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CIRCLE

Contents

Syllabus

Equation of a circle in various forms, equations of tangent, normal and chord. Parametric equations of a circle, intersection of a circle with a straight line or a circle, equation of a circle through the points of intersection of two circles and those of a circle and a straight line

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CIRCLE

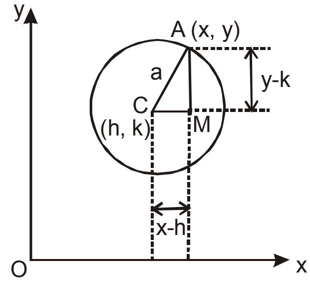
KEY CONCEPTS

DEFINITION & STANDARD RESULTS

1. **DEFINITION :** A circle is the locus of a point which moves in a plane so that its distance from a fixed point 'C' is always constant. The fixed point is called centre of the circles & constant distance is radius of the circle.

2. **EQUATION OF A CIRCLE** :
Equation of circle represents relation between co-ordintes (x, y) of moving point and some constants .
The value of constants depend upon the position of fixed point (centre) about which the point moves.

EQUATION OF A CIRCLE IN VARIOUS FORM :



(a) CENTRAL FORM :-The circle with centre (h, k) & radius 'a' has the equation ;

$(x - h)^2 + (y - k)^2 = a^2$

As in triangle ACM,

$CM^2 + AM^2 = AC^2$

$\therefore \quad (x - h)^2 + (y - k)^2 = a^2$

(b) GENERAL FORM : General equation $x^2 + y^2 + 2gx + 2fy + c = 0$, this equation can be written as-

$(x^2 + 2gx + g^2) + (y^2 + 2fy + f^2) = g^2 + f^2 - c$

OR $\quad (x + g)^2 + (y + f)^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2 \qquad ...(1)$

Comparing eq (1) with

$(x - h)^2 + (y - k)^2 = a^2$

$\therefore \quad h = -g,\ k = -f,\ a = \sqrt{g^2 + f^2 - c}$

so equation (1) represent a circle with **centre (–g, –f)** and **radius** $\sqrt{g^2 + f^2 - c}$

If $g^2 + f^2 - c > 0$ radius of circle is real

If $g^2 + f^2 - c < 0$ radius of circle is not real & hence it represent a circle with real centre & imaginary radius

If $g^2 + f^2 - C = 0$ radius of circle is zero so we can say that circle becomes a point (–g, –f)., or we can say the circle is a point circle

Note : (1) The general equation of a circle contains three arbitrary constants, g, f & c which corresponds to the fact that a unique circle passes through three non collinear points.

(2) The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ also represents a circle, If following conditions are satisfied :

(i) a = b (co-efficient of x^2 and y^2 should be same)

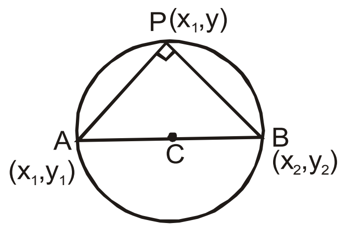
(ii) h = 0(co-efficient of xy should be zero)

(c) DIAMETERIC FORM :

Let A (x_1, y_1) and B (x_2, y_2) are two ends of diameter of a circle. P is any point on circle with co-ordinates (x, y)

$\angle APB = 90^\circ$ (as angle made in semi circle is right angle)

$\therefore$ slope of AP x slope of BP= –1



$$\frac{y - y_1}{x - x_1} \times \frac{y - y_2}{x - x_2} = -1$$

$(y - y_1)(y - y_2) = -(x - x_1)(x - x_2)$

so $\quad (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

is required equation.

Note: This will be the circle of least radius passing through (x_1, y_1) & (x_2, y_2).

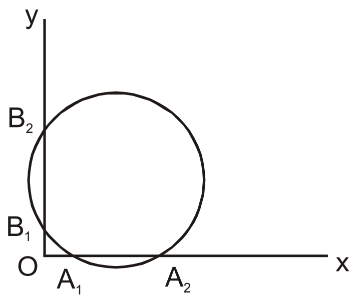
2 **INTERCEPTS MADE BY A CIRCLE ON THE AXES :**

Let equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

intercept on x-axis is A_1A_2 ; intercept on y-axis is B_1B_2

Intercept on x-axis :

Ordinates of A_1 & A_2 are zero (y = 0) and abscissa are x_1 and x_2. So equation of circle on which A_1 and A_2 lies get reduce to $x^2 + 2gx + c = 0$



Here x_1 and x_2 are roots of above equation

$\therefore \quad x_1 + x_2 = -2g \qquad x_1 x_2 = c$

Intercepts on x-axis in $A_1 A_2 = x_2 - x_1$

$A_1 A_2 = x_2 - x_1 = \sqrt{(x_1 + x_2)^2 - 4x_1x_2} = \sqrt{(-2g)^2 - 4c}$

$\therefore \quad A_1 A_2 = 2\sqrt{g^2 - c}$

Intercept on y-axis :

abscissa of B_1 B_2 are zero (x = 0) ordinates of B_1 B_2 are y_1 , y_2 respectively as B_1 and B_2 lie on circle so equation of circle get reduced to $y^2 + 2fy + c = 0$

as y_1 & y_2 lie on circle so they are roots of above equation

hence $\quad y_1 + y_2 = -2f \qquad y_1 y_2 = c$

Hence $\quad B_1 B_2 = y_2 - y_1 = \sqrt{(y_1 + y_2)^2 - 4y_1y_2} = \sqrt{(-2f)^2 - 4c} = 2\sqrt{f^2 - c}$

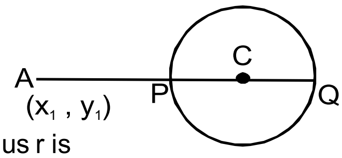
$\therefore$ **Intercept on x-axis =** $2\sqrt{g^2 - c}$; **Intercept on y-axis =** $2\sqrt{f^2 - c}$

Note : (i) If $g^2 > c$ then circle cut x-axis at two distinct, real points.
(ii) If $g^2 < c$ then circle do not cut x-axis at real points.
(iii) If $g^2 = c$ then circle touches x-axis.
(iv) If $f^2 > c$ then circle cut y-axis at two distinct, real point.
(v) If $f^2 = c$ then circle touches y-axis.
(vi) If $f^2 < c$ then circle do not cut y-axis at real points.

4 POSITION OF A POINT w.r.t. A CIRCLE :

The point (x_1 , y_1) is inside on or outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ according as

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$ or = or > 0



Note : The greatest & the least distance of a point A from a circle with centre C & radius r is AC + r & AC – r respectively.

