CONTINUITY & DERIVABILITY

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Syllabus

Continuity of a function, limit and continuity of the sum, difference, product and quotient of two functions, continuity of composite functions, intermediate value property of continuous functions. Derivative of a function, derivative of the sum difference, product and quotient of two functions

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Definition:
If the graph of a function has no break or jump, then it is said to be continuous function. A function which is not continuous is called a discontinuous function.

Continuity of a Function at a point:
A Function $f(x)$ is said to be continuous at some point $x=a$ of its domain if

$$\lim_{x \to a} f(x) = f(a)$$

i.e., If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

i.e., If $f(a^-) = f(a^+) = f(a)$

i.e., If $\{\text{LHL at } x=a\} = \{\text{RHL at } x= a\} = \{\text{value of the function at } x = a \}$.

REMEMBER:
- In case of dis-continuity of the second kind the non-negative difference between the value of the RHL at $x = a$ & LHL at $x = a$ is called THE JUMP OF DISCONTINUITY. A function having a finite number of jumps in a given interval is called a PIECE WISE CONTINUOUS or SECTIONALLY CONTINUOUS function in this interval.
- All Polynomials, Trigonometrical functions, exponential & Logarithmic functions are continuous in their domains.

Note: From the adjacent graph note that
- $f$ is continuous at $x = -1$
- $f$ has isolated discontinuity at $x = 1$
- $f$ has missing point discontinuity at $x = 2$
- $f$ has non-removable (finite type) discontinuity at the origin.

Continuity in an Interval:
(a) A function $f$ is said to be continuous in $(a, b)$ if $f$ is continuous at each & every point $\in (a, b)$.

(b) A function $f$ is said to be continuous in a closed interval $[a, b]$ if:
   (i) $f$ is continuous in the open interval $(a, b)$,
   (ii) $f$ is right continuous at ‘a’ i.e. $\lim_{x \to a^+} f(x) = f(a) = a$ finite quantity and
   (iii) $f$ is left continuous at ‘b’ i.e. $\lim_{x \to b^-} f(x) = f(b) = a$ finite quantity.

(c) All Polynomial functions, Trigonometrical functions, Exponential and Logarithmic functions are continuous at every point of their respective domains.
On the basis of above facts continuity of a function should be checked at the following points

(i) Continuity of a function should be checked at the points where definition of a function changes.

(ii) Continuity of \( f(x) \) and \( [f(x)] \) should be checked at all points where \( f(x) \) becomes integer.

(iii) Continuity of \( \text{sgn}(f(x)) \) should be checked at the points where \( f(x) = 0 \) (if \( f(x) \neq 0 \) in any open interval containing \( a \), then \( x = a \) is not a point of discontinuity)

(iv) In case of composite function \( f(g(x)) \) continuity should be checked at all possible points of discontinuity of \( g(x) \) and at the points where \( g(x) = c \), where \( x = c \) is a possible point of discontinuity of \( f(x) \).

**Algebra on continuity :**

(i) If \( f \) & \( g \) are two functions which are continuous at \( x = c \), then the functions defined by:

\[ F_1(x) = f(x) \pm g(x) ; \quad F_2(x) = K f(x), K \text{ is any real number} ; \quad F_3(x) = f(x).g(x) \]

are also continuous at \( x = c \). Further, if \( g \) (c) is not zero, then \( F_4(x) = \frac{f(x)}{g(x)} \) is also continuous at \( x = c \).

(ii) If \( f(x) \) is continuous & \( g(x) \) is discontinuous at \( x = a \), then the product function \( \phi (x) = f(x).g(x) \) may or may not be continuous but sum or difference function \( \phi (x) = f(x) \pm g(x) \) will necessarily be discontinuous at \( x = a \).

**E.g.** \( f(x) = x \) & \( g(x) = \begin{cases} \sin \frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \)

(iii) If \( f(x) \) and \( g(x) \) both are discontinuous at \( x = a \), then the product function \( \phi (x) = f(x).g(x) \) is not necessarily be discontinuous at \( x = a \).

**E.g.** \( f(x) = g(x) = \begin{cases} 1 & , \quad x \geq 0 \\ -1 & , \quad x < 0 \end{cases} \)

and atmost one out of \( f(x) + g(x) \) and \( f(x) - g(x) \) is continuous at \( x = a \).

**Continuity of composite functions :**

If \( f \) is continuous at \( x = c \) and \( g \) is continuous at \( x = f(c) \), then the composite \( g[f(x)] \) is continuous at \( x = c \). eg. \( f(x) = \frac{x \sin x}{x^2 + 2} \) & \( g(x) = |x| \) are continuous at \( x = 0 \), hence the composite function \( (gof)(x) = \frac{x \sin x}{x^2 + 2} \) will also be continuous at \( x = 0 \).

**Intermediate Value Theorem :**

A function \( f \) which is continuous in \([a, b]\) possesses the following properties:

(i) If \( f(a) \) & \( f(b) \) possess opposite signs, then there exists at least one solution of the equation \( f(x) = 0 \) in the open interval \((a, b)\).

(ii) If \( K \) is any real number between \( f(a) \) & \( f(b) \), then there exists at least one solution of the equation \( f(x) = K \) in the open interval \((a, b)\).
**SINGLE POINT CONTINUITY:**

Functions which are continuous only at one point are said to exhibit single point continuity.

\[ e.g. f(x) = \begin{cases} 
  x & \text{if } x \in \mathbb{Q} \\
  -x & \text{if } x \notin \mathbb{Q}
\end{cases} \quad \text{and} \quad g(x) = \begin{cases} 
  x & \text{if } x \in \mathbb{Q} \\
  0 & \text{if } x \notin \mathbb{Q}
\end{cases} \]

are both continuous only at \( x = 0 \).

(Two fold meaning of derivability)

---

**Geometrical meaning of derivative**

Slope of the tangent drawn to the curve at \( x = a \) if it exists.

**Physical meaning of derivative**

(functions which are differentiable) Instantaneous rate of change of function

**Note:** "Tangent at a point 'A' is the the limiting case of secant through A."

**EXISTENCE OF DERIVATIVE:**

Right hand & Left hand Derivatives:

By definition: \[ f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \]

(i) The right hand derivative of \( f \) at \( x = a \) denoted by \( f'(a^+) \) is defined by:

\[ f'(a^+) = \lim_{h \to 0^+} \frac{f(a + h) - f(a)}{h}, \]

provided the limit exists & is finite.

(ii) The left hand derivative of \( f \) at \( x = a \) denoted by \( f'(a^-) \) is defined by:

\[ f'(a^-) = \lim_{h \to 0^-} \frac{f(a - h) - f(a)}{h}, \]

provided the limit exists & is finite.

\( f \) is said to be derivable at \( x = a \) if \( f'(a^+) = f'(a^-) = a \) finite quantity.

This geometrically means that a unique tangent with finite slope can be drawn at \( x = a \) as shown in the figure.

**Properties of differentiable functions**

(i) The sum, difference, product, quotient (Dr \( \neq 0 \)) and composite of two differentiable function is always a, differentiable function.

(ii) If a function is differentiable at some point, then it is necessarily continuous at that point, but its converse is not true. Hence

\[ \text{differentiability} \quad \Rightarrow \quad \text{continuity} \]

\[ \text{continuity} \quad \Rightarrow \quad \text{differentiability} \]

(iii) If a function is discontinuous at some point then it is not differentiable at that point.

**Graphical Definition of Derivability**

The function \( y = f(x) \) is derivable if its graph is always smooth i.e., there should be no break or corner.
THEOREM: If \( f(x) \) and \( g(x) \) both are derivable at \( x = a \), then

1. \( f(x) \pm g(x) \) will be differentiable at \( x = a \).
2. \( f(x) \cdot g(x) \) will be differentiable at \( x = a \).
3. \( \frac{f(x)}{g(x)} \) will be differentiable at \( x = a \) if \( g(a) \neq 0 \).

Note that:

(a) If \( f(x) \) is differentiable at \( x = a \) and \( g(x) \) is not differentiable at \( x = a \), then
   
   (i) Sum or difference must be non derivable at \( x = 0 \). (Can be proved by contradiction). For example note and verify that
       
       \[ f(x) = \cos |x| \text{ (which is differentiable at } x = 0) \]
       
       and \( g(x) = |x| \text{ (which is non derivable at } x = 0) \), then the functions
       
       \( \cos |x| + |x| \) and \( \cos |x| - |x| \) are not derivable at \( x = 0 \).

   (ii) However product function \( f(x) \cdot g(x) \) can be differentiable.
       
       e.g. \( f(x) = x \) and \( g(x) = |x| \); \( F(x) = f(x) \cdot g(x) \) is differentiable at \( x = 0 \).

(b) If \( f(x) \) and \( g(x) \) are both non derivable at \( x = a \), then nothing definite can be said about the sum or difference function w.r.t. differentiability.
e.g. Let \( f(x) = \sin |x| \) (not derivable at \( x = 0 \))
and \( g(x) = |x| \) (not derivable at \( x = 0 \))

then \( f(x) = \sin |x| - |x| \) is derivable at \( x = 0 \)

\[
\begin{align*}
\sin x - x & \text{ for } x \geq 0 \\
-x - x & \text{ for } x < 0
\end{align*}
\]

and \( f(x) = \sin |x| + |x| \) is not derivable at \( x = 0 \).

(ii) Similarly product of two non derivable function can be differentiable.

\( \text{e.g. } f(x) = |x| \text{ and } g(x) = -|x| \)

(c) Derivative of continuous function need not be a continuous function.

\( \text{e.g. } \) A function \( f \) is defined as

\[
f(x) = \begin{cases} 
  x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\
  0 & \text{if } x = 0
\end{cases}
\]

Show that \( f \) is differentiable at all \( x \in \mathbb{R} \). Also show that \( f' \) is not continuous at \( x = 0 \). Thus a function \( f \) that is differentiable at every point of \( \mathbb{R} \) need not have a continuous derivative.

**Differentiability over an Interval :**

\( f(x) \) is said to be differentiable over an open interval if it is differentiable at each point of the interval and \( f(x) \) is said to be differentiable over a closed interval \( [a, b] \) if:

(i) for the points \( a \) and \( b \), \( f'(a') \) and \( f'(b') \) exist finitely

(ii) for any point \( c \) such that \( a < c < b \), \( f'(c') \) & \( f'(c') \) exist finitely and are equal.

All polynomial, exponential, logarithmic and trigonometric (inverse trigonometric not included) functions are differentiable in their domain.

**Note :** Derivability should be checked at following points

(i) At all points where continuity is required to be checked.

(ii) At the critical points of modulus and inverse trigonometric function.

**Important formula :**

For finding limit \( \lim_{h \to 0} \frac{f(a+g(h))-f(a+p(h))}{g(h)-p(h)} = f'(a) \),

if \( \lim_{h \to 0} p(h) = \lim_{h \to 0} g(h) = 0 \) and \( f(x) \) is differentiable at \( x = a \).
PART - I : OBJECTIVE QUESTIONS

* Marked Questions are having more than one correct option.

Section (A) : Continuity

A-1. If \( f(x) = \frac{x}{2} \) is discontinuous at \( x = a \), then

(A) \( a \in N \)  
(B) \( a \in W \)  
(C) \( \frac{a}{2} \in Z \)  
(D) \( a \in Q \)

A-2. If \( f(x) = \begin{cases} 
  x^2, & \text{when } x \leq 0 \\
  1, & \text{when } 0 < x < 1 \\
  \frac{1}{x}, & \text{when } x \geq 1 
\end{cases} \), then \( f(x) \) is

(A) continuous at \( x = 0 \) but not at \( x = 1 \)  
(B) continuous at \( x = 1 \) but not at \( x = 0 \)  
(C) continuous at \( x = 0 \) and \( x = 1 \)  
(D) discontinuous at \( x = 0 \) and \( x = 1 \)

A-3. Function \( f(x) = \begin{cases} 
  \sin \frac{1}{x}, & x \neq 0 \\
  0, & x = 0 
\end{cases} \) is discontinuous at

(A) origin  
(B) all points  
(C) no where  
(D) only at origin

A-4. If \( f(x) = \begin{cases} 
  x^2 + 2, & \text{when } x < 1 \\
  4x - 1, & \text{when } 1 \leq x \leq 3 \\
  x^2 + 5, & \text{when } x > 3 
\end{cases} \), then correct statement is

(A) \( \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) \)  
(B) \( f(x) \) is continuous at \( x = 3 \)  
(C) \( f(x) \) is continuous at \( x = 1 \)  
(D) \( f(x) \) is continuous at \( x = 1 \) and 3

A-5. Function \( f(x) = \frac{1}{\log|x|} \) is discontinuous at

(A) one point  
(B) two points  
(C) three points  
(D) infinite number of points

A-6. If \( f(x) = \begin{cases} 
  \frac{x^2 - (a+2)x + 2a}{x^2 - 2}, & x \neq 2 \\
  \frac{1}{2}, & x = 2 
\end{cases} \), then \( a \) is equal to

(A) 0  
(B) 1  
(C) -1  
(D) 2
A-7. Determine the value of constant $k$ so that the function $f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x}, & x \neq 0 \\ \frac{\cos kx}{x^2}, & x = 0 \end{cases}$ is continuous at $x = 0$

(A) $k = \pm 2$  (B) $k = \pm 4$  (C) $k = \pm 1$  (D) None of these

A-8. Which of the following function has finite number of points of discontinuity (where $[$.] denotes greatest integer)

(A) $\tan x$  (B) $|x|/x$  (C) $x + [x]$  (D) $\sin \left[ \pi x \right]$ 

A-9. If $f(x)$ is continuous function and $g(x)$ is a discontinuous function then correct statement is

(A) $f(x) + g(x)$ is continuous function  (B) $f(x) - g(x)$ is continuous function
(C) $f(x) + g(x)$ is discontinuous function  (D) $f(x) \cdot g(x)$ is discontinuous function

A-10. Let $f(x) = \left[ \frac{x + \frac{1}{2}}{2} \right]$, when $-2 \leq x \leq 2$, where $[.]$ represents greatest integer function. Then

(A) $f(x)$ is continuous at $x = 2$  (B) $f(x)$ is continuous at $x = 1$
(C) $f(x)$ is continuous at $x = -1$  (D) $f(x)$ is discontinuous at $x = 0$

A-11. If $f(x) = \begin{cases} \tan^{-1}(\tan x); & x \leq \frac{\pi}{4} \\ \pi [x] + 1; & x > \frac{\pi}{4} \end{cases}$, then jump of discontinuity is

where $[.]$ denotes greatest integer function

(A) $\frac{\pi}{4} - 1$  (B) $\frac{\pi}{4} + 1$  (C) $1 - \frac{\pi}{4}$  (D) $1 - \frac{\pi}{4}$

A-12. If function $f(x) = \frac{\sqrt{x^2 + 1} - \sqrt[3]{x^2 + 1}}{x}$, is continuous function, then $f(0)$ is equal to

(A) 2  (B) 1/4  (C) 1/6  (D) 1/3

A-13. If function $f(x) = \frac{(27 - 2x)^{1/3} - 3}{9 - 3(243 + 5x)^{1/3}}$ is continuous at $x = 0$, then $f(0)$ is equal to

(A) 2  (B) 4  (C) 6  (D) 2/3

A-14. Function $f(x) = |\sin x| + |\cos x| + |x|$ is discontinuous at

(A) $x = 0$  (B) $x = \frac{\pi}{2}$  (C) $x = \pi$  (D) no where

A-15. Function $f(x) = 3|x - 1| - 2|x - 3|$ is discontinuous at

(A) $x = 1, 3$  (B) $x = 2$  (C) no where  (D) everywhere except $x = 1, 3$
A-16. The value of p for which the function \( f(x) = \begin{cases} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{p}\right) \log\left(1 + \frac{x^2}{3}\right)} & , x \neq 0 \\ (12) (\log 4)^3 & , x = 0 \end{cases} \) may be continuous at \( x = 0 \), is

(A) 1  
(B) 2  
(C) 3  
(D) None of these

A-17. If \( f(x) = \begin{cases} \frac{x}{k} & , x \neq 0 \\ \text{is continuous at } x = 0 , \text{ then } k \text{ is equal to} \\ \end{cases} \)

(A) 2a + b  
(B) 2a – b  
(C) b – 2a  
(D) a + b

A-18. If \( f(x) = \begin{cases} \frac{0x}{x - 3} & , x \geq 1 \\ \frac{0}{x + 3} & , x < 1 \end{cases} \), then \( f(x) \) is -

(A) continuous at \( x = 1 \) but not at \( x = 3 \)  
(B) continuous at \( x = 3 \) but not at \( x = 1 \)  
(C) continuous at \( x = 1 \) and \( x = 3 \)  
(D) discontinuous at \( x = 1 \) and \( x = 3 \)

Section (B) : Derivability

B-1. Function \( f(x) = |x| + |x - 1| \) is not differentiable at

(A) \( x = 1 \), \( -1 \)  
(B) \( x = 0 \), \( -1 \)  
(C) \( x = 0 \), \( 1 \)  
(D) \( x = 1 \), \( 2 \)

B-2. Which of the following functions is not differentiable at \( x = 0 \)

(A) \( x \cdot |x| \)  
(B) \( x^3 \)  
(C) \( e^{-x} \)  
(D) \( x + |x| \)

B-3. Let \( f(x) = x \left\{ \sqrt{x} - \sqrt{x + 1} \right\} \). Then \( f \) is -

(A) continuous but not differentiable at \( x = 0 \)  
(B) discontinuous at \( x = 0 \)  
(C) continuous and differentiable both at \( x = 0 \)  
(D) None of these

B-4. Let \( f(x) = \frac{|x|}{\sin x} \) for \( x \neq 0 \) & \( f(0) = 1 \) then,

(A) \( f(x) \) is continuous & differentiable at \( x = 0 \)  
(B) \( f(x) \) is continuous & not differentiable at \( x = 0 \)  
(C) \( f(x) \) is discontinuous & not differentiable at \( x = 0 \)  
(D) none

B-5. The function \( f(x) \) is defined as follows \( f(x) = \begin{cases} \frac{-x}{x^2} & , 0 \leq x \leq 1 \\ x^3 - x + 1 & , x > 1 \end{cases} \) then \( f(x) \) is:

(A) derivable & cont. at \( x = 0 \)  
(B) derivable at \( x = 1 \) but not continuous at \( x = 1 \)  
(C) neither derivable nor cont. at \( x = 1 \)  
(D) not derivable at \( x = 0 \) but continuous at \( x = 1 \)
B-6. If \( f(x) = \begin{cases} \frac{ax}{x^2 - bx + 3} & \text{for } x < 2 \\ \frac{ax^2 - bx + 3}{x \geq 2} \end{cases} \) be differentiable for all \( x \), then
\[
\begin{align*}
(A) & \quad a = \frac{3}{2} \quad \text{and} \quad b = \frac{9}{2} \\
(B) & \quad a = \frac{1}{4} \quad \text{and} \quad b = \frac{3}{4} \\
(C) & \quad a = \frac{3}{4} \quad \text{and} \quad b = \frac{9}{4} \\
(D) & \quad a = 3, \quad b = 9
\end{align*}
\]

B-7. If \( f(x) = \begin{cases} e^x & \text{for } x < 1 \\ a - bx & \text{for } x \geq 1 \end{cases} \) is differentiable for \( x \in \mathbb{R} \), then:
\[
\begin{align*}
(A) & \quad a = 1, \quad b = e - 1 \\
(B) & \quad a = 0, \quad b = e \\
(C) & \quad a = 0, \quad b = -e \\
(D) & \quad a = e, \quad b = 1
\end{align*}
\]

B-8. A function \( f \) defined as \( f(x) = x [x] \) for \(-1 \leq x \leq 3\) where \([x]\) defines the greatest integer \( \leq x \) is:
\[
\begin{align*}
(A) & \quad \text{continuous at all points in the domain of } f \text{ but non-derivable at a finite number of points} \\
(B) & \quad \text{discontinuous at all points & hence non-derivable at all points in the domain of } f \\
(C) & \quad \text{discontinuous at a finite number of points but not derivable at all points in the domain of } f \\
(D) & \quad \text{discontinuous & also non-derivable at a finite number of points of } f.
\end{align*}
\]

B-9. If \( f(x) = \begin{cases} x(3e^{1/x} + 4) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases} \), then \( f(x) \) is
\[
\begin{align*}
(A) & \quad \text{continuous as well differentiable at } x = 0 \\
(B) & \quad \text{continuous but not differentiable at } x = 0 \\
(C) & \quad \text{neither differentiable at } x = 0 \text{ nor continuous at } x = 0 \\
(D) & \quad \text{none of these}
\end{align*}
\]

B-10. If \( f(x) = \frac{x}{\sqrt{x + 1} - \sqrt{x}} \) be a real valued function, then
\[
\begin{align*}
(A) & \quad f(x) \text{ is continuous, but } f'(0) \text{ does not exist} \\
(B) & \quad f(x) \text{ is differentiable at } x = 0 \\
(C) & \quad f(x) \text{ is not continuous at } x = 0 \\
(D) & \quad f(x) \text{ is not differentiable at } x = 0
\end{align*}
\]

B-11. The function \( f(x) = \sin^{-1}(\cos x) \) is:
\[
\begin{align*}
(A) & \quad \text{discontinuous at } x = 0 \\
(B) & \quad \text{continuous at } x = 0 \\
(C) & \quad \text{differentiable at } x = 0 \\
(D) & \quad \text{none of these}
\end{align*}
\]

B-12. If \( f(x) = \begin{cases} \frac{x^2 - 1}{x^2 + 1} & \text{for } 0 < x \leq 2 \\ \frac{1}{4} (x^3 - x^2) & \text{for } 2 < x \leq 3 \\ \frac{9}{4} \left( |x - 4| + |2 - x| \right) & \text{for } 3 < x < 4 \end{cases} \), then:
\[
\begin{align*}
(A) & \quad f(x) \text{ is differentiable at } x = 2 \text{ & } x = 3 \\
(B) & \quad f(x) \text{ is non-differentiable at } x = 2 \text{ & } x = 3 \\
(C) & \quad f(x) \text{ is differentiable at } x = 3 \text{ but not at } x = 2 \\
(D) & \quad f(x) \text{ is differentiable at } x = 2 \text{ but not at } x = 3.
\end{align*}
\]

B-13. The number of points at which the function \( f(x) = \max \{a - x, a + x, b\}, -\infty < x < \infty, \ 0 < a < b \) cannot be differentiable is:
\[
\begin{align*}
(A) & \quad 1 \\
(B) & \quad 2 \\
(C) & \quad 3 \\
(D) & \quad \text{none of these}
\end{align*}
\]
Section (C) : Miscellaneous

C-1. If \( f(x) \) is differentiable everywhere, then:
(A) \( \frac{1}{2} f \) is differentiable everywhere
(B) \( f^{1/2} \) is differentiable everywhere
(C) \( f^{1/2} \) is not differentiable at some point
(D) \( f + \frac{1}{2} f \) is differentiable everywhere

C-2. If for all values of \( x \) & \( y \): \( f(x + y) = f(x)f(y) \) and \( f(5) = 2 \), \( f'(0) = 3 \), then \( f'(5) \) is:
(A) 3  
(B) 4  
(C) 5  
(D) 6

C-3. The derivative of every even function
(A) is always an even function  
(B) is always an odd function
(C) May be an even function  
(D) May be an odd function

PART - II : MISCELLANEOUS OBJECTIVE QUESTIONS

Comprehension # 1

Let \( f(x) = \begin{cases} \frac{1}{x}g(x) & , \ x \leq 0 \\ x + ax^2 - x^3 & , \ x > 0 \end{cases} \), where \( g(t) = \lim_{x \to 0} (1 + a \tan x)^{tx} \), \( a \) is positive constant, then

1. If \( a \) is even prime number, then \( g(2) = \)
(A) \( e^2 \)  
(B) \( e^3 \)  
(C) \( e^4 \)  
(D) none of these

2. Set of all values of \( a \) for which function \( f(x) \) is continuous at \( x = 0 \)
(A) \((-1, 10)\)  
(B) \((-\infty, \infty)\)  
(C) \((0, \infty)\)  
(D) none of these

3. If \( f(x) \) is differentiable at \( x = 0 \), then \( a \in \)
(A) \((-5, -1)\)  
(B) \((-10, 3)\)  
(C) \((0, \infty)\)  
(D) none of these

Comprehension # 2

Let \( f : \mathbb{R} \to \mathbb{R} \) be a function defined as,

\[
 f(x) = \begin{cases} 1 - |x| & , \ |x| \leq 1 \\ 0 & , \ |x| > 1 \end{cases}
\]
and \( g(x) = f(x - 1) + f(x + 1) \), \( \forall x \in \mathbb{R} \). Then

4. The value of \( g(x) \) is:
   
(A) \( g(x) = \begin{cases} 0 & , \ x \leq -3 \\ 2+x & , -3 \leq x \leq -1 \\ -x & , -1 < x \leq 0 \\ x & , 0 < x \leq 1 \\ 2-x & , 1 < x \leq 3 \\ 0 & , x > 3 \end{cases} \)

(B) \( g(x) = \begin{cases} 0 & , \ x \leq -2 \\ 2+x & , -2 \leq x \leq -1 \\ -x & , -1 < x \leq 0 \\ x & , 0 < x \leq 1 \\ 2-x & , 1 < x \leq 2 \\ 0 & , x > 2 \end{cases} \)

(C) \( g(x) = \begin{cases} 0 & , \ x \leq 0 \\ 2+x & , 0 < x < 1 \\ -x & , 1 \leq x \leq 2 \\ x & , 2 < x \leq 3 \\ 2-x & , 3 \leq x < 4 \\ 0 & , 4 \leq x \end{cases} \)

(D) none of these
5. The function g(x) is continuous for, x ∈
   (A) R – {0, 1, 2, 3, 4}   (B) R – {–2, –1, 0, 1, 2}   (C) R
   (D) none of these

6. The function g(x) is differentiable for, x ∈
   (A) R   (B) R – {–2, –1, 0, 1, 2}   (C) R – {0, 1, 2, 3, 4}
   (D) none of these

Match The column

7. Let \([\cdot]\) denotes the greatest integer function.

<table>
<thead>
<tr>
<th>Column – I</th>
<th>Column – II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) If (P(x) = [2 \cos x], x \in [-\pi, \pi]), then (P(x)) is discontinuous at exactly 7 points</td>
<td></td>
</tr>
<tr>
<td>(B) If (Q(x) = [2 \sin x], x \in [-\pi, \pi]), then (Q(x)) is discontinuous at exactly 4 points</td>
<td></td>
</tr>
<tr>
<td>(C) If (R(x) = [2 \tan x/2], x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]), then (R(x)) has non-removable discontinuities</td>
<td></td>
</tr>
<tr>
<td>(D) If (S(x) = \left[\frac{3 \csc x}{3}\right], x \in \left[\frac{\pi}{2}, 2\pi\right]), then (S(x)) is continuous at infinitely many values</td>
<td></td>
</tr>
</tbody>
</table>

8. Statement - 1 f(x) = |x| \cos x is not differentiable at x = 0
   Statement - 2 Every absolute value functions are not differentiable.
   (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
   (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (C) Statement-1 is True, Statement-2 is False
   (D) Statement-1 is False, Statement-2 is True

9. Statement - 1 f(x) = \text{Sgn} (\cos x) is not differentiable at x = \frac{\pi}{2}
   Statement - 2 \(g(x) = [\cos x]\) is not differentiable at x = \frac{\pi}{2}
   where \([\cdot]\) denotes greatest integer function
   (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
   (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (C) Statement-1 is True, Statement-2 is False
   (D) Statement-1 is False, Statement-2 is True

10. Statement - 1 \(f(x) = |x – 2| + x^2 - 5x + 6 \frac{x}{x-1} + \tan x\) is continuous function within the domain of f(x).
    Statement - 2 All absolute valued polynomial function, Rational polynomial function, trigonometric functions are continuous within their domain.
    (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
    (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
    (C) Statement-1 is True, Statement-2 is False
    (D) Statement-1 is False, Statement-2 is True
PART - I : OBJECTIVE QUESTIONS

1. A point (x, y), where function \( f(x) = \lfloor \sin x \rfloor \) in \((0, 2\pi)\) is not continuous, is \( (\lfloor . \rfloor \) denotes greatest integer \( \leq x)\).
   (A) (3, 0)  (B) (2, 0)  (C) (1, 0)  (D) (4, -1)

2. The function \( f(x) \) is defined by \( f(x) = \begin{cases} \log(4x-3)(x^2 - 2x + 5) & \text{if } \frac{3}{2} < x < 1 \text{ or } x > 1 \\ 4 & \text{if } x = 1 \end{cases} \), is continuous at \( x = 1 \)
   (A) is continuous at \( x = 1 \)
   (B) is discontinuous at \( x = 1 \) since \( f(1^+) \) does not exist though \( f(1^-) \) exists
   (C) is discontinuous at \( x = 1 \) since \( f(1^-) \) does not exist though \( f(1^+) \) exists
   (D) is discontinuous since neither \( f(1^-) \) nor \( f(1^+) \) exists.

3. Function \( f(x) = \lfloor x \rfloor \cos \left( \frac{2x-1}{2} \right) \pi \) is discontinuous at
   (A) every \( x \)  (B) no \( x \)  (C) every integral point  (D) every non-integral point

4. If \( f(x) = \begin{cases} x/2 - 1 & \text{if } 0 \leq x < 1 \\ 1/2 & \text{if } 1 \leq x < 2 \end{cases} \), \( g(x) = (2x + 1)(x - k) + 3, \ 0 \leq x < \infty \), then \( g( f(x) ) \), will be continuous
   at \( x = 1 \) if \( k \) is equal to
   (A) 1/2  (B) 1/6  (C) 11/6  (D) 13/6

5. If \( f(x) = \begin{cases} \frac{\sin[x]}{[x]} + 1, & x > 0 \\ \frac{\cos \pi [x/2]}{[x]} - 1, & x < 0 \end{cases} \), is a continuous function at \( x = 0 \), then the value of \( K \) (\( \lfloor . \rfloor \) denotes greatest integer function) is
   (A) 0  (B) 1  (C) -1  (D) None of these

6. \( f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-2}, & 0 \leq x \leq 1 \end{cases} \) is continuous in the interval \([-1, 1]\), then \( 'p' \) is equal to:
   (A) -1  (B) -1/2  (C) 1/2  (D) 1

7. Let \( f(x) = \begin{cases} \frac{1 + \cos 2\pi x}{1 - \sin \pi x}, & x < \frac{1}{2} \\ \frac{p}{\sqrt{2}x - 1}, & x = \frac{1}{2} \end{cases} \). If \( f(x) \) is discontinuous at \( x = \frac{1}{2} \), then
   (A) \( p \in \mathbb{R} - \{4\} \)  (B) \( p \in \mathbb{R} - \left\{ \frac{1}{4} \right\} \)  (C) \( p \in \mathbb{R}_0 \)  (D) \( p \in \mathbb{R} \)
8. If \( f(x) = \frac{a \cos x - \cos bx}{x^2} \), \( x \neq 0 \) and \( f(0) = 4 \) is continuous at \( x = 0 \) then the ordered pair \((a, b)\) is

(A) \((\pm 1, 3)\)  \hspace{1cm} (B) \((1, \pm 3)\)  \hspace{1cm} (C) \((-1, -3)\)  \hspace{1cm} (D) \((1, 3)\)

9. If \( f(x) = \frac{\ln(e^{x^2} + 2\sqrt{x})}{\tan \sqrt{x}} \) is continuous at \( x = 0 \), then \( f(0) \) must be equal to:

(A) 0  \hspace{1cm} (B) 1  \hspace{1cm} (C) \(e^2\)  \hspace{1cm} (D) 2

10. \( y = f(x) \) is a continuous function such that its graph passes through \((a, 0)\). Then \( \lim_{x \to a} \frac{\log_a(1 + 3f(x))}{2f(x)} \) is:

(A) 1  \hspace{1cm} (B) 0  \hspace{1cm} (C) \(\frac{3}{2}\)  \hspace{1cm} (D) \(\frac{2}{3}\)

11. Consider \( f(x) = \lim_{n \to \infty} \frac{x^n - \sin x^n}{x^n + \sin x^n} \) for \( x > 0, x \neq 1 \) \( f(1) = 0 \) then

(A) \( f \) is continuous at \( x = 1 \)
(B) \( f \) has a finite discontinuity at \( x = 1 \)
(C) \( f \) has an infinite or oscillatory discontinuity at \( x = 1 \)
(D) \( f \) has a removable type of discontinuity at \( x = 1 \)

12. Let \( f(x) = \left\{ \begin{array}{ll} x^2 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{array} \right. \) then:

(A) \( f \) is discontinuous for all \( x \)
(B) discontinuous for all \( x \) except at \( x = 0 \)
(C) discontinuous for all \( x \) except at \( x = 1 \) or \(-1\)
(D) none

13. \( f \) is a continuous function on the real line. Given that \( x^2 + (f(x) - 2)x - \sqrt{3} \cdot f(x) + 2\sqrt{3} - 3 = 0 \). Then the value of \( f(\sqrt{3}) \)

(A) cannot be determined  \hspace{1cm} (B) is \(2(1 - \sqrt{3})\)
(C) is zero  \hspace{1cm} (D) is \(\frac{2(\sqrt{3} - 2)}{\sqrt{3}}\)

14. Let \( f(x) = [2 + 3 \sin x] \) (where \([\ ]\) denotes the greatest integer function) \( x \in (0, \pi) \). Then number of points at which \( f(x) \) is discontinuous is:

(A) 0  \hspace{1cm} (B) 4  \hspace{1cm} (C) 5  \hspace{1cm} (D) infinite

15. The function \( f(x) \) is defined by \( f(x) = \begin{cases} \log_{(\sqrt{4x-3})}(x^2 - 2x + 5), & \text{if } \frac{3}{2} < x < 1 \text{ or } x > 1 \\ 4, & \text{if } x = 1 \end{cases} \)

(A) is continuous at \( x = 1 \)
(B) is discontinuous at \( x = 1 \) since \( f(1^+) \) does not exist though \( f(1^-) \) exists
(C) is discontinuous at \( x = 1 \) since \( f(1^-) \) does not exist though \( f(1^+) \) exists
(D) is discontinuous since neither \( f(1^+) \) nor \( f(1^-) \) exists.
16. The function \( f(x) = \lim_{t \to \infty} \left( \frac{(1 + \sin \pi x)^t - 1}{(1 + \sin \pi x)^t + 1} \right) \) is

(A) everywhere continuous  
(B) discontinuous at all integer values of \( x \)  
(C) continuous at \( x = 0 \)  
(D) none of these

17. If \( f(x) = \begin{cases} \sqrt{x} \left( 1 + \frac{\sin 1}{x} \right), & x > 0 \\ -\sqrt{x} \left( 1 + \frac{\sin 1}{x} \right), & x < 0 \\ 0, & x = 0 \end{cases} \), then \( f(x) \) is

(A) continuous as well differentiable at \( x = 0 \)  
(B) continuous at \( x = 0 \), but not differentiable at \( x = 0 \)  
(C) neither continuous at \( x = 0 \) nor differentiable at \( x = 0 \)  
(D) none of these

More than one choice type

18. Which of the following function(s) not defined at \( x = 0 \) has/have removable discontinuity at the origin?

(A) \( f(x) = \frac{1}{1 + 2 \cot x} \)  
(B) \( f(x) = \cos \left( \frac{\sin x}{x} \right) \)  
(C) \( f(x) = x \sin \frac{\pi}{x} \)  
(D) \( f(x) = \frac{1}{x} \)

19. Let \( f(x) = \begin{cases} \tan^2 \{x\} \left( \frac{x^2}{x^2 - [x]^2} \right)^2 & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ \sqrt{\{x\} \cot \{x\}} & \text{for } x < 0 \end{cases} \) where \([x]\) is the step up function and \( \{x\} \) is the fractional part of \( x \), then:

(A) \( \lim_{x \to 0^+} f(x) = 1 \)  
(B) \( \lim_{x \to 0^-} f(x) = 1 \)  
(C) \( \cot^{-1} \left( \lim_{x \to 0^-} \frac{f(x)}{(x - 1)} \right)^2 = 1 \)  
(D) \( f \) is continuous at \( x = 1 \)

20. If \( f(x) = \cos \left( \frac{\pi}{x} \right) \cos \left( \frac{\pi}{2} (x - 1) \right) \); where \([x]\) is the greatest integer function of \( x \), then \( f(x) \) is continuous at:

(A) \( x = 0 \)  
(B) \( x = 1 \)  
(C) \( x = 2 \)  
(D) none of these

21. The function, \( f(x) = \left[ |x| \right] - \left[ |x| \right] \) where \([x]\) denotes greatest integer function

(A) is continuous for all positive integers  
(B) is discontinuous for all non positive integers  
(C) has finite number of elements in its range  
(D) is such that its graph does not lie above the x-axis.
22. If \( f(x) = \frac{1}{2}x - 1 \), then on the interval \([0, \pi]\)

(A) \( \tan(f(x)) \) and \( \frac{1}{f(x)} \) are both continuous

(B) \( \tan(f(x)) \) and \( \frac{1}{f(x)} \) are both discontinuous

(C) \( \tan(f(x)) \) and \( f^{-1}(x) \) are both continuous

(D) \( \tan(f(x)) \) is continuous but \( \frac{1}{f(x)} \) is not.

23. Given \( f(x) = \begin{cases} 3 & \text{if } x > 0 \\ \cot^{-1}\left(\frac{2x^3 - 3}{x^2}\right) & \text{if } x < 0 \end{cases} \), where \( (\cdot.\) & \([.\)] denotes the fractional part and the integral part functions respectively, then which of the following statement does not hold good.

(A) \( f(0^-) = 0 \)

(B) \( f(0^+) = 3 \)

(C) \( f(0) = 0 \Rightarrow \) continuity of \( f \) at \( x = 0 \)

(D) \( \) irremovable discontinuity of \( f \) at \( x = 0 \)

24. Let \( f(x) = [x] + \sqrt{x - [x]} \), where \([\cdot\)] denotes the greatest integer function. Then

(A) \( f(x) \) is continuous on \( \mathbb{R}^+ \)

(B) \( f(x) \) is continuous on \( \mathbb{R} \)

(C) \( f(x) \) is continuous on \( \mathbb{R} - 1 \)

(D) discontinuous at \( x = 1 \)

25.* Which of the following function(s) not defined at \( x = 0 \) has/have non–removable discontinuity at the origin?

(A) \( f(x) = \frac{1}{1 + 2^x} \)

(B) \( f(x) = \tan^{-1}\frac{1}{x} \)

(C) \( f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} \)

(D) \( f(x) = \frac{1}{\ln|x|} \)

26.* Which of the following function(s) defined below has/have single point continuity.

(A) \( f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \not\in \mathbb{Q} \end{cases} \)

(B) \( g(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 1 - x & \text{if } x \not\in \mathbb{Q} \end{cases} \)

(C) \( h(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \not\in \mathbb{Q} \end{cases} \)

(D) \( k(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ -x & \text{if } x \not\in \mathbb{Q} \end{cases} \)

**Derivability**

27. The set of all points where the function \( f(x) = \frac{x}{1 + |x|} \) is differentiable is :

(A) \( (-\infty, \infty) \)

(B) \([0, \infty) \)

(C) \((-\infty, 0) \cup (0, \infty) \)

(D) \((0, \infty) \)

(E) none

28. The number of points at which the function, \( f(x) = |x - 0.5| + |x - 1| + \tan x \) does not have a derivative in the interval \((0, 2)\) is :

(A) 1

(B) 2

(C) 3

(D) 4

29. Let \( f(x) = x^3 \) and \( g(x) = |x| \), Then at \( x = 0 \), the composite functions

(A) \( \text{gof is derivable but fog is not} \)

(B) \( \text{fog is derivable but gof is not} \)

(C) \( \text{gof and fog both are derivable} \)

(D) \( \text{neither fog nor fog is derivable} \)

30. Which of the following functions defined below are NOT differentiable at the indicated point ?

(A) \( f(x) = \begin{cases} x^2 & \text{if } -1 \leq x < 0 \\ -x^2 & \text{if } 0 \leq x \leq 1 \end{cases} \) at \( x = 0 \)

(B) \( g(x) = \begin{cases} x & \text{if } -1 \leq x < 0 \\ \tan x & \text{if } 0 \leq x \leq 1 \end{cases} \) at \( x = 0 \)

(C) \( h(x) = \begin{cases} \sin 2x & \text{if } x \leq 0 \\ 2x & \text{if } x > 0 \end{cases} \) at \( x = 0 \)

(D) \( k(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } 1 < x \leq 2 \end{cases} \) at \( x = 1 \)
31. Number of point(s) where the function \( f(x) = (x-1)\left|x^2 - 3x + 2\right| + \sin|x| \) is non-derivable is (are)–

(A) 1  
(B) 2  
(C) 3  
(D) 4

32. \( f(x) = \begin{cases} 
  x^2 + 2x + 3 & x \leq 2 \\
  \frac{a}{\pi} \sin(\pi x) + b & x > 2 
\end{cases} \)

If \( f(x) \) is derivable \( \forall x \in \mathbb{R} \) then–

(A) \( 2a + b\pi = 7 \)  
(B) \( b + 2\pi = 3 \)  
(C) \( 2a + b\pi = 13 \)  
(D) none of these

33. Let \( f(x) = \begin{cases} 
  4x^2 + 2[x]x & \text{if } -\frac{1}{2} \leq x < 0 \\
  ax^2 - bx & \text{if } 0 \leq x \leq \frac{1}{2} 
\end{cases} \)

where \([x]\) denotes the greatest integer function. Then

(A) \( f(x) \) is continuous in \( \left[-\frac{1}{2}, \frac{1}{2}\right] \) if \( a = 4 \) and \( b = 0 \).

(B) \( f(x) \) is continuous and differentiable in \( \left[-\frac{1}{2}, \frac{1}{2}\right] \) if \( f(a) = 4, b = 2 \).

(C) \( f(x) \) is continuous and differentiable in \( \left[-\frac{1}{2}, \frac{1}{2}\right] \) for all \( a \), provided \( b = 2 \).

(D) for no choice of \( a \) and \( b \), \( f(x) \) is differentiable in \( \left[-\frac{1}{2}, \frac{1}{2}\right] \).

34. Identify the correct statement.

(A) If \( f(x) \) is derivable at \( x = a \), \( f(x) \) will also be derivable at \( x = a \).

(B) If \( f(x) \) is discontinuous at \( x = a \), \( f(x) \) will also be discontinuous at \( x = a \).

(C) If \( f(x) \) is continuous at \( x = a \), \( f(x) \) too will be continuous at \( x = a \).

(D) If \( f(x) \) is continuous at \( x = a \), \( f(x) \) too will be continuous at \( x = a \).

35. Consider \( f(x) = \frac{2(\sin x - \sin^3 x) + \sin x - \sin^3 x}{2(\sin x - \sin^3 x) - \sin x - \sin^3 x} \), \( x \neq \frac{\pi}{2} \) for \( x \in (0, \pi) \) if \( f(\pi/2) = 3 \) where \([\ ]\) denotes the greatest integer function then,

(A) \( f \) is continuous & differentiable at \( x = \pi/2 \)

(B) \( f \) is continuous but not differentiable at \( x = \pi/2 \)

(C) \( f \) is neither continuous nor differentiable at \( x = \pi/2 \)

(D) none of these

36. If \( f(x) = \begin{cases} 
  x + \{x\} + x\sin\{x\} & \text{for } x \neq 0 \\
  0 & \text{for } x = 0 
\end{cases} \), where \( \{.\} \) denotes the fractional part function, then:

(A) \( f \) is continuous & differentiable at \( x = 0 \)  
(B) \( f \) is continuous but not differentiable at \( x = 0 \)  
(C) \( f \) is continuous & differentiable at \( x = 2 \)  
(D) none of these.
37. Given \( f(x) = \begin{cases} \log_a |x| + [-x]|^y & \text{for } x \neq 0; a > 1 \\ 0 & \text{for } x = 0 \end{cases} \)

where \([.]\) represents the integral part function, then:

(A) \( f \) is continuous but not differentiable at \( x = 0 \)

(B) \( f \) is continuous & differentiable at \( x = 0 \)

(C) the differentiability of 'f' at \( x = 0 \) depends on the value of \( a \)

(D) \( f \) is continuous & differentiable at \( x = 0 \) and for \( a = e \) only.

38. If \( f: \mathbb{R} \to \mathbb{R} \) be a differentiable function, such that \( f(x + 2y) = f(x) + f(2y) + 4xy \forall x, y \in \mathbb{R} \), then

(A) \( f'(1) = f'(0) + 1 \)

(B) \( f'(1) = f'(0) - 1 \)

(C) \( f'(0) = f'(1) + 2 \)

(D) \( f'(0) = f'(1) - 2 \)

39. Given \( f(x) = \begin{cases} \frac{x^2 e^{2(x-1)}}{4} & \text{for } 0 \leq x \leq 1 \\ a \text{sgn}(x+1) \cos(2x-2) + bx^2 & \text{for } 1 < x \leq 2 \end{cases} \)

\( f(x) \) is differentiable at \( x = 1 \) provided:

(A) \( a = -1, b = 2 \)

(B) \( a = 1, b = -2 \)

(C) \( a = -3, b = 4 \)

(D) \( a = 3, b = -4 \)

40. Let \( f''(x) \) be continuous at \( x = 0 \) and \( f''(0) = 4 \) then value of \( \lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} \) is

(A) 11

(B) 2

(C) 12

(D) none of these

41. Let \( f: \mathbb{R} \to \mathbb{R} \) be a function such that \( f \left( \frac{x + y}{3} \right) = \frac{f(x) + f(y)}{3} \), \( f(0) = 0 \) and \( f'(0) = 3 \), then

(A) \( \frac{f(x)}{x} \) is differentiable in \( \mathbb{R} \)

(B) \( f(x) \) is continuous but not differentiable in \( \mathbb{R} \)

(C) \( f(x) \) is continuous in \( \mathbb{R} \)

(D) \( f(x) \) is bounded in \( \mathbb{R} \)

42. Suppose that \( f \) is a differentiable function with the property that \( f(x + y) = f(x) + f(y) + xy \) and

\( \lim_{h \to 0} \frac{1}{h} f(h) = 3 \), then

(A) \( f \) is a linear function

(B) \( f(x) = 3x + x^2 \)

(C) \( f(x) = 3x + \frac{x^2}{2} \)

(D) none of these

43. If a differentiable function \( f \) satisfies \( f \left( \frac{x + y}{3} \right) = \frac{4 - 2(f(x) + f(y))}{3} \) \( \forall x, y \in \mathbb{R} \), then \( f(x) \) is equal to

(A) \( \frac{1}{7} \)

(B) \( \frac{2}{7} \)

(C) \( \frac{8}{7} \)

(D) \( \frac{4}{7} \)
More than one choice

44. \( f(x) = |x| + \sin x \left[ \frac{\pi}{2} \right] \). It is:

(A) continuous nowhere
(B) continuous everywhere
(C) differentiable nowhere
(D) differentiable everywhere except at \( x = 0 \)

45. The function \( f: \mathbb{R} - \{-1, 1\} \to \mathbb{R}, f(x) = \frac{x}{1-|x|} \) is such that:

(A) it is continuous at the origin
(B) it is not derivable at the origin
(C) the range of the function is \( \mathbb{R} \)
(D) it is increasing everywhere in its domain

46. If \( f(x) = \sum_{k=0}^{n} a_k |x|^k \), where \( a_k \)'s are real constants, then \( f(x) \) is

(A) continuous at \( x = 0 \) for all \( a_k \)
(B) differentiable at \( x = 0 \) for all \( a_k \in \mathbb{R} \)
(C) differentiable at \( x = 0 \) for all \( a_{2k+1} = 0 \)
(D) none of these

47. Given that the derivative \( f'(a) \) exists. Indicate which of the following statement(s) is/are always True

(A) \( f'(a) = \lim_{h \to 0} \frac{f(h) - f(a)}{h} \)
(B) \( f'(a) = \lim_{h \to 0} \frac{f(a) - f(a-h)}{h} \)
(C) \( f'(a) = \lim_{t \to 0} \frac{f(a+2t) - f(a)}{2t} \)
(D) \( f'(a) = \lim_{t \to 0} \frac{f(a+2t) - f(a+t)}{2t} \)

48. Let \([x]\) denote the greatest integer less than or equal to \( x \). If \( f(x) = [x \sin \pi x] \), then \( f(x) \) is:

(A) continuous at \( x = 0 \)
(B) continuous in \((-1, 0)\)
(C) differentiable at \( x = 1 \)
(D) differentiable in \((-1, 1)\)

49. If \( f(x) = 3 \cdot (2x+3)^{2/3} + 2x + 3 \) then,

(A) \( f(x) \) is continuous but not differentiable at \( x = -3/2 \)
(B) \( f(x) \) is differentiable at \( x = 0 \)
(C) \( f(x) \) is continuous at \( x = 0 \)
(D) \( f(x) \) is differentiable but not continuous at \( x = -3/2 \)

50. Let \( f(x) \) be defined in \([-2, 2] \) by \( f(x) = \begin{cases} \max (4 - x^2, 1 + x^2) & -2 \leq x \leq 0 \\ \min (4 - x^2, 1 + x^2) & 0 < x \leq 2 \end{cases} \) then \( f(x) \)

(A) is continuous at all points.
(B) has a point of discontinuity
(C) is not differentiable only at one point
(D) is not differentiable at more than one point

51.* \( f(x) = (\sin^{-1}x)^2 \cdot \cos (1/x) \) if \( x \neq 0; f(0) = 0 \), \( f(x) \) is:

(A) continuous nowhere in \(-1 \leq x \leq 1 \)
(B) continuous everywhere in \(-1 \leq x \leq 1 \)
(C) differentiable nowhere in \(-1 \leq x \leq 1 \)
(D) differentiable everywhere in \(-1 < x < 1 \)

52. If \( f(x) = \frac{1}{2} x - 1 \), then on the interval \([0, \pi]\)

(A) \( \tan (f(x)) \) and \( \frac{1}{f(x)} \) are both continuous
(B) \( \tan (f(x)) \) and \( \frac{1}{f(x)} \) are both discontinuous
(C) \( \tan (f(x)) \) and \( f^{-1}(x) \) are both continuous
(D) \( \tