin\left(\frac{x+\sin x}{2}\right), \forall x \in [0, \pi]$, which satisfies $f(x) + f(2\pi - x) = \pi$, $\forall x \in [\pi, 2\pi]$ and $f(x) = f(4\pi - x)$ for all $x \in [2\pi, 4\pi]$, then

4. If α is the length of the largest interval on which f(x) is increasing, then α =

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π

5. If f(x) is symmetric about $x = \beta$, then β =

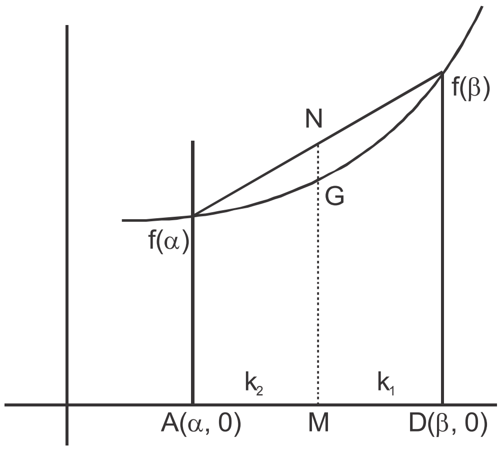
(A) $\frac{\alpha}{2}$ (B) α (C) $\frac{\alpha}{4}$ (D) 2α

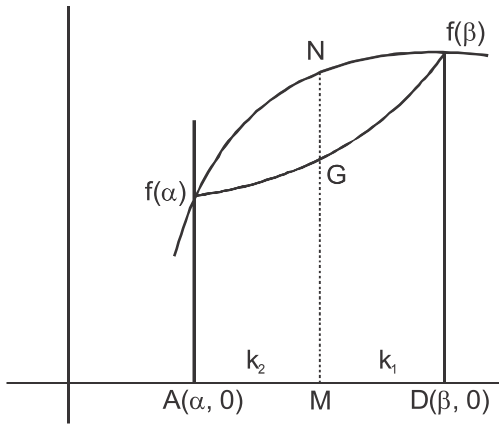
6. Maximum value of f(x) on $[0, 4\pi]$ is:

(A) $\frac{\beta}{2}$ (B) β (C) $\frac{\beta}{4}$ (D) 2β

Comprehension # 3

For a double differentiable function f(x) if $f''(x) \geq 0$ then f(x) is concave upward and if $f''(x) \leq 0$ then f(x) is concave downward





Here $M\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}, 0\right)$

If f(x) is a concave downward in [a, b] and $\alpha, \beta \in [a, b]$ then $\frac{k_1 f(\alpha) + k_2 f(\beta)}{k_1 + k_2} \leq f\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}\right)$; where $k_1, k_2 \in R^+$ then answer the following

7. Which of the following is true

(A) $\frac{\sin\alpha + \sin\beta}{2} > \sin\left(\frac{\alpha+\beta}{2}\right); \alpha, \beta\ (0, \pi)$

(B) $\frac{\sin\alpha + \sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right); \alpha, \beta \in (\pi, 2\pi)$

(C) $\frac{\sin\alpha + \sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right)$; $\alpha, \beta \in (0, \pi)$

(D) none of these

8. Which of the following is true

(A) $\dfrac{2^{\alpha}+2^{\beta+1}}{3} \le 2^{\frac{\alpha+2\beta}{3}}$ (B) $\dfrac{2\ell n\alpha+\ell n\beta}{3} \ge \ell n\left(\dfrac{2\alpha+\beta}{3}\right)$

(C) $\dfrac{\tan^{-1}\alpha+\tan^{-1}\beta}{2} \le \tan^{-1}\left(\dfrac{\alpha+\beta}{2}\right)$ a, b $\in R^{-}$ (D) $\dfrac{e^{\alpha}+2e^{\beta}}{3} \ge e^{\frac{\alpha+2\beta}{3}}$

9. Let α, β and γ are three distinct real numbers and f" (x) < 0. Also f(x) is increasing function and let $A = \dfrac{r^{-1}(\alpha)+f^{-1}(\beta)+f^{-1}(\gamma)}{3}$ and $B = f^{-1}\left(\dfrac{\alpha+\beta+\gamma}{3}\right)$, then order relation between A and B is – (given $f^{-1}(x)$)

(A) A > B (B) A < B (C) A = B (D) none of these

ASSERTION/REASONING

10. **Statement 1:-** The curves $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ and $\dfrac{x^2}{1+a^2}-\dfrac{y^2}{1-b^2}=1$ are orthogonal, for b $\in$ (–1, 1)

Statement 2:- $ax^2 + by^2 = 1$ and $Ax^2 + By^2 = 1$ are orthogonal iff ab (A – B) = AB (a – b).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True
(E) Statement-1 is False, Statement-2 is False

11. **Statement 1:-** If f(x) is increasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.
Statement 2:- If f(x) is decreasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True
(E) Statement-1 is False, Statement-2 is False

12. **Statement 1:-** e^{π} is bigger than π^{e}.
Statement 2:- $f(x) = x^{1/x}$ is a increasing function when x $\in$ [e, ∞)

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True
(E) Statement-1 is False, Statement-2 is False

13. **Statement 1:-** ABC is given triangle having respective sides a, b, c. D, E, F are points of the sides BC, CA, AB respectively so that AFDE is a parallelogram. The maximum area of the parallelogram is $\dfrac{1}{4}$ bcsin A.

Statement 2:- Maximum value of $2kx - x^2$ is at x = k.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True
(E) Statement-1 is False, Statement-2 is False

14. Let $f(x) = x^{50} - x^{20}$
Statement 1:- Global maximum of f(x) in [0, 1] is 0.
Statement 2:- x = 0 is a stationary point of f(x).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True
(E) Statement-1 is False, Statement-2 is False

TRUE/FALSE

15. Curve $y^2 = 4ax$ and $y = e^{-\frac{x}{2a}}$ are orthogonal curves.

16. The function $y = \frac{2x^2 - 1}{x^4}$ is neither increasing nor decreasing.

17. The function $x^{100} + \sin x - 1$ is strictly increasing in [0, 1]

18. If f(x) is strictly increasing real function defined on R and C is a real constant, then number of solutions of f(x) = c is always equal to one.

19. Let f(x) = x ; $x \in (0, 1)$. f(x) does not has any point of local maxima/minima

20. f(x) = {x} has maximum at x = 6 (here {.} denotes fractional part function).

21. A cylinder of a given volume which is open at the top; has minimum total surface area when its height is equal to the radius of its base.

FILL IN THE BLANKS

22. The normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at the point P (0, – 3) is tangent to the curve at the point(s) ....

23. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at point (2, –1) is ....

24. A kite is 300 m high and there are 500 m of cord out. If the wind moves the kite horizontally at the rate of 5 km/hr. directly away from the person who is flying it, the rate at which the cord is being paid is ......

25. Let f be the function $f(x) = \cos x - \left(1 - \frac{x^2}{2}\right)$ then f(x) is increasing in the interval ....

26. If $f(x) = \frac{\left(\sin^{-1} x + \tan^{-1} x\right)}{\pi} + 2\sqrt{x}$ then the range of f(x) is ....

27. The minimum and maximum values of y in $4x^2 + 12xy + 10y^2 - 4y + 3 = 0$ are respectively .....and ......

Match the Column :

28. **Column-I** **Column-II**

	Column-I		Column-II
(A)	If θ is angle between the curves y = [|sin x| + |cos x|] ([.] denote GIF) and $x^2 + y^2 = 5$ then $\text{cosec}^2\theta$ is	(p)	$\frac{5}{4}$
(B)	Length of subnormal to $x = \sqrt{2}\cos t$, $y = -3\sin t$ at $t = \frac{-\pi}{4}$ is	(q)	2
(C)	If [a, b], (b < 1) is largest interval in which $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 19$ is strictly increasing then $\frac{a}{b}$ is	(r)	$\frac{8}{3}$
(D)	If a + b = 8 , a , b > 0 then minimum value of $\frac{a^3 + b^3}{48}$ is	(s)	$\frac{9}{2}$

29. **Column-I** **Column-II**

	Column-I		Column-II
(A)	$f(x) = \frac{\sin x}{e^x}, x \in [0, \pi]$	(p)	Conditions is Rolle's theorem are satisfied.
(B)	$f(x) = \text{sgn}\left((e^x - 1)\ \ell nx\right), x \in \left[\frac{1}{2}, \frac{3}{2}\right]$	(q)	Conditions in LMVT are satisfied
(C)	$f(x) = (x-1)^{2/5}, x \in [0, 3]$	(r)	At least one condition in Rolle's theorem is not satisfied.
(D)	$f(x) = \begin{cases} x\left(\frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}\right) & , x \in [-1, 1] - \{0\} \\ 0 & , x = 0 \end{cases}$	(s)	At least one condition in LMVT is not satisfied

30. **Column-I** **Column-II**

	Column-I		Column-II
(A)	A rectangle is inscribed in an equilateral triangle of side 4cm. Square of maximum area of such a rectangle is	(p)	65
(B)	The volume of a rectangular closed box is 72 and the base sides are in the ratio 1:2. The least total surface area is	(q)	36
(C)	Maximum value of $\left(\sqrt{-3 + 4x - x^2} + 4\right)^2 + (x - 5)^2$ (where $1 \le x \le 3$) is	(r)	12
(D)	The sides of a rectangle of greatest perimeter which is inscribed in a semicircle of radius $\sqrt{5}$ are a and b. Then $a^3 + b^3 =$	(s)	108

EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

1. If tangents are drawn from the origin to the curve y = sin x, then their points of contact lie on the curve

(A) $x - y = xy$ (B) $x + y = xy$ (C) $x^2 - y^2 = x^2y^2$ (D) $x^2 + y^2 = x^2y^2$

2. Let f(x) = $\begin{cases} -x^2 & ,x < 0 \\ x^2 + 8 & ,x \geq 0 \end{cases}$ Equation of tangent line touching both branches of y = f(x) is

(A) y = 4x + 1 (B) y= 4x + 4 (C) y = x + 4 (D) y = x + 1

3. If g (x) is a curve which is obtained by the reflection of f(x) = $\frac{e^x - e^{-x}}{2}$ by the line y = x then

(A) g(x) has more than one tangent parallel to x-axis

(B) g(x) has more than one tangent parallel to y -axis

(C) y = – x is a tangent to g(x) at (0, 0)

(D) g (x) has no extremum

4. Equation of normal drawn to the graph of the function defined as f(x) = $\frac{\sin x^2}{x}, x \neq 0$ and f(0) = 0 at the origin is

(A) x + y = 0 (B) x – y = (C) y = 0 (D) x = 0

5. The line $\frac{x}{a} + \frac{y}{b}$ = 1 touches the curve y = $be^{-x/a}$ at the point

(A) (– a, 2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(a, \frac{b}{e}\right)$ (D) (0, b)

6. All points on curve $y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ at which tangents are parallel to the axis of x, lie on a

(A) circle (B) parabola (C) line (D) none of these

7. The ordinate of y = (a/ 2) $(e^{x/a} + e^{-x/a})$ is the geometric mean of the length of the normal and the quantity.

(A) a/2 (B) a (C) e (D) none of these

8. Let f(x) = $x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function in the set of real numbers R. Then a & b satisfy the condition.

(A) $a^2 - 3b - 15 > 0$ (B) $a^2 - 3b + 15 \leq 0$ (C) $a^2 + 3b - 15 < 0$ (D) a > 0 & b > 0

9. If f(x) = $a^{\{a^{|x|}\,\mathrm{sgn}\,x\}}$; g(x) = $a^{[a^{|x|}\,\mathrm{sgn}\,x]}$ for a > 1, a ≠ 1 and x ∈ R, where {} & [ ] denote the fractional part and integral part functions respectively, then which of the following statements holds goods for the function h(x), where (ln a) h(x) = (ln f(x) + ln g(x)).

(A) ‘h' is even and increasing (B) ‘h' is odd and decreasing

(C) ‘h' is even and decreasing (D) ‘h' is odd and increasing

10. If f(x) = (x – 4) (x – 5) (x – 6) (x – 7) then,

(A) f'(x) = 0 has four roots.

(B) three roots of f'(x) = 0 lie in (4, 5) ∪ (5, 6) ∪ (6, 7).

(C) the equation f'(x) = 0 has only one real root

(D) three roots of f'(x) = 0 lie in (3, 4) ∪ (4, 5) ∪ (5, 6)

11. If f: [1, 10] → [1, 10] is a non-decreasing function and g: [1, 10] → [1, 10] is a non-increasing function. Let h(x) = f(g(x)) with h(1) = 1, then h(2)

(A) lies in (1, 2) (B) is more than 2 (C) is equal to 1 (D) is not defined

12. If $f(x) = \dfrac{x^2}{2-2\cos x}$; $g(x) = \dfrac{x^2}{6x-6\sin x}$ where 0 < x < 1, then

(A) both 'f' and 'g' are increasing functions (B) 'f' is decreasing & 'g' is increasing function

(C) 'f' is increasing & 'g' is deceasing function (D) both 'f' & 'g' are decreasing function

13. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5 & , x \le 1 \\ -2x + \log_2(b^2 - 2) & , x > 1 \end{cases}$ the set of values of b for which f(x) has greatest value at x = 1 is given by:

(A) $1 \le b \le 2$ (B) b = {1, 2} (C) $b \in (-\infty, -1)$ (D) $\left[-\sqrt{130}, -\sqrt{2}\right) \cup \left(\sqrt{2}, \sqrt{130}\right]$

14. The set of values of p for which the extremum of the function $f(x) = x^3 - 3px^2 + 3(p^2-1)x + 1$ lie in the interval (–2 , 4), is:

(A) (–3, 5) (B) (– 3, 3) (C) (–1, 3) (D) (–1, 4)

15. Four points A, B, C, D lie in that order on the parabola $y = ax^2 + bx + c$. The coordinates of A, B & D are known as A(–2, 3); B (–1, 1) and D(2, 7). The coordinates of C for which the area of the quadrilateral ABCD is greatest, is

(A) (1/2, 7/4) (B) (1/2, – 7/4) (C) (–1/2, 7/4) (D) none

16. In a regular triangular prism the distance from the centre of one base to one of the vertices of the other base is ℓ. The altitude of the prism for which the volume is greatest, is :

(A) $\dfrac{\ell}{2}$ (B) $\dfrac{\ell}{\sqrt{3}}$ (C) $\dfrac{\ell}{3}$ (D) $\dfrac{\ell}{4}$

17. The lower corner of a leaf in a book is folded over so as to just reach the inner edge of the page. The fraction of width folded over if the area of the folded part is minimum is:

(A) 5/8 (B) 2/3 (C) 3/4 (D) 4/5

18. If x_1 and x_2 are abscissa of two points on the curve $f(x) = x - x^2$ in the interval [0, 1], then maximum value of the expression $(x_1 + x_2) - (x_1^2 + x_2^2)$ is

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{4}$ (C) 1 (D) 2

19. The maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b is

(A) 2 (ab) (B) $\dfrac{1}{2}(a+b)^2$ (C) $\dfrac{1}{2}(a^2+b^2)$ (D) none of these

20. Least value of the function, $f(x) = 2^{x^2} - 1 + \frac{2}{2^{x^2}+1}$ is:

(A) 0 (B) 3/2 (C) 2/3 (D) 1

More than one choice type

21. If P is a point on the curve $5x^2 + 3xy + y^2 = 2$ and O is the origin, then OP has

(A) minimum value $\frac{1}{2}$

(B) minimum value $\frac{2}{\sqrt{11}}$

(C) maximum value $\sqrt{11}$

(D) maximum value 2

22. For the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$, at point (2, – 1)

(A) length of subtangent is 7/6

(B) slope of tangent = 6/7

(C) length of tangent = $\sqrt{(85)}/6$

(D) none of these

23. Let $f(x) = x^{m/n}$ for $x \in R$ where m and n are integers, m even and n odd and 0 < m < n. Then

(A) f(x) decreases on $(-\infty, 0]$

(B) f(x) increases on $[0, \infty)$

(C) f(x) increases on $(-\infty, 0]$

(D) f(x) decreases on $[0, \infty)$

24. Let f and g be two differentiable functions defined on an interval I such that $f(x) \geq 0$ and $g(x) \leq 0$ for all $x \in I$ and f is strictly decreasing on I while g is strictly increasing on I then

(A) the product function fg is strictly increasing on I

(B) the product function fg is strictly decreasing on I

(C) fog (x) is monotonically increasing on I

(D) fog (x) is monotonically decreasing on I

25. Let $\phi(x) = (f(x))^3 - 3(f(x))^2 + 4f(x) + 5x + 3\sin x + 4\cos x \ \forall \ x \in R$, then

(A) ϕ is increasing whenever f is increasing

(B) ϕ is increasing whenever f is decreasing

(C) ϕ is decreasing whenever f is decreasing

(D) ϕ is decreasing if $f'(x) = -11$

26. For the function $f(x) = x^4(12\ell nx - 7)$

(A) the point (1, – 7) is the point of inflection

(B) $x = e^{1/3}$ is the point of minima

(C) the graph is concave downwards in (0, 1)

(D) the graph is concave upwards in $(1, \infty)$

27. The curve $y = \frac{x+1}{x^2+1}$ has

(A) x = 1 , as point of inflection

(B) $x = -2 + \sqrt{3}$, as point of inflection

(C) x = –1 , as point of minimum

(D) $x = -2 - \sqrt{3}$, as point of inflection

28. Let $f(x) = 40/(3x^4 + 8x^3 - 18x^2 + 60)$. Which of the following statements(s) about f(x) is (are) correct?

(A) f(x) has local minima at x = 0

(B) f(x) has local maxima at x = 0.

(C) absolute maximum value of f(x) is not defined

(D) f(x) is local maxima at x = – 3, x = 1

PART - II : SUBJECTIVE QUESTIONS

1. A light shines from the top of a pole 50 ft. high. A ball is dropped from the same height from a point 30 ft. away from the light. How fast is the shadow of the ball moving along the ground 1/2 sec. later? [Assume the ball falls a distance s = 16 t^2 ft. in 't' sec.]

2. A variable ΔABC in the xy plane has its orthocenter at vertex 'B', a fixed vertex 'A' at the origin and the third vertex 'C' restricted to lie on the parabola y = $1+\frac{7x^2}{36}$. The point B starts at the point (0, 1) at time t = 0 and moves upward along the y axis at a constant velocity of 2cm/sec. How fast is the area of the triangle increasing when t = $\frac{7}{2}$ sec.

3. Find equation of line which is tangent at a point of curve $4x^3 = 27y^2$ and normal at other point.

4. The tangent to curve $y = x - x^3$ at point P meets the curve again at Q. Prove that one point of trisection of PQ lies on y-axis. Find locus of other point of trisection.

5. Find the equation of the common tangent to the parabolas $y = x^2 + 4x + 8$ and $y = x^2 + 8x + 4$, also find the coordinates of point of contact.

6. In the curve $x^a y^b = K^{a+b}$ prove that the portion of the tangent intercepted between the coordinate axes is divided at its point of contact into segments which are in a constant ratio. (All the constants being positive)

7. If f: $[0, \infty) \to R$ is the function defined by f(x) = $\frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$, then whether f(x) is injective or not.

8. If $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$ monotonically increases for $\forall$ $x \in R$, then find range of values of a

9. Find the set of values of p for which the equation $|\ell nx| - px = 0$ possess three distinct roots.

10. Find the set of all values of the parameter 'a' for which the function
$f(x) = \sin 2x - 8(a + 1)\sin x + (4a^2 + 8a - 14)x$ increases for all $x \in R$ and has no critical points for all $x \in R$.

11. If $ax^2 + (b/x) \geq c$ for all positive x where a > 0 and b > 0 then show that $27ab^2 \geq 4c^3$.

12. Prove that $e^x + \sqrt{1+e^{2x}} \geq (1+x) + \sqrt{2+2x+x^2}$ $\forall$ $x \in R$

13. Find which of the two is large $\ell n(1+x)$ or $\frac{\tan^{-1}x}{1+x}$

14. Find the values of 'a' for which the function f(x) = $\frac{a}{3}x^3 + (a + 2)x^2 + (a - 1)x + 2$ possess a negative point of minimum.

15. Find the polynomial f(x) of degree 6, which satisfies $\lim_{x\to 0}\left(1+\frac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at x = 1 and local minimum at x = 0 and x = 2.

16. The three sides of a trapezium are equal each being 6 cms long, find the area of the trapezium when it is maximum.

17. A sheet of poster has its area 18 m^2. The margin at the top & bottom are 75 cms. and at the sides 50 cms. What are the dimensions of the poster if the area of the printed space is maximum ?

18. From a fixed point A on the circumference of a circle of radius 'a', let the perpendicular AY fall on the tangent at a point P on the circle, prove that the greatest area which the ΔAPY can have is $3\sqrt{3}\frac{a^2}{8}$ sq. units.

19. Find the set of values (s) of 'a' for which the function f(x) = $\frac{ax^3}{3}$ + (a+ 2)x^2 + (a – 1) x + 2 possess a negative point of inflection.

20. Let f' (sin x) < 0 and f'' (sin x) > 0, $\forall$ x $\in \left(0, \frac{\pi}{2}\right)$ and g(x) = f(sin x) + f(cos x), then find the intervals of monotonicity of g(x).

21. Using Rolle's theorem show that the derivative of the function f(x) = $\begin{cases} x\sin\frac{\pi}{x} & \text{for } x>0 \\ 0 & \text{for } x=0 \end{cases}$ vanishes at an infinite set of points of the interval (0, 1).

22. A function f is differentiable in the interval $0 \le x \le 5$ such that f(0) = 4 & f(5) = – 1. If g (x) - $\frac{f(x)}{x+1}$, then prove that there exists some c $\in$ (0, 5) such that g'(c) = – $\frac{5}{6}$.

23. Let f(x) and g(x) be differentiable functions having no common zeros so that f(x) g'(x) $\ne$ f' (x) g(x). Prove that between any two zeros of f(x), there exist atleast one zero of g(x).

24. f is continuous in [a, b] and differentiable in (a, b) (where a > 0) such that $\frac{f(a)}{a} = \frac{f(b)}{b}$. Prove that there exist $x_0 \in$ (a, b) such that f' (x_0) = $\frac{f(x_0)}{x_0}$.

25. If ϕ (x) is a differentiable function $\forall$ x $\in$ R and a $\in R^+$ such that $\phi(0) = \phi(2a)$, $\phi(a) = \phi(3a)$ and $\phi(0) \ne \phi(a)$ then show that there is at least one root of equation $\phi'(x + a) = \phi'(x)$ in (0, 2a)

26. Let $-1 \le p \le 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval $\left[\frac{1}{2}, 1\right]$ and identify it.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

1. In [0, 1] Lagranges Mean Value theorem is NOT applicable to **[IIT-JEE-2003, Scr. (3, –1)/84]**

(A) $f(x) = \begin{cases} \frac{1}{2} - x & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2 & x \geq \frac{1}{2} \end{cases}$ (B) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(C) $f(x) = x\,|x|$ (D) $f(x) = |x|$

2. Using the relation $2(1 - \cos x) < x^2$, $x \neq 0$ or otherwise, prove that $\sin(\tan x) \geq x$, $\forall\ x \in \left[0, \frac{\pi}{4}\right]$

[IIT-JEE-2003, Main (4, 0)/60]

3. Let $f:[0, 4] \to R$ is a differentiable function **[IIT-JEE-2003, Main (4, 0)/60]**
For some $a, b \in (0, 4)$, show that $f^2(4) - f^2(0) = 8f(a)\,f'(b)$

4. For the circle $x^2 + y^2 = r^2$, find the value of 'r' for which the area enclosed by the tangents drawn from they point P (6, 8) to the circle and the chord of contact is maximum. **[IIT-JEE-2003, Main (2, 0)/60]**

5. If f is differentiable and strictly increasing in a neighborhood of '0', then

$\lim\limits_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)} =$ **[IIT-JEE-2004, Scr. (3, –1)/84]**

(A) 0 (B) 1 (C) –1 (D) 2

6. If $f(x) = x^{\alpha}\,\ell nx$ and $f(0) = 0$, If Rolle's theorem can be applied to f in [0, 1], then value of α can be

[IIT-JEE-2004, Scr. (3, –1)/84]

(A) –2 (B) –1 (C) 0 (D) 1/2

7. $P(x) = 51\,x^{101} - 2323x^{100} - 45x + 1035$
Using Rolle's theorem, prove that P(x) = 0 has at least one root in $(45^{1/100}, 46)$ **[IIT-JEE-2004, Mains (2, 0)/60]**

8. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ **[IIT-JEE-2004, Scr. (3, –1)/84]**
(A) f(x) is a strictly increasing function (B) f(x) has a local maxima
(C) f(x) is a strictly decreasing function (D) f(x) is bounded

9. Prove that $\sin x + 2x \geq \frac{3x(x+1)}{\pi}, x \in \left[0, \frac{\pi}{2}\right]$. Justify the inequalities used in the relation.

[IIT-JEE-2004, Mains (4, 0)/60]

10. If $|f(x_1) - f(x_2)| \le (x_1 - x_2)^2$, for all $x_1, x_2 \in R$. Find the equation of tangent to the curve y = f(x) at the point (1, 2)

[IIT-JEE-2005, Mains (2, 0)/60]

11. If f(x) be a twice differentiable function such that $f(x) = x^2$ for x = 1, 2, 3, then **[IIT-JEE-2005, Scr. (3, –1)/84]**

(A) $f''(x) = 2 \ \forall \ x \in [1, 3]$ (B) $f''(x) = 2$ for some $x \in (1, 3)$

(C) $f''(x) = 3 \ \forall \ x \in (2, 3)$ (D) $f''(x) = f'(x)$ for $x \in (2, 3)$

12. If P(x) be a polynomial of degree 3 satisfying P(–1) = 10, P(1) = –6 and P(x) has maxima at x = –1 and P'(x) has minima at x = 1. Find the distance between the local maxima and local minima of the curve.

[IIT-JEE-2005, Mains (4, 0)/60]

13. If f(x) is a twice differentiable function such that f(a) = 0, f(b) = 2, f(c) = –1, f(d) = 2, f(e) = 0, where a < b < c < d < e, then the minimum number of zeroes of $g(x) = (f'(x))^2 + f''(x)\, f(x)$ in the interval [a, e] is

[IIT-JEE-2006, 6/184]

14*. f(x) is cubic polynomial which has local maximum at x = –1, if f(2) = 18, f(1) = –1 and f'(x) has local minima at x = 0, then **[IIT-JEE-2006, (5, –1)/184]**

(A) the distance between (–1, 2) and (a, f(a)), where x = a is the point of local minima is $2\sqrt{5}$.

(B) f(x) is increasing for $x \in [1, 2\sqrt{5})$

(C) f(x) has local minima at x = 1

(D) the value of f(0) = 5

15. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$ **[IIT-JEE-2007, Paper-1 (3, –1)/81]**

(A) on the left of x = c (B) on the right of x = c (C) at no point (D) at all points

Comprehension #1

If a continuous function f defined on the real line R, assumes positive and negative values in R then the equation f(x) = 0 has a root in R. For example, if it is known that a continuous function f on R is positive at consider $f(x) = ke^x - x$ for all real x where k is a real constant.

16. The line y = x meets $y = ke^x$ for $k \le 0$ at **[IIT-JEE-2007, Paper-2 (4, –1)/81]**

(A) no point (B) one point (C) two points (D) more than two points

17. The positive value of k for which $ke^x - x = 0$ has only one root is **[IIT-JEE-2007, Paper-2 (4, –1)/81]**

(A) $\frac{1}{e}$ (B) 1 (C) e (D) $\log_e 2$

18. For k > 0, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is

[IIT-JEE-2007, Paper-2 (4, –1)/81]

(A) $\left(0, \frac{1}{e}\right)$ (B) $\left(\frac{1}{e}, 1\right)$ (C) $\left(\frac{1}{e}, \infty\right)$ (D) (0, 1)

19. Let f(x) = 2 + cos x for all real x. **[IIT-JEE-2007, Paper-2 (3, –1)/81]**

Statement-1 : For each real t, there exists a point c in $[t, t + \pi]$ such that f'(c) = 0

Statement-2: $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

20. Let the function g: $(-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then. g is

[IIT-JEE-2008, Paper-2 (3, –1)/163]

(A) even and is strictly increasing in $(0, \infty)$ (B) odd and is strictly decreasing in $(-\infty, \infty)$
(C) odd and is strictly increasing in $(-\infty, \infty)$ (D) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

21. The total number of local maxima and local minima of the function f(x) = $\begin{cases}(2+x)^3 & ,-3 < x \le -1 \\ x^{2/3} & ,-1 < x < 2\end{cases}$ is

[IIT-JEE-2008, Paper-1 (3, –1)/82]

(A) 0 (B) 1 (C) 2 (D) 3

22. For the function $f(x) = x\cos\frac{1}{x}, x \ge 1$, **[IIT-JEE-2009, Paper-2 (4, –1)/80]**

(A) for at least one x in the interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

(B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x + 2)-f(x)>2$

(D) f' (x) is strictly decreasing in the interval $[1, \infty)$

23. Let p(x) be a polynomial of degree 4 having extremum at x = 1,2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right) = 2$. Then the value of p(2) is

[IIT-JEE-2009, Paper-2 (4, –1)/80]

24. Let f be a function defined on R (the set of all real numbers) such that
$f'(x) = 2010\,(x-2009)\,(x-2010)^2\,(x-2011)^3\,(x-2012)^4$, for all $x \in R$.

If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ell n\,(g(x))$, for all $x \in R$, then the number of points in R at which g has a local maximum is **[IIT-JEE-2010, Paper-2 (3, 0)/79]**

25. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

[IIT-JEE-2010, Paper-1 (3, –1)/84]

(A) $a = b$ and $c \ne b$ (B) $a = c$ and $a \ne b$ (C) $a \ne b$ and $c \ne d$ (D) $a = b = c$

26. Match the statements given in Column-I with the intervals/union of intervals given in Column-II

[IIT-JEE-2011, Paper-2 (8, 0)/80]

Column-I **Column-II**

(A) The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right); z \text{ is a complex number}, |z| = 1, z \ne \pm 1\right\}$ is (p) $(-\infty, -1) \cup (1, \infty)$

(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is (q) $(-\infty, 0) \cup (0, \infty)$

(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$ (r) $[2, \infty)$

then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is

(D) If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then f(x) is increasing in (s) $(-\infty, -1] \cup [1, \infty)$

(t) $(-\infty, 0] \cup [2, \infty)$

27. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **[IIT-JEE-2011, Paper-2 (4, 0)/80]**

28. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 p(3) = 2, then p'(0) is **[IIT-JEE-2012, Paper-1 (4, 0)/70]**

29. Let f: IR $\rightarrow$ IR be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is **[IIT-JEE-2012, Paper-1 (4, 0)/70]**

30. The number of points in $(-\infty, \infty)$, for which $x^2 - x \sin x - \cos x = 0$, is **[JEE Advanced 2013]**

(A) 6 (B) 4 (C) 2 (D) 0

31. Let $f(x) = x \sin \pi x$, $x > 0$. Then for all natural numbers n, f' (x) vanishes at **[JEE Advanced 2013]**

(A) a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

(B) a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(C) a unique point in the interval (n , n + 1)

(D) two points in the interval (n , n + 1)

32. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are **[JEE Advanced 2013]**

(A) 24 (B) 32 (C) 45 (D) 60

33. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$

[JEE Advanced 2013]

34. The function $f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$ has a local minimum or a local maximum at x =

(A) –2 (B) $\frac{-2}{3}$ (C) 2 (D) $\frac{2}{3}$

Paragraph for Questions 35 and 36

Let f: [0, 1] $\rightarrow$ R (the set of all real numbers) be a function. Suppose the function f is twice differentiable, f(0) = f(1) = 0 and satisfies $f''(x) - 2f'(x) + f(x) \ge e^x$, $x \in [0, 1]$.

35. If the function $e^{-x}f(x)$ assumes its minimum in the interval [0, 1] at $x = \frac{1}{4}$, which of the following is true?

(A) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$ (B) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$

(C) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$ (D) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

36. Which of the following is true for 0 < x < 1 ?

(A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. The real number x when added to its reciprocal givens the minimum sum at x equals **[AIEEE-2003]**
(A) 2 (B) 1 (C) – 1 (D) – 2

2. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where a > 0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals **[AIEEE-2003]**
(A) 3 (B) 1 (C) 2 (D) 1/2

3. If 2a + 3b + 6c =0, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval **[AIEEE-2004]**
(A) (0 ,1) (B) (1, 2) (C) (2, 3) (D) (1, 3)

4. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa, is

(A) (2, 4) (B) (2, – 4) (C) $\left(-\frac{9}{8}, \frac{9}{2}\right)$ (D) $\left(\frac{9}{8}, \frac{9}{2}\right)$

5. The normal to the curve $x = a(1 + \cos\theta)$, $y = a\sin\theta$ at 'θ' always passes through the fixed point **[AIEEE-2004]**
(A) (a, 0) (B) (0, a) (C) (0, 0) (D) (a, a)

6. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is **[AIEEE-2004]**
(A) $\pi/2$ (B) $\pi/6$ (C) $\pi/4$ (D) $\pi/3$

7. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched? **[AIEEE-2005]**

Interval	**Function**
(A) $(-\infty, -4]$	$x^3 + 6x^2 + 6$
(B) $\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(C) $[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(D) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$

8. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is- **[AIEEE-2005]**

(A) $\frac{5}{6\pi}$ cm/min (B) $\frac{1}{54\pi}$ cm/min (C) $\frac{1}{50\pi}$ cm/min (D) $\frac{1}{36\pi}$ cm/min

9. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is **[AIEEE-2005]**

(A) 2ab (B) ab (C) $\sqrt{ab}$ (D) $\frac{a}{b}$

10. Let f(x) be differentiable for all x. If f(1) = – 2 and f' (x) $\geq$ 2 for x $\in$ (1, 6) then **[AIEEE-2005]**

(A) f(6) = 5 (B) f(6) < 5 (C) f(6) > 8 (D) f(6) < 8

11*. The normal to the curve x = a (cos θ + θ sin θ), y = a (sin θ – θ cos θ) at any point 'θ' is such that

(A) it is at a constant distance from the origin (B) it passes through $\left(\frac{a\pi}{2},-a\right)$

(C) it makes angle $\pi/2 + \theta$ with the x-axis (D) it passes through the origin

12. The function f(x) = $\frac{x}{2}+\frac{2}{x}$ has a local minimum at **[AIEEE-2006]**

(A) x = – 2 (B) x = 0 (C) x = 1 (D) x = 2

13. A value of c for which the conclusion of Mean value Theorem holds for the function f(x) = $\log_e x$ on the interval [1, 3] is **[AIEEE-2007]**

(A) 2 $\log_3 e$ (B) $\frac{1}{2}$ $\log_e 3$ (C) $\log_3 e$ (D) $\log_e 3$

14. The function f(x) = $\tan^{-1}$(sin x + cos x) is an increasing function in **[AIEEE-2007]**

(A) (π/4, π/2) (B) (– π/2, π/4) (C) (0, π/2) (D) (–π/2, π/2)

15. Suppose the cubic x^3 – px + q = 0 has three distinct real roots where p > 0 and q > 0. Then, which one of the following holds? **[AIEEE-2007]**

(A) Minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ (B) Minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(C) Minima a both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ (D) Maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

16. The shortest distance between the line y –x = 1 and the curve x = y^2 is **[AIEEE-2007]**

(A) $\frac{3\sqrt{2}}{8}$ (B) $\frac{2\sqrt{3}}{8}$ (C) $\frac{3\sqrt{2}}{5}$ (D) $\frac{\sqrt{3}}{4}$

17. Given P(x) = $x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of P' (x) = 0. If P (–1) < P(1), then in the interval [–1, 1] **[AIEEE-2009]**

(A) P (–1) is the minimum and P(1) is the maximum of P

(B) P (–1) is not minimum but P(1) is the maximum of P

(C) P (–1) is the minimum and P(1) is not the maximum of P
(D) neither P(–1) is the minimum nor P(1) is the maximum of P

18. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is **[AIEEE-2010]**

(A) y = 1 (B) y = 2 (C) y = 3 (D) y = 0

19. Let f: R → R be defined by $f(x) = \begin{cases} k - 2x & ,\text{if } x \le -1 \\ 2x + 3 & ,\text{if } x > -1 \end{cases}$

If f has a local minimum at x = –1, then a possible value of k is- **[AIEEE-2010]**

(A) 0 (B) $-\frac{1}{2}$ (C) –1 (D) 1

20. Let f: R → R be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$ **[AIEEE-2010]**

Statement-1 : $f(c) = \frac{1}{3}$, for some c ∈ R.

Statement-2: $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all x ∈ R.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is False
(C) Statement-1 is False, Statement-2 is True
(D) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.

21. The shortest distance between line y – x = 1 and curve $x = y^2$ is : **[AIEEE-2011]**

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{3\sqrt{2}}{8}$ (C) $\frac{8}{3\sqrt{2}}$ (D) $\frac{4}{\sqrt{3}}$

22. Let f be a function defined by $f(x) = \begin{cases} \frac{\tan x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$ **[AIEEE-201, II]**

Statement-1: x = 0 is point of minima of f
Statement-2: f' (0) = 0.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

23. A spherical balloon is filled with 4500 π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72 π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is: **[AIEEE-2012]**

(A) $\frac{9}{7}$ (B) $\frac{7}{9}$ (C) $\frac{2}{9}$ (D) $\frac{9}{2}$

24. Let a, b ∈ R be such that the function f given by $f(x) = \ell n |x| + bx^2 + ax, x \ne 0$ has extreme values at x = – 1 and x = 2.
Statement-1 : f has local maximum atx = – 1 and at x = 2.

statement-2: $a = \frac{1}{2}$ and $b = \frac{-1}{4}$. **[AIEEE-2012]**

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

25. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]

(A) lies between 1 and 2
(B) lies between 2 and 3
(C) lies between – 1 and 0.
(D) does not exist **[JEE Mains 2013]**

26. If $y = \sec(\tan^{-1} x)$ then $\frac{dy}{dx}$ at x = 1 is equal to : **[JEE Mains 2013]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) 1 (D) $\sqrt{2}$

EXERCISE # 4

NCERT BOARD QUESTIONS

1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

2. If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.

3. A kite is moving horizontally at a height of 151.5 meters. If the speed of kite is 10 m/s, how fast is the string being let out; when the kite is 250 m away from the boy who is flying the kite? the height of boy is 1.5 m.

4. Two men A and B start with velocities v at the same time from the junction of two roads inclined at 45º to each other . If they travel by different roads, find the rate at which they are being separated..

5. Find an angle θ, $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine.

6. Find the approximate value of $(1.999)^5$.

7. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

8. A man, 2m tall, walks at the rate of $1\frac{2}{3}$ m/s towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is the tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{3}$ m from the base of the light?

9. A swimming pool is to be drained for cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 200 (10 - t)^2$. How fast is the water running out at the end of 5 seconds? What is the average rate atwhich the water flows out during the first 5 seconds?

10. The volume of a cube increases at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

11. x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the areas of second square with respect to the area of first square.

12. Find the condition that the curves $2x = y^2$ and $2xy = k$ intersect orthogonally.

13. Prove that the curves $xy = 4$ and $x^2 + y^2 = 8$ touch each other.

14. Find the co-ordinates of the point on the curve $\sqrt{x}+\sqrt{y}$ = 4 which tangent is equally inclined to the axes.

15. Find the angle of intersection of the curves $y = 4 - x^2$ and $y = x^2$.

16. Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touch each other at the point (1, 2).

17. Find the equation of the normal lines to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

18. At what points on the curves $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to the y-axis?

19. Show that the line $\frac{x}{a}+\frac{y}{b} = 1$, touches the curve $y = b.e^{\frac{-x}{a}}$ at the point where the curve intersects the axis of y.

20. Show that $f(x) = 2x + \cot^{-1}x + \log\left(\sqrt{1+x^2} - x\right)$ is increasing in R.

21. Show that $a \geq 1$, $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing in R.

22. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left(0, \frac{\pi}{4}\right)$.

23. At what point, the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is maximum ? Also find the maximum slope.

24. Prove that $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $x = \frac{\pi}{6}$.

Long Answer (L.A.)

25. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

26. Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also find the corresponding local maximum and local minimum values.

27. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300/- per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Re 1/- one subscriber will discontinue the service. Find what increase will bring maximum profit?

28. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

29. An open box with square base is to be made of a given quantity of card board of area c^2. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

30. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume at large as possible, when revolved about one of its sides, Also find the maximum volume.

31. If the sum of the surface areas of cube and a sphere is constant, what is the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum?

32. AB is a diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is isosceles.

33. A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom cost Rs5/cm^2 and the material for the sides costs Rs 2.50/cm^2. Find the least cost of the box.

34. The sum of the surface areas of a rectangular parallelepiped with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes.

Objective Type Questions

Choose the correct answer from the given four options in each of the following questions 35- 39:

35. The sides of an equilateral triangle are increasing at the rate of 2cm/sec. The rate at which the area increases, when side is 10 cm is:

(A) 10 cm^2/s (B) $\sqrt{3} cm^2/s$ (C) $\sqrt{10}\ cm^2/s$ (D) $\frac{10}{3}\ cm^2/s$

36. A ladder, 5 meter long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10cm/sec then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 metres from the wall is:

(A) $\frac{1}{10}$ radian/sec (B) $\frac{1}{20}$ radian/sec (C) 20 radian/sec (D) 10 radian/sec

37. The curve $y = x^{\frac{1}{5}}$ has at (0, 0)

(A) a vertical tangent (parallel to y-axis) (B) a horizontal tangent (parallel to x-axis)
(C) an oblique tangent (D) no tangent

38. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$ is

(A) $3x - y = 8$ (B) $3x + y + 8 = 0$ (C) $x + 3y \pm 8 = 0$ (D) $x + 3y = 0$

39. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1, 1) then the value of a is :

(A) 1 (B) 0 (C) – 6 (D) .6

40. If $y = x^4 - 10$ and if x changes from 2 to 1.99, what is the change in y

(A) .32 (B) .032 (C) 5.68 (D) 5.968

41. The equation of tangent of the curve $y(1 + x^2) = 2 - x$, where it crosses x-axis is :

(A) $x + 5y = 2$ (B) $x - 5y = 2$ (C) $5x - y = 2$ (D) $5x + y = 2$

42. The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to x-axis are :

(A) (2, – 2), (–2, – 34) (B) (2, 34), (–2, 0) (C) (0, 34), (–2, 0) (D) (2, 2), (–2, 34)

43. The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets x-axis at:

(A) (0, 1) (B) $\left(-\frac{1}{2}, 0\right)$ (C) (2, 0) (D) (0, 2)

44. The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, –1) is:

(A) $\frac{22}{7}$ (B) $\frac{6}{7}$ (C) $\frac{-6}{7}$ (D) – 6

45. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

46. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is :
(A) $[-1, \infty)$ (B) $[-2, -1]$ (C) $(-\infty, -2]$ (D) $[-1, 1]$

47. Let the f: $R \to R$ be defined by $f(x) = 2x + \cos x$, then f:
(A) has a minimum at $x = \pi$ (B) has a maximum , at $x = 0$
(C) is a decreasing function (D) is an increasing function

48. $y = x(x-3)^2$ decreases for the values of x given by:
(A) $1 < x < 3$ (B) $x < 0$ (C) $x > 0$ (D) $0 < x < \frac{3}{2}$

49. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly
(A) increasing in $\left(\pi, \frac{3\pi}{2}\right)$ (B) decreasing in $\left(\frac{\pi}{2}, \pi\right)$
(C) decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (D) decreasing in $\left[0, \frac{\pi}{2}\right]$

50. Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$
(A) sin 2x (B) tan x (C) cos x (D) cos 3x

51. The function $f(x) = \tan x - x$
(A) always increases (B) always decreases
(C) never increases (D) sometimes increases and sometimes decreases

52. If x is real, the minimum value of $x^2 - 8x + 17$ is
(A) – 1 (B) 0 (C) 1 (D) 2

53. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0, 9] is
(A) 126 (B) 0 (C) 135 (D) 160

54. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has
(A) two points of local maximum (B) two points of local minimum
(C) one maxima and one minima (D) no maxima or minima

55. The maximum value of sin x . cos x is
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $2\sqrt{2}$

56. At $x = \frac{5\pi}{6}$, $f(x) = 2\sin 3x + 3\cos 3x$ is :
(A) maximum (B) minimum (C) zero (D) neither maximum nor minimum

57. Maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is :
(A) 0 (B) 12 (C) 16 (D) 32

58. $f(x) = x^x$ has a stationary point at
(A) $x = e$ (B) $x = \frac{1}{e}$ (C) $x = 1$ (D) $x = \sqrt{e}$

59. The maximum value of $\left(\frac{1}{x}\right)^x$ is :
(A) e (B) e^e (C) $e^{\frac{1}{e}}$ (D) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

Fill in the blanks in each of the following exercises 60 to 64:

60. The curves $y = 4x^2 + 2x - 8$ and $y = x^3 - x + 13$ touch each other at the point ....

61. The equation of normal to the curve $y = \tan x$ at (0, 0) is ....

62. The values of a for which the function $f(x) = \sin x - ax + b$ increases on R are ....

63. The function $f(x) = \frac{2x^2 - 1}{x^4}$, $x > 0$, decreases in the interval ....

64. The least value of the function $f(x) = ax + \frac{b}{x}$ $(a > 0, b > 0, x > 0)$ is ....

ANSWERS

EXERCISE # 1

PART # I

A-1.	(B)	**A-2.**	(C)	**A-3.**	(A)	**A-4.**	(C)	**B-1.**	(A)	**B-2.**	(C)	**B-3.**	(D)
B-4.	(B)	**B-5*.**	(C, D)	**B-6*.**	(A, B)	**B-7.**	(B)	**B-8.**	(A)	**B-9*.**	(A,D)	**C-1.**	(A)
C-2.	(B)	**C-3*.**	(A, B)	**C-4*.**	(A, D)	**C-5*.**	(B, C)	**D-1.**	(C)	**D-2.**	(A)	**D-3.**	(C)
D-4.	(B)	**D-5*.**	(A)	**D-6*.**	(B, D)	**D-7*.**	(A, C)	**D-8*.**	(A,B,C)	**D-9.**	(D)	**D-10.**	(D)
D-11.	(C)	**D-12.**	(A)	**D-13.**	(C)	**E-1.**	(D)	**E-2.**	(C)	**E-3.**	(D)	**E-4*.**	(A, C, D)
F-1.	(A)	**F-2.**	(C)	**F-3.**	(B)								

PART # II

A-1. (i) – 2 cm/min (ii) 2 cm^2/min **A-2.** $2x^2 - 3x + 1$ **A-3.** (i) 2 km/hr (ii) 6km/h

A-4. $7.68\,\pi cm^3$ **B-1.** y = x **B-2.** a = 1 , b = – 2 **B-4.** 2x + y = 4, y = 2x **B-5.** (9/4, 3/8)

B-6. $\frac{\pi}{3}$ **B-7.** –1 **B-8.** (– 6 , 3) **B-9.** 10 **B-10.** 2 : 1

C-2. (i) M.D. in $(-\infty, -3]$, M.I. in [–3, 0] M.D . in [0, 2], MI. in $[2, \infty)$ (ii) M.D. in $\left(0, \frac{1}{\sqrt{3}}\right]$, M.I. in $\left[\frac{1}{\sqrt{3}}, \infty\right)$

C-3. (i) Neither increasing nor decreasing, increasing
(ii) at x = – 2 decreasing
at x = 0 decreasing
at x = 3 neither increasing nor decreasing
at x = 5 increasing

C-4. $(-\infty, -3]$ **C-5.** $a \in R^+$ **C-8.** 2 sin x + tan x , 0

D-1. (i) local max. at x = 1, local min. at x = 6 (ii) local max. at $x = -\frac{1}{5}$, local min. at x = – 1

(iii) local min. at $x = \frac{1}{e}$, No local maxima

D-2. (i) max = 8, min. = – 8 (ii) max = $\sqrt{2}$, min = – 1 (iii) max. = 8, min. = – 10
(iv) max. = 25, min = – 39 (v) max. a x = $\pi/6$, max. value = 3/4; min at x = 0 and $\pi/2$. min. value = 1/2

D-3. local max at x = 1, local min at x = 2. **D-5.** $b \in (0, e]$ **D-6.** f = 191 **D-8.** $\frac{4\pi r^3}{3\sqrt{3}}$

D-10. 110 m, $\frac{220}{\pi}$m **D-11.** 32 sq. units **D-12.** 12 cm, 6 cm **E-1.** (i) 3 (ii) 1

1.	(B)	**2.**	(D)	**3.**	(C)	**4.**	(C)	**5.**	(B)	**6.**	(A)	**7.**	(C)
8.	(D)	**9.**	(A)	**10.**	(A)	**11.**	(D)	**12.**	(C)	**13.**	(A)	**14.**	(B)
15.	True	**16.**	True	**17.**	True	**18.**	False	**19.**	True	**20.**	False	**21.**	True

22. (1, –1) (–1, –5) **23.** 6/7 **24.** 4km/hr **25.** $[0, \infty)$ **26.** $\left[0, \frac{11}{4}\right]$ **27.** 3, 1

28. **$(A \to p), (B \to s), (C \to q), (D \to r)$** **29.** **$(A \to p,q), (B \to r, s), (C \to r,s), (D \to r,s)$**

30. **$(A \to r), (B \to s), (C \to q), (D \to p)$**

EXERCISE # 2

PART # I

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(D)	**6.**	(B)	**7.**	(B)
8.	(B)	**9.**	(D)	**10.**	(B)	**11.**	(C)	**12.**	(C)	**13.**	(D)	**14.**	(C)
15.	(A)	**16.**	(B)	**17.**	(B)	**18.**	(A)	**19.**	(B)	**20.**	(D)	**21.**	(B, D)
22.	(A,B,C)	**23.**	(A, B)	**24.**	(A, D)	**25.**	(A, D)	**26.**	(A,B,C,D)	**27.**	(A, B,D)	**28.**	(A, C, D)

PART # II

1. – 1500 ft/sec **2.** $\frac{66}{7}\text{cm}^2/\text{sec}$ **3.** $y = \sqrt{2}x - 2,\ y = -\sqrt{2}x + 2\sqrt{2}$ **4.** $y = x - 5x^3$

5. y = 8x + 4; point of contact (2, 20) and (0, 4) **7.** Injective **8.** $[0, \infty)$ **9.** $p \in (0, 1/e)$

10. $a < -\left(2+\sqrt{5}\right)$ or $a > \sqrt{5}$ **13.** $\ell n\,(1+x)$ **14.** $(1, \infty)$ **15.** $f(x) = 2x^4 - \frac{12}{5}x^5 + \frac{2}{3}x^6$

16. 27 $\sqrt{3}$ sq. cms **17.** width 2$\sqrt{3}$ m length 3$\sqrt{3}$ m **19.** $(-\infty, -2) \cup (0, \infty)$

20. Increasing when $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, decreasing when $x \in \left(0, \frac{\pi}{4}\right)$ **26.** $\cos\left(\frac{1}{3}\cos^{-1} p\right)$

EXERCISE # 3

PART # I

1.	(A)	**4.**	5	**5.**	(C)	**6.**	(D)	**8.**	(A)	**10.**	y = 2	**11.**	(B)
12.	$4\sqrt{65}$	**13.**	6	**14*.**	(B, C)	**15.**	(A)	**16.**	(B)	**17.**	(A)	**18.**	(A)
19.	(B)	**20.**	(C)	**21.**	(C)	**22.**	(B, C, D)			**23.**	0	**24.**	1
25.	(D)	**26.**	**(A $\rightarrow$ s), (B $\rightarrow$ t), (C $\rightarrow$ r), (D $\rightarrow$ r)**					**27.**	2	**28.**	9	**29.**	5
30.	(C)	**31.**	(BC)	**32.**	(AC)			**33.**	9	**34.**	(A, B)	**35.**	(C)
36.	(D)												

PART # II

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(D)	**5.**	(A)	**6.**	(A)	**7.**	(B)
8.	(C)	**9.**	(A)	**10.**	(D)	**11*.**	(A, C)	**12.**	(D)	**13.**	(A)	**14.**	(B)
15.	(A)	**16.**	(A)	**17.**	(B)	**18.**	(C)	**19.**	(C)	**20.**	(D)	**21.**	(B)
22.	(B)	**23.**	(C)	**24.**	(B)	**25.**	(D)	**26.**	(A)				

EXERCISE # 4

3. 8m/s **4.** $\left(\sqrt{2-\sqrt{2}}\right)\nu$ unit / sec **5.** $\theta = \frac{\pi}{3}$ **6.** 31.92 **7.** $0.018\pi cm^3$

8. $2\frac{2}{3}$ m/s towards light, –1 m/s **9.** 2000 litres/s, 3000 litre/s **11.** $2x^3 - 3x + 1$

12. $k^2 = 8$ **14.** (4, 4) **15.** $\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$ **17.** x + 3y = ± 8 **18.** (3, 2), (–1, 2)

23. (1, – 16), max. slope = 12

26. x = 1 is the point of local maxima; local maximum = 0
x = 3 is the point of local minima ; local minimum = – 28 x = 0 is the point of inflection.

27. Rs 100 **30.** 6cm, 12 cm, 864 πcm^3 **31.** 1 : 1 **33.** Rs 1920 **34.** $\frac{2}{3}x^3\left(1+\frac{2\pi}{27}\right)$

35.	(C)	**36.**	(B)	**37.**	(A)	**38.**	(C)	**39.**	(D)	**40.**	(A)	**41.**	(A)
42.	(D)	**43.**	(B)	**44.**	(B)	**45.**	(C)	**46.**	(B)	**47.**	(D)	**48.**	(A)
49.	(B)	**50.**	(C)	**51.**	(A)	**52.**	(C)	**53.**	(B)	**54.**	(C)	**55.**	(B)
56.	(A)	**57.**	(B)	**58.**	(B)	**59.**	(C)	**60.**	(3, 34)	**61.**	x + y = 0		

62. $(-\infty, -1)$ **63.** $(1. \infty)$ **64.** $2\sqrt{ab}$