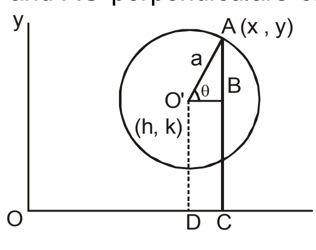
5 LINE & A CIRCLE :

Let L = 0 be a line & S = 0 be a circle. If r is the radius of the circle & p is the length of the perpendicular from the centre on the line, then :

(i) $p > r \Leftrightarrow$ the line does not meet the circle i.e. passes out side the circle.
(ii) $p = r \Leftrightarrow$ the line touches the circle.
(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.
(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

6 PARAMETRIC EQUATIONS OF A CIRCLE :

Let equation of given circle is $(x - h)^2 + (y - k)^2 = a^2$. O' D and AC perpendiculars on x-axis O'B is perpendicular on AC



Let $\quad \angle BO'A = \theta$ and O'A = a

Hence $\quad O'B = a\cos\theta \qquad AB = a\sin\theta$

Now $\quad$ x = OC = OD + DC = OD + O'B

$\therefore \quad x = h + a\cos\theta$

Similarly $\quad y = BC + AB = k + a\sin\theta$

So $\mathbf{x = h + a\cos\theta}$ and $\mathbf{y = k + a\sin\theta}$ are parametric equations of circle $(x - k)^2+(y - k)^2 = a^2$

Special case : If centre of circle is at origin then h = k = 0 , therefore parametric equations gets reduced to $x = a\cos\theta$, $y = a\sin\theta$

Note : The equation of a straight line joining two point α & β on the circle $x^2 + y^2 = a^2$ is

$$x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$$

7 TANGENT & NORMAL:

(a) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at its point (x_1, y_1) is, $x x_1 + y y_1 = a^2$.
Hence the equation of a tangent at $(a \cos \alpha, a \sin \alpha)$ is ; $x \cos\alpha + y \sin\alpha = a$
The point of intersection of the tangents at the points P(α) and Q(B) is

$$\left(\frac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \frac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$$

(b) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point (x_1, y_1) is:
$x x_1 + y y_1 + g (x + x_1) + f (y + y_1) + c = 0$

(c) $y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2 (1 + m^2)$ and the point of contact is $\left(-\frac{a^2m}{c}, \frac{a^2}{c}\right)$

(d) If a line is normal / orthogonal to a circle then it must pass through the centre of the circle. Using bthis fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is

$$y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$$

8 A FAMILY OF CIRCLES :

(a) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ & $S_2 = 0$ is given by $S_1 + K S_2 = 0$ $(K \neq -1)$

(b) The equation of the family of circles passing through the point of intersection of a circle S = 0 & a line L = 0 is given by S + KL = 0

(c) The equation of a family of circles passing through two given points (x_1, y_1) & (x_2, y_2) can be written in the form :

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+K\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \text{ where K is a parameter}$$

(d) The equation of a family of circles touching a fixed line $y - y_1 = m (x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K [y - y_1 - m (x - x_1)] = 0$, where K is a parameter.
In case the line through (x_1, y_1) is parallel to y-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K (x - x_1) = 0$. Also if line is parallel to x-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K (y - y_1) = 0$

(e) Family of circles circumscribing a triangle whose sides are given by $L_1 = 0= 0$; $L_2= 0$ & $L_3 = 0$ is given by ;
$L_1 L_2 + \lambda L_2 L_3 + \mu L_3 L_1 = 0$ provided co-efficient of xy = 0 & co-efficient of x^2 = co-fficient of y^2.

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines
$L_1 = 0, L_2 = 0, L_3 = 0$ & $L_4 = 0$ provided co-efficient of x^2 = co-fficient of y^2 and co-efficent of xy = 0

9 LENGTH OF A TANGENT AND POWER OF A POINT :

The length of a tangent from an external point (x_1, y_1) to the circle $S \equiv x^2+y^2+2gx+2fy+c=0$ is given by
$L = \sqrt{x_1^{\ 2} + y_1^{\ 2} + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$

Square of length of the tangent from the point P is also called **THE POWER OF POINT** w.r.t. a circle. Power of a point remains constant w.r.t.. a circle.

Note : Power of a point P is positive, negative or zero according as the point 'P' is outside, inside or on the circle respectively.

10 DIRECTOR CIRCLE :

The locus of the point of intersection of two perpendicular tangents is called the **DIRECTOR CIRCLE** of the given circle. The director circle of a circle is the concentric circle having radius equal to $\sqrt{2}$ times the original circle.

11 EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT :

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point

M (x_1, y_1) is $(y - y_1) = -\dfrac{x_1 + g}{y_1 + f}(x - x_1)$. This on simplication can be put in the form

$x x_1 + y y_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$

Note : The shortest chord of a circle passing through a point 'M' inside the circle , is one chord whose middle point is M.

12 CHORD OF CONTACT :

If two tangents PT_1 & PT_2 are drawn from the point P (x_1, y_1) to the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact $T_1 T_2$ is

$x x_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

REMEMBER :

(a) Chord of contact exists only if the point 'P' is not inside.

(b) Length of chord of contact $T_1 T_2 = \dfrac{2LR}{\sqrt{R^2 + L^2}}$

(c) Area of the triangle formed by the pair of the tangents & its chord of contact = $= \dfrac{RL^3}{R^2 + L^2}$

where R is the radius of the circle & L is the length of the tangent form (x_1, y_1) on $S = 0$

(d) Angle between the pair of tangent from $(x_1, y_1) = \tan^{-1}\left(\dfrac{2RL}{L^2 - R^2}\right)$; where R = radius, L = length of tangent

(e) Equation of the circle circumscribing the triangle $PT_1 T_2$ is $(x - x_1)(x + g) + (y - y_1)(y + f) = 0$

(f) The joint equation of a pair of tangents drawn from the point A (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $SS_1 = T^2$

where $S \equiv x^2 + y^2 + 2gx + 2fy + c$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$

13 POLE & POLAR :

(i) If through a point P in the plane of the circle, there be drawn any straight line to meet the circle inQ and R, the locus of the point of intersection of the tangents at Q & R is called the **POLAR OF THE POINT P ;** also P is called the **POLE OF THE POLAR**

(ii) The equation to the polar nof a point P (x_1, y_1) w.r.t. the circle $x^2 + y^2 = a^2$ is given by $\mathbf{xx_1 + yy_1 = a^2}$, & if the circle is general then the equation of the polar becomes $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$. Note that if the point (x_1, y_1) be on the circle then the chord of contact, tangent & polar will be represented by the same equation

(iii) Pole of a given line $Ax + By + C = 0$ w.r.t. any circle $x^2 + y^2 = a^2$ is $\left(-\dfrac{Aa^2}{C}, -\dfrac{Ba^2}{C}\right)$

(iv) If the polar of a point P pass through a point Q, then the polar of Q passes through P.

(v) Two lines L_1 & L_2 are conjugate of each other if Pole of L_1 lies on L_2 & vice versa Similarly two points P & Q are said to be conjugate of each other if the polar of P passes through Q & vice versa.

14 COMMON TANGENTS TO TWO CIRCLES :

(i) Where the two circles neither intersect nor touch each other, there are FOUR common tangents, two of them are transverse & the others are direct common tangents.

(ii) When they intersect there are two common tangents, both of them being direct.

(iii) When they touch each other :

(a) EXTERNALLY : there are three common tangents, two direct and one is the tangent at the point of contact.

(b) INTERNALLY : only one common tangent possible at their point of contact.

(iv) Length of an external common tangent & internal common tangent to the two circles is given by :

$$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2} \;\; \& \;\; L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$$

Where d = distance between the centres of the two circles. r_1 & r_2 are the radii of two circles.

(v) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii. Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

15 RADICAL AXIS & RADICAL CENTRE :

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of two circles $S_1 = 0$ & $S_2 = 0$ is given by

$S_1 - S_2 = 0$ i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$

Note :

(a) If two circles intersects, then the radicla axis is the common chord of the two circles

(b) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis need not always pass through the mid point of the line joining the centres of the two circles.

(e) Radical axis bisects a common tangent between the two circles.

(f) The common point of intersection of the radical axes of three circles taken two at a time is called the radical centre of three circles.

(g) A system of circles, every two each have the same radical axis, is called a coaxal system.

(h) Pairs of circles which do not have radical axis are cocentric.

16 ORTHOGONALITY OF TWO CIRCLES :

Two circles $S_1 = 0$ & $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Note : **(a)** Locus of the centre of a variable circle orthogonal to two fixed circles is the radical axis between the two fixed circles.

(b) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0$, $S_2 = 0$ & $S_3 = 0$ are cocurrent is a circle which is orthogonal to all the three circles

(c) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$, where K is a parameter.

In case the line through (x_1, y_1) is parallel to y-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K(x - x_1) = 0$

Also if line is parallel to x-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K(y - y_1) = 0$

(d) Family of circles circumscribing a triangle whose sides are given by $L_1 = 0 = 0$; $L_2 = 0$ & $L_3 = 0$ is given by ; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided co-efficient of xy = 0 & co-efficient of x^2 = co-fficient of y^2.

(e) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ & $L_4 = 0$ is $L_1L_3 + \lambda L_2L_4 = 0$, provided co-efficient of x^2 = co-fficient of y^2.and co-efficent of xy = 0

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

** Marked Questions are having more than one correct option.*

Section (A) : Equation of circle, Parametric equation, position of a point

A-1. The radius of the circle passing through the points (1, 2), (5, 2) & (5, – 2) is:

(A) $5\sqrt{2}$ (B) $2\sqrt{5}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

A-2. The centres of the circles $x^2 + y^2 - 6x - 8y - 7 = 0$ and $x^2 + y^2 - 4x - 10y - 3 = 0$ are the ends of the diameter of the circle

(A) $x^2 + y^2 - 5x - 9y + 26 = 0$ (B) $x^2 + y^2 + 5x - 9y + 14 = 0$

(C) $x^2 + y^2 + 5x - y - 14 = 0$ (D) $x^2 + y^2 + 5x + y + 14 = 0$

A-3. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x–axis in points whose abscissa are roots of the equation:

(A) $x^2 + ax + b = 0$ (B) $x^2 - ax + b = 0$ (C) $x^2 + ax - b = 0$ (D) $x^2 - ax - b = 0$

A-4. The intercepts made by the circle $x^2 + y^2 - 5x - 13y - 14 = 0$ on the x-axis and y-axis are respectively

(A) 9, 13 (B) 5, 13 (C) 9, 15 (D) none

A-5. Equation of line passing through mid point of intercepts made by circle $x^2 + y^2 - 4x - 6y = 0$ on co-ordinate axis is

(A) $3x + 2y - 12 = 0$ (B) $3x + y - 6 = 0$ (C) $3x + 4y - 12 = 0$ (D) $3x + 2y - 6 = 0$

A-6*. Equations of circles which pass through the points (1, –2) and (3, – 4) and touch the x-axis is

(A) $x^2 + y^2 + 6x + 2y + 9 = 0$ (B) $x^2 + y^2 + 10x + 20y + 25 = 0$

(C) $x^2 + y^2 - 6x + 4y + 9 = 0$ (D) none

A-7*. The equation of circles passing through (3, –6) touching both the axes an

(A) $x^2 + y^2 - 6x + 6y + 9 = 0$ (B) $x^2 + y^2 + 6x - 6y + 9 = 0$

(C) $x^2 + y^2 + 30x - 30y + 225 = 0$ (D) $x^2 + y^2 - 30x + 30y + 225 = 0$

Section (B) : Line and circle, tangent, pair of tangent

B-1. Find the co-ordinates of a point p on line x + y = – 13, nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$

(A) (– 6, – 7) (B) (– 15, 2) (C) (– 5, – 6) (D) (– 7, – 6)

B-2. The number of tangents that can be drawn from the point (8, 6) to the circle $x^2 + y^2 - 100 = 0$ is

(A) 0 (B) 1 (C) 2 (D) none

B-3. The two lines through (2, 3) from which the circle $x^2 + y^2 = 25$ intercepts chords of length 8 units have equations

(A) 2x + 3y = 13, x + 5y = 17 (B) y = 3, 12x + 5y = 39

(C) x = 2, 9x – 11y = 51 (D) none of these

B-4. The line 3x + 5y + 9 = 0 w.r.t. the circle $x^2 + y^2 - 4x + 6y + 5 = 0$ is

(A) chord (B) diameter (C) tangent (D) none

B-5. The tangent lines to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line 4x + 3y + 5 = 0 are given by:

(A) 4x + 3y – 7 = 0, 4x + 3y + 15 = 0 (B) 4x + 3y – 31 = 0, 4x + 3y + 19 = 0

(C) 4x + 3y – 17 = 0, 4x + 3y + 13 = 0 (D) none of these

B-6. The condition so that the line $(x + g) \cos\theta + (y + f) \sin\theta = k$ is a tangent to $x^2 + y^2 + 2gx + 2fy + c = 0$ is

(A) $g^2 + f^2 = c + k^2$ (B) $g^2 + f^2 = c^2 + k$ (C) $g^2 + f^2 = c^2 + k^2$ (D) $g^2 + f^2 = c + k$

B-7. The tangent to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ at

(A) (–2, 1) (B) (–3, 0) (C) (–1, –1) (D) (3, –1)

B-8. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) none

B-9. A line segment through a point P cuts a given circle in 2 points A & B, such that PA = 16 & PB = 9, find the length of tangent from points to the circle

(A) 7 (B) 25 (C) 12 (D) None of these

B-10. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is:

(A) $\sqrt{q-p}$ (B) $\sqrt{p-q}$ (C) $\sqrt{q+p}$ (D) none

B-11. The equation of the diameter of the circle $(x - 2)^2 + (y + 1)^2 = 16$ which bisects the chord cut off by the circle on the line $x - 2y - 3 = 0$ is

(A) $x + 2y = 0$ (B) $2x + y - 3 = 0$ (C) $3x + 2y - 4 = 0$ (D) none

Section (C) : Normal, Director circle, chord of contact, pole & polar, chord with mid point

C-1. The equation of normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ which passes through (1, 1) is

(A) $3x + y - 4 = 0$ (B) $x - y = 0$ (C) $x + y = 0$ (D) none

C-2. The co-ordinates of the middle point of the chord cut off on $2x - 5y + 18 = 0$ by the circle $x^2 + y^2 - 6x + 2y - 54 = 0$ are

(A) (1, 4) (B) (2, 4) (C) (4, 1) (D) (1, 1)

C-3. The locus of the mid point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:

o`Ùk $x^2+y^2=4$ dh ml thok ds e/; fcUnq dk fcUnq iFk tks ewy fcUnq ij ledks.k cukrh gSa] gksxk–

(A) $x + y = 2$ (B) $x^2 + y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x + y = 1$

C-4. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point

(A) (1, 2) (B) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (C) (2, 4) (D) none

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre

D-1. Number of common tangents of the circles $(x + 2)^2+(y-2)^2 = 49$ and $(x - 2)^2 + (y + 1)^2 = 4$ is:

(A) 0 (B) 1 (C) 2 (D) 3

D-2. The equation of the common tangent to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ at their point of contact is

(A) $12x + 5y + 19 = 0$ (B) $5x + 12y + 19 = 0$ (C) $5x - 12y + 19 = 0$ (D) $12x - 5y + 19 = 0$

D-3. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is

(A) $x^2 + y^2 - 16x - 18y - 4 = 0$ (B) $x^2 + y^2 - 7x + 11y + 6 = 0$

(C) $x^2 + y^2 + 2x - 8y + 9 = 0$ (D) none of these

Section (E) : Family of circles, Locus, Miscellaneous

E-1. The locus of the centre of the circle which bisects the circumferences of the circles $x^2 + y^2 = 4$ & $x^2 + y^2 - 2x + 6y + 1 = 0$ is:

(A) a straight line (B) a circle (C) a parabola (D) none of these

E-2. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then p + q is equal to:

(A) 25 (B) 100 (C) 10 (D) 48

PART - II : SUBJECTIVE QUESTIONS

Section (A) : Equation of circle, Parametric equation, position of a point

A-1. Find the equation of the circle that passes through the points (1, 0), (– 1, 0) and (0, 1).

A-2. ABCD is a square whose side is a ; taking AB and AD as axes, prove that the equation to the circle circumscribing the square is $x^2 + y^2 = a(x + y)$.

A-3. Find the equation to the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the axes.

A-4. Find equation of circle which touches x & y axis & $\perp^r$ distance of centre of circle from $3x + 4y + 11 = 0$ is 5. Given that circle lies in I^{st} quadrant.

A-5. Find the equation to the circle which touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y.

A-6. Find the cartesian equations of the circle, $x = -3 + 2 \sin \theta$, $y = 4 + 2 \cos \theta$

Section (B) : Line and circle, tangent, pair of tangent

B-1. Find the points of intersection of the line $x - y + 2 = 0$ and the circle $3x^2 + 3y^2 - 29x - 19y + 56 = 0$. Also determine the length of the chord intercepted.

B-2. Show that the line $7y - x = 5$ touches the circle $x^2 + y^2 - 5x + 5y = 0$ and find the equation of the other parallel tangent.

B-3. Find the equation of the tangents to the circle $x^2 + y^2 = 4$ which make an angle of 60º with the x-axis.

B-4. Show that two tangents can be drawn from the point (9, 0) to the circle $x^2 + y^2 = 16$; also find the equation of the pair of tangents and the angle between them.

B-5. If the length of the tangent from (f, g) to the circle $x^2 + y^2 = 6$ be twice the length of the tangent from (f, g) to the circle $x^2 + y^2 + 3x + 3y = 0$ then will $f^2 + g^2 + 4f + 4g + 2 = 0$?

Section (C) : Normal, Director circle, chord of contact, pole & polar, chord with mid point

C-1. Find the equation of the normal to the circle $x^2 + y^2 = 5$ at the point (1, 2)

C-2. Find the equation of director circle of the circle $(x + 4)^2 + y^2 = 8$

C-3. Find the equation of the chord of the circle $x^2 + y^2 + 6x + 8y + 9 = 0$ whose middle point is (– 2, – 3).

C-4. Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is $\dfrac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$.

C-5. Find the polar of the point (– 2, 3) with respect to the circle $x^2 + y^2 - 4x - 6y + 5 = 0$.

C-6. Prove that the polars of the point (1, – 2) with respect to the circles whose equations are $x^2 + y^2 + 6y + 5 = 0$ and $x^2 + y^2 + 2x + 8y + 5 = 0$ coincide. Prove also that there is another point the polars of which with respect to these circles are the same and find the coordinates.

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre

D-1. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$

D-2. Show that the circles $x^2 + y^2 - 2x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 6 = 0$ cut each other orthogonally.

D-3. Find the equation of the circle passing through the origin and cutting the circles $x^2 + y^2 - 4x + 6y + 10 = 0$ and $x^2 + y^2 + 12y + 6 = 0$ at right angles.

D-4. Given the three circles $x^2 + y^2 - 16x + 60 = 0$, $3x^2 + 3y^2 - 36x + 81 = 0$ and $x^2 + y^2 - 16x - 12y + 84 = 0$, find (A) the point from which the tangents to them are equal in length, and (B) this length.

D-5. Tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it is met by the circle $x^2 + y^2 - 5x + 3y - 2 = 0$; find the point of intersection of these tangents.

Section (E) : Family of circles, Locus, Miscellaneous

E-1. Find the equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

E-2. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of a circle with this chord as diameter.

PART - III : MISCELLANEOUS OBJECTIVE QUESTIONS

MATCH THE COLUMN

1. **Column – I** | **Column – II**

	Column – I		Column – II
(A)	Number of values of a for which the common chord of the circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin is	(p)	4
(B)	A chord of the circle $(x - 1)^2 + y^2 = 4$ lies along the line $y = 22\sqrt{3}\,(x - 1)$. The length of the chord is equal to	(q)	2
(C)	The number of circles touching all the three lines $3x + 7y = 2$, $21x + 49y = 5$ and $9x + 21y = 0$ are	(r)	0
(D)	If radii of the smallest and largest circle passing through the point $\left(\sqrt{3}, \sqrt{2}\right)$ and touching the circle $x^2 + y^2 - 2\sqrt{2}\,y - 2 = 0$ are r_1 and r_2 respectively, then the mean of r_1 and r_2 is	(s)	1

2.

	Column – I		Column – II
(A)	Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is	(p)	1
(B)	Number of indirect common tangents of the circles $x^2 + y^2 - 4x - 10y + 4 = 0$ & $x^2 + y^2 - 6x - 12y - 55 = 0$ is	(q)	2
(C)	Number of common tangents of the circles $x^2 + y^2 - 2x - 4y = 0$ & $x^2 + y^2 - 8y - 4 = 0$ is	(r)	3
(D)	Number of direct common tangents of the circles $x^2 + y^2 + 2x - 8y + 13 = 0$ & $x^2 + y^2 - 6x - 2y + 6 = 0$ is	(s)	0

COMPREHENSION

Comprehension # 1

Let S_1, S_2, S_3 be the circles $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ and $x^2 + y^2 + 5x - 8y + 15 = 0$, then

3. Point from which length of tangents to these three circles is same is
(A) (1, 0) (B) (3, 2) (C) (10, 5) (D) (– 2, 1)

4. Equation of circle S_4 which cut orthogonally to all given circle is
(A) $x^2 + y^2 - 6x + 4y - 14 = 0$ (B) $x^2 + y_2 + 6x + 4y - 14 = 0$
(C) $x^2 + y^2 - 6x - 4y + 14 = 0$ (D) $x^2 + y^2 - 6x - 4y - 14 = 0$

5. Radical centre of circles S_1, S_2, & S_4 is
(A) $\left(-\frac{3}{5}, -\frac{8}{5}\right)$ (B) (3, 2) (C) (1, 0) (D) $\left(-\frac{4}{5}, -\frac{3}{2}\right)$

Comprehension # 2

Two circles are
$S_1 \equiv (x + 3)^2 + y^2 = 9$
$S_2 \equiv (x - 5)^2 + y^2 = 16$
with centres C_1 & C_2

6. A direct common tangent is drawn from a point P which touches S_1 & S_2 at Q & R, respectively. Find the ratio of area of ΔPQC_1 & ΔPRC_2.

(A) 3 : 4 (B) 9 : 16 (C) 16 : 9 (D) 4 : 3

7. From point 'A' on S_2 which is nearest to C_1, a variable chord is drawn to S_1. The locus of mid point of the chord.

(A) circle (B) Diameter of s_1 (C) Arc of a circle (D) chord of s_1 but not diameter

8. Locus of 7 cuts the circle S_1 at B & C, then line segment BC subtends an angle on the major are of BC.

(A) $\cos^{-1}\frac{3}{4}$ (B) $\frac{\pi}{2} - \tan^{-1}\frac{4}{3}$ (C) $\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{3}{4}$ (D) $\frac{\pi}{2}\cot^{-1}\frac{4}{3}$

ASSERTION / REASONING

9. **STATEMENT-1 :** Number of circles through the three points A(3, 5), B(4, 6), C(5, 7) is 1

STATEMENT-2 : Through three non collinear points in a plane, one and only one circle can be drawn.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10. **STATEMENT-1 :** The length of intercept made by the circle $x^2 + y^2 - 2x - 2y = 0$ on the x-axis is 2.

STATEMENT-2 : $x^2 + y^2 - \alpha x - \beta y = 0$ is a circle which passes through origin with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ and radius $\sqrt{\frac{\alpha^2 + \beta^2}{2}}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

11. **STATEMENT-1 :** If three circles which are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.

STATEMENT-2 : Radical axis for two intersecting circles is the common chord.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12. **STATEMENT - 1 :** If a line L = 0 is tangent to the circle S = 0, then it will also be a tangent to the circle $S + \lambda L = 0$.

STATEMENT - 2 : If a line touches a circle, then perpendicular distance of the line from the centre of the circle is equal to the radius of the circle.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

1. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ & $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abcd is equal to:

(A) 4 (B) 16 (C) 1 (D) none

2. From the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn & extended to a point M such that AM = 2 AB. The equation of the locus of M is:

(A) $x^2 + 8x + y^2 = 0$ (B) $x^2 + 8x + (y - 3)^2 = 0$
(C) $(x - 3)^2 + 8x + y^2 = 0$ (D) $x^2 + 8x + 8y^2 = 0$

3. Two thin rods AB & CD of lengths 2a & 2b move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:

(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 - y^2 = a^2 - b^2$ (C) $x^2 + y^2 = a^2 - b^2$ (D) $x^2 - y^2 = a^2 + b^2$

4. The co–ordinate of the point on the circle $x^2 + y^2 - 12x - 4y + 30 = 0$, which is farthest from the origin are:

(A) (9, 3) (B) (8, 5) (C) (12, 4) (D) none

5. The value of 'c' for which the set,
$\{(x, y) \mid x^2 + y^2 + 2x \le 1\} \cap \{(x, y) \mid x - y + c \ge 0\}$ contains only one point in common is:

(A) $(-\infty, -1] \cup [3, \infty)$ (B) $\{-1, 3\}$ (C) $\{-3\}$ (D) $\{-1\}$

6. Let x & y be the real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M & m respectively, then the numerical value of M – m is:

(A) 2 (B) 8 (C) 15 (D) none of these

7. The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their point of contact is:

(A) $\frac{192}{25}$ (B) 192 (C) 25 (D) 250

8. The distance between the chords of contact of tangents to the circle; $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin & the point (g, f) is:

(A) $\sqrt{g^2+f^2}$ (B) $\frac{\sqrt{g^2+f^2-c}}{2}$ (C) $\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$ (D) $\frac{\sqrt{g^2+f^2+c}}{2\sqrt{g^2+f^2}}$

9*. A circle passes through the point $\left(3, \sqrt{\frac{7}{2}}\right)$ and touches the line pair $x^2 - y^2 - 2x + 1 = 0$. centre of circles lies inside the circle $x^2 + y^2 - 8x + 10y + 15 = 0$. Co–ordinates of the centre of the circle are:

(A) (4, 0) (B) (5, 0) (C) (6, 0) (D) (0, 4)

10. If tangent at (1, 2) to the circle c_1: $x^2 + y^2 = 5$ intersects the circle c_2: $x^2 + y^2 = 9$ at A & B and tangents at A & B to the second circle meet at point C, then the co–ordinates of C are:

(A) (4, 5) (B) $\left(\frac{9}{15}, \frac{18}{5}\right)$ (C) (4, – 5) (D) $\left(\frac{9}{5}, \frac{18}{5}\right)$

11. A point A(2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ & AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is:

(A) $(x + g)(x - 2) + (y + f)(y - 1) = 0$ (B) $(x + g)(x - 2) - (y + f)(y - 1) = 0$
(C) $(x - g)(x + 2) + (y - f)(y + 1) = 0$ (D) none

12. The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\frac{\pi}{3}$ radians at its circumference is:

(A) $(x - 2)^2 + (y + 3)^2 = 6.25$ (B) $(x + 2)^2 + (y - 3)^2 = 6.25$
(C) $(x + 2)^2 + (y - 3)^2 = 18.75$ (D) $(x + 2)^2 + (y + 3)^2 = 18.75$

13. The locus of the centers of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is:

(A) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$ (C) $2x + 5y + 11 = 0$ (D) none

14. If the circle C_1: $x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co–ordinates of the centre of C_2 are:

(A) $\left(\pm\frac{9}{5}, \pm\frac{12}{5}\right)$ (B) $\left(\pm\frac{9}{5}, \mp\frac{12}{5}\right)$ (C) $\left(\pm\frac{12}{5}, \pm\frac{9}{5}\right)$ (D) $\left(\pm\frac{12}{5}, \mp\frac{9}{5}\right)$

15. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, then the angle between the tangents is:

(A) α (B) 2α (C) $\frac{\alpha}{2}$ (D) none

16. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:

(A) 36 (B) 9 (C) 18 (D) 4

17. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ & $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each then:

(A) $f_1g_1 = f_2g_2$ (B) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$ (C) $f_1f_2 = g_1g_2$ (D) none

18. Two circles whose radii are equal to 4 and 8 intersect at right angles. The length of their common chord is:

(A) $\frac{16}{\sqrt{5}}$ (B) 8 (C) $4\sqrt{6}$ (D) $\frac{8\sqrt{5}}{5}$

19. A circle touches a straight line $\ell x + my + n = 0$ & cuts the circle $x^2 + y^2 = 9$ orthogonally. The locus of centres of such circles is:

(A) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$ (B) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

(C) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 + 9)$ (D) none of these

20. The circle $x^2 + y^2 = 4$ cuts the circle $x^2 + y^2 + 2x + 3y - 5 = 0$ in A & B. Then the equation of the circle on AB as a diameter is:

(A) $13(x^2 + y^2) - 4x - 6y - 50 = 0$ (B) $9(x^2 + y^2) + 8x - 4y + 25 = 0$

(C) $x^2 + y^2 - 5x + 2y + 72 = 0$ (D) none of these

21. The length of the tangents from any point on the circle $15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles $5x^2 + 5y^2 - 24x + 32y + 75 = 0$ and $5x^2 + 5y^2 - 48x + 64y + 300 = 0$ are in the ratio

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) none of these

22. The normal at the point (3, 4) on a circle cuts the circle at the point (–1, –2). Then the equation of the circle is

(A) $x^2 + y^2 + 2x - 2y - 13 = 0$ (B) $x^2 + y^2 - 2x - 2y - 11 = 0$

(C) $x^2 + y^2 - 2x + 2y + 12 = 0$ (D) $x^2 + y^2 - 2x - 2y + 14 = 0$

Multiple choice

23. The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through two fixed points, (k is parameter)

(A) $\left(1+\sqrt{3}, 0\right)$ (B) $\left(-1+\sqrt{3}, 0\right)$ (C) $\left(-\sqrt{3}-1, 0\right)$ (D) $\left(1-\sqrt{3}, 0\right)$

24. The equation of the circle which touches both the axes and the line $\frac{x}{3} + \frac{y}{4} = 1$ and lies in the first quadrant is $(x - c)^2 + (y - c)^2 = c^2$ where c is

(A) 1 (B) 2 (C) 4 (D) 6

PART - II : SUBJECTIVE QUESTIONS

1. On the line joining (1, 0) and (3, 0) an equilateral triangle is drawn having its vertex in the first quadrant. Find the equation to the circles described on its sides as diameter.

2. One of the diameters of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A & B are the points (–3, 4) & (5, 4) respectively. Then find the area of the rectangle.

3. A variable circle passes through the point A (a, b) & touches the x–axis; show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

4. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B (1, 7) & D (4, – 2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

5. Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the midpoints of the portions of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$.

6. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x–axis, can be drawn to the circle from $\left(a, \frac{b}{2}\right)$.

7. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.

8. Find the equations to the four common tangents to the circles $x^2 + y^2 = 25$ and $(x - 12)^2 + y^2 = 9$.

9. Find the equation of the circle which cuts each of the circles, $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ & $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter.

10. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2 : 1.

11. Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it.

12. Find the values of a for which the point (2a, a + 1) is an interior point of the larger segment of the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ made by the chord whose equation is $x - y + 1 = 0$.

13. If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$. Prove that $\ell x + my + 1 = 0$ touches a definite circle. Find the centre & radius of the circle.

14. A circle touches the line y = x at a point P such that $OP = 4\sqrt{2}$ where O is the origin. The circle contains the point (–10, 2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. Find the equation of the circle.

15. Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distances 'd' from a point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + \frac{d^2}{2} = 0$.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

*** Marked Questions are having more than one correct option.**

1. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then **[IIT - 1999, 2]**

(A) $p^2 = q^2$ (B) $p^2 = 8q^2$ (C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$

2*. Let L_1 be a straight line passing through the origin and L_2 be the straight line x + y = 1. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 ? **[IIT - 1999, 3]**

(A) x + y = 0 (B) x – y = 0 (C) x + 7y = 0 (D) x – 7y = 0

3. Let T_1, T_2 be two tangents drawn from (– 2, 0) onto the circle C: $x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time. **[IIT - 1999,10]**

4. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3, 4) and (–4, 3) respectively, the $\angle$ QPR is equal to **[IIT - 2000]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

5. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is **[IIT - 2000]**

(A) 2 or $-\frac{3}{2}$ (B) – 2 or $-\frac{3}{2}$ (C) 2 or $\frac{3}{2}$ (D) – 2 or $\frac{3}{2}$

6. Let PQ and RS be tangents at the extremities of diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals **[IIT- 2001, 1]**

(A) $\sqrt{PQ\,.\,RS}$ (B) $\frac{PQ + RS}{2}$ (C) $\frac{2PQ + RS}{PQ + RS}$ (D) $\frac{\sqrt{PQ^2 + RS^2}}{2}$

7. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then, locus of the centroid of the triangle PAB as P moves on the circles is **[IIT- 2001, 1]**

(A) a parabola (B) a circle (C) an ellipse (D) a pair of straight line

8. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. **[IIT 2001, 5]**

9. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is **[IIT- 2002, 3]**

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

10. If a > 2b > 0, then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is **[IIT- 2002, 3]**

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$ (C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

11. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is

(A) (4, 7) (B) (2, 9) (C) (7, 4) (D) (9, 2)

[JEE '2003 (Scr) 3]

12. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1) then the radius of the circle is **[IIT- 2004]**

(A) 3 (B) 2 (C) 3/2 (D) $\sqrt{1}$

13. Find the equation of circle touching the line 2x + 3y + 1 = 0 at (1, – 1) and cutting orthogonally the circle having line segment joining (0, 3) and (– 2, – 1) as diameter. **[IIT - 2004]**

14. A circle is given by $x^2 + (y - 1)^2 = 1$. Another circle C touches it externally and also the x-axis, then the locus of its centre is **[IIT- 2005]**

(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (B) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{(x, y) : y \le 0\}$

(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

15. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. **[IIT - 2005]**

16. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$. **[IIT- 2007]**

STATEMENT-1 : The tangents are mutually perpendicular.

because

STATEMENT-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

17. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2 CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is **[IIT- 2007]**

(A) 3 (B) 2 (C) $\frac{3}{2}$ (D) 1

18. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents **[IIT-2008]**

(A) four straight lines, when c = 0 and a, b are of the same sign
(B) two straight lines and a circle, when a = b and c is of sign opposite to that of a
(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
(D) a circle and an ellipse, when a nad b are of the same sign and c is of sign opposite to that of a

19*. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, the

[JEE '2008 (4, 0) out of 82]

(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$ (B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$ (D) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$

Comprehension #1

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ

20. The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (B) $(x - 2\sqrt{3})^2 + (y + \frac{1}{2})^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

21. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$ (C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

22. Equations of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1$, $y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x$, $y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1$, $y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x$, $y = 0$ **[JEE '2008 (4, –1) out of 82]**

23. Consider

$L_1 : 2x + 3y + p - 3 = 0$

$L_2 : 2x + 3y + p + 3 = 0$,

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.

Statement-1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

Statement-2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement -1 is true, Statement - 2 is true ; Statement - 2 is correct explanation for Statement - 1

(B) Statement -1 is true, Statement - 2 is true ; Statement - 2 is NOT correct explanation for Statement - 1

(C) Statement -1 is true, Statement - 2 is false.

(D) Statement -1 is false, Statement - 2 is true **[JEE '2008 (3, –1) out of 81]**

24. Tangents drawan from the point P(1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is **[JEE '2009 (3, –1) out of 81]**

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

25. The centres of two circle C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is. **[JEE '2009 (4, 0) out of 81]**

26. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is **[JEE '2010, Paper2 (3, 0),79]**

[**Note** : [k] denotes the largest integer less than or equal to k]

27. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point **[IIT-JEE - 2011, Paper-2]**

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

28. The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right)\left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$

then the number of point (s) in S lying inside the smaller part is **[IIT-JEE - 2011, Paper-2]**

29. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line 4x – 5y = 20 to the circle $x^2 + y^2 = 9$ is : **[JEE - 2012]**

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question Nos. 30 to 31

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point P($\sqrt{3}$, 1). A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$. **[JEE - 2012]**

30. A common tanagent of the two circles is :

(A) x = 4 (B) y = 2 (C) $x + \sqrt{3}y = -1$ (D) $x + 2\sqrt{2}y = 6$

31. A possible equation of L is :

(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$ (C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

32*. circle (s) touching x – axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y –axis is (are) **[IIT-JEE- 2013]**

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. The greatest distance of the point P (10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is- **[AIEEE 2002]**

(1) 10 unit (2) 15 unit (3) 5 unit (4) None of these

2. The equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ which make equal intercepts on the positive coordinate axes, is- **[AIEEE 2002]**

(1) x + y = 2 (2) $x + y = 2\sqrt{2}$ (3) x + y = 4 (4) x + y = 8

3. If the chord y = mx + 1 of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45º at the major segment of the circle, then value of m is **[AIEEE 2002]**

(1) $2 \pm \sqrt{2}$ (2) $-2 \pm \sqrt{2}$ (3) $-1 \pm \sqrt{2}$ (4) none of these

4. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is **[AIEEE 2002]**

(1) $4 \le x^2 + y^2 \le 64$ (2) $x^2 + y^2 \le 25$ (3) $x^2 + y^2 \ge 25$ (4) $3 \le x^2 + y^2 \le 9$

5. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is

(1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, -\sqrt{2}\right)$ (3) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, \frac{3}{2}\right)$ **[AIEEE 2002]**

6. The equation of circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is :

(1) $x^2 + y^2 = a^2$ (2) $x^2 + y^2 = 4a^2$ (3) $x^2 + y^2 = 16a^2$ (4) $x^2 + y^2 = 9a^2$ **[AIEEE 2002]**

7. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then- **[AIEEE 2003]**

(1) $2 < r < 8$ (2) $r < 2$ (3) $r = 2$ (4) $r > 2$

8. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq unit. Then, the equation of the circle is- **[AIEEE 2003]**

(1) $x^2 + y^2 + 2x - 2y = 62$ (2) $x^2 + y^2 + 2x - 2y = 47$
(3) $x^2 + y^2 - 2x + 2y = 47$ (4) $x^2 + y^2 - 2x + 2y = 62$

9. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is - **[AIEEE 2004]**

(1) $2ax + 2by + (a^2 + b^2 + 4) = 0$ (2) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(3) $2ax - 2by + (a^2 + b^2 + 4) = 0$ (4) $2ax - 2by - (a^2 + b^2 + 4) = 0$

10. A variable circle passes through the fixed point A(p, q) and touches x-axis. The locus of the other end of the diameter through A is- **[AIEEE 2004]**

(1) $(x - p)^2 = 4qy$ (2) $(x - q)^2 = 4py$ (3) $(y - p)^2 = 4qx$ (4) $(y - q)^2 = 4px$

11. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is- **[AIEEE 2004]**

(1) $x^2 + y^2 - 2x + 2y - 23 = 0$ (2) $x^2 + y^2 - 2x - 2y - 23 = 0$
(3) $x^2 + y^2 + 2x + 2y - 23 = 0$ (4) $x^2 + y^2 - 2x - 2y - 23 = 0$

12. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is- **[AIEEE 2004]**

(1) $x^2 + y^2 - x - y = 0$ (2) $x^2 + y^2 - x + y = 0$ (3) $x^2 + y^2 + x + y = 0$ (4) $x^2 + y^2 + x - y = 0$

13. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then the line $5x + by - a = 0$ passes through P and Q for - **[AIEEE 2005]**

(1) exactly two values of a (2) infinitely many values of a
(3) no value of a (4) exactly one value of a

14. A circle touches the x-axis and also touches the circle with cetnre at (0, 3) and radius 2. The locus of the centre of the circle is- **[AIEEE 2005]**

(1) a parabola (2) a hyperbola (3) a circle (4) an ellipse

15. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is : **[AIEEE 2006]**

(1) $x^2 + y^2 + 2x - 2y - 62 = 0$ (2) $x^2 + y^2 - 2x + 2y - 62 = 0$
(3) $x^2 + y^2 - 2x + 2y - 47 = 0$ (4) $x^2 + y^2 + 2x - 2y - 47 = 0$

16. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre, is : **[AIEEE 2006]**

(1) $x^2 + y^2 = 1$ (2) $x^2 + y^2 = \frac{27}{4}$ (3) $x^2 + y^2 = \frac{9}{4}$ (4) $(x^2 + y^2 = \frac{3}{2}$

17. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval **[AIEEE 2007]**

(1) $0 < k < 1 < 2$ (2) $k \geq 1/2$ (3) $-1/2 \leq k \leq 1/2$ (4) $k \leq 1/2$

18. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(1) (3, –4) (2) (–3, 4) (3) (–3, –4) (4) (3, 4) **[AIEEE 2008]**

19. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for : **[AIEEE 2009]**

(1) all except one value of p (2) all except two values of p
(3) exactly one value of p (4) all values of p

20. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[AIEEE 2010]**

(1) $-35 < m < 15$ (2) $15 < m < 65$ (3) $35 < m < 85$ (4) $-85 < m < -35$

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (c > 0) touch each other if **[AIEEE 2011]**

(1) $|a| = c$ (2) $a = 2c$ (3) $|a| = 2c$ (4) $2|a|c$

22. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is **[AIEEE 2012]**

(1) $\frac{10}{3}$ (2) $\frac{3}{5}$ (3) $\frac{6}{5}$ (4) $\frac{5}{3}$

23. The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point :

(1) (– 5, 2) (2) (2, – 5) (3) (5, – 2) (4) (– 2, 5)

[IIT-JEE Mains 2013]

EXERCISE # 4

NCERT BOARD QUESTIONS

1. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

2. Show that the point (x, y) given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that $-1 \le t \le 1$ where a is any given real numbers.

3. If a circle passes through the point (0, 0) (a, 0), (0, b) then fidn the coordinates ofits centre.

4. Find the equation of the circle which touches x-axis and whose centre is (1, 2).

5. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7$ are tangents to a circle, then find the radius of the circle.

6. Find the equation of a circle which touches both the axes and the lie $3x - 4y + 8 = 0$ and lies in the third quadrant.

7. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinate of the other end of the diameter.

8. Find the equation of the circle having (1, – 2) as its centre and passing through $3x + y = 14$, $2x + 5y = 18$.

9. If the line $y = \sqrt{3}\, x + k$ touches the circle $x^2 + y^2 = 16m$ then fidn the value of k.

10. Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has double of its area.

11. If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

12. Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$.

13. Find the equation of a circle whose centre is (3, – 1) and which cuts off a chord of length 6 units on the line $2x - 5y + 18 = 0$.

14. Find the equation of a circle of radius 5 which is touching another circle
$x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5).

15. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line $y = x - 1$.

ANSWERS

EXERCISE # 1

PART # I

A-1.	(D)	**A-2.**	(A)	**A-3.**	(B)	**A-4.**	(C)	**A-5.**	(D)	**A-6*.**	(B,C)	**A-7*.**	(A,D)
B-1.	(A)	**B-2.**	(B)	**B-3.**	(B)	**B-4.**	(B)	**B-5.**	(B)	**B-6.**	(A)	**B-7.**	(D)
B-8.	(C)	**B-9.**	(C)	**B-10.**	(A)	**B-11.**	(B)	**C-1.**	(A)	**C-2.**	(A)	**C-3.**	(C)
C-4.	(B)	**D-1.**	(B)	**D-2.**	(B)	**D-3.**	(A)	**E-1.**	(A)	**E-2.**	(C)		

PART # II

A-1. $x^2 + y^2 = 1$ **A-3.** $x^2 + y^2 \pm 3x \pm 4y = 0$ **A-4.** $x^2 + y^2 - 4x - 4y + 4 = 0$

A-5. $x^2 + y^2 \pm 6\sqrt{2}y \pm 6x + 9 = 0$ **A-6.** $(x + 3)^2 + (y - 4)^2 = 4$ **B-1.** $(1, 3), (5, 7), 4\sqrt{2}$

B-2. $x - 7y - 45 = 0$ **B-3.** $\sqrt{3}x - y \pm 4 = 0$

B-4. $16x^2 - 65y^2 - 288x + 1296 = 0, \tan^{-1}\left(\frac{8\sqrt{65}}{49}\right)$ **B-5.** Yes

C-1. $2x - y = 0$ **C-2.** $(x + 4)^2 + y^2 = 16$ **C-3.** $x + y + 5 = 0$

C-5. $x = 0$ **C-6.** $(2, -1)$ **D-1.** $x = 0$, $3x + 4y = 10$, $y = 4$, and $3y = 4x$.

D-3. $2(x^2 + y^2) - 7x + 2y = 0$ **D-4.** $\left(\frac{33}{4}, 2\right)$; $\frac{1}{4}$ **D-5.** $\left(6, -\frac{18}{5}\right)$

E-1. $x^2 + y^2 - 17x - 19y + 50 = 0$ **E-2.** $x^2 + y^2 - 2x - 4y = 0$.

PART # III

1. (A) → (q), (B) → (p), (C) → (r), (D) → (s) **2.** (A) → (r), (B) → (s), (C) → (p),(D) → (q)

3.	(B)	**4.**	(D)	**5.**	(A)	**6.**	(B)	**7.**	(C)	**8.**	(A)	**9.**	(D)
10.	(C)	**11.**	(B)	**12.**	(A)								

EXERCISE # 2

PART # I

1.	(C)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(D)	**6.**	(B)	**7.**	(A)
8.	(C)	**9*.**	(A, C)	**10.**	(D)	**11.**	(A)	**12.**	(B)	**13.**	(A)	**14.**	(B)
15.	(B)	**16.**	(C)	**17.**	(B)	**18.**	(A)	**19.**	(A)	**20.**	(A)	**21.**	(A)
22.	(B)	**23*.**	(A,D)	**24*.**	(A, D)								

PART # II

1. $x^2 + y^2 - 3x - \sqrt{3}y + 2 = 0$; $x^2 + y^2 - 5x - \sqrt{3}y + 6 = 0$; $x^2 + y^2 - 4x + 3 = 0$

2. 32 sq. unit **4.** 75 sq. units **6.** $(a^2 > 2b^2)$ **7.** $x^2 + y^2 - 2x - 2y = 0$

8. $2x - \sqrt{5}y - 15 = 0$, $2x + \sqrt{5}y - 15 = 0$, $x - \sqrt{35}y - 30 = 0$, $x + \sqrt{35}y - 30 = 0$

9. $x^2 + y^2 - 4x - 6y - 4 = 0$ **10.** $4x - 3y - 25 = 0$ **OR** $3x + 4y - 25 = 0$

11. $x^2 + y^2 + x - 6y + 3 = 0$ **12.** $a \in (0, 9/5)$

13. Centre $\equiv (3, 0)$, (radius) $= \sqrt{5}$ **14.** $x^2 + y^2 + 18x - 2y + 32 = 0$

EXERCISE # 3

PART # I

1. (D) **2*.** (B, C) **3.** $c_1: (x - 4)^2 + y^2 = 9$; $c_2: \left(x + \frac{4}{3}\right)^2 + y^2 = \frac{1}{9}$ **4.** (C)

5. (A) **6.** (A) **7.** (B) **8.** $9 + 3\sqrt{10}$ **9.** (C) **10.** (A)

11. (A) **12.** (A) **13.** $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ **14.** (D) **15.** $\sqrt{5}$

16.	(A)	**17.**	(B)	**18.**	(B)	**19*.**	(B, D)	**20.**	(D)	**21.**	(A)	**22.**	(D)
23.	(C)	**24.**	(B)	**25.**	8	**26.**	3	**27.**	(D)	**28.**	1	**29.**	(A)
30.	(D)	**31.**	(A)	**32.**	(A, C)								

PART # II

1.	(2)	**2.**	(2)	**3.**	(4)	**4.**	(2)	**5.**	(2)	**6.**	(2)	**7.**	(1)
8.	(3)	**9.**	(2)	**10.**	(1)	**11.**	(1)	**12.**	(1)	**13.**	(3)	**14.**	(1)
15.	(3)	**16.**	(3)	**17.**	(2)	**18.**	(3)	**19.**	(1)	**20.**	(1)	**21.**	(1)
22.	(1)	**23.**	(3)										

EXERCISE # 4

PART # I NCERT QUESTIONS

1. $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ **3.** $\left(\frac{a}{2}, \frac{b}{2}\right)$ **4.** $x^2 + y^2 - 2x - 4y + 1 = 0$ **5.** $\frac{3}{4}$

6. $x^2 + y^2 + 4x + 4y + 4 = 0$ **7.** (1, 2) **8.** $x^2 + y^2 - 2x + 4y - 20 = 0$ **9.** $k \pm 8$

10. $x^2 + y^2 - 6x + 12y - 15 = 0$ **11.** $x^2 + y^2 - 2x + 2y = 47$ **12.** $x^2 + y^2 - 4x - 1010y + 25 = 0$

13. $(x - 3)^2 + (y + 1)^2 = 38$ **14.** $x^2 + y^2 - 18x - 16y + 120 = 0$ **15.** $x^2 + y^2 - 8x - 6y ++ 16 = 0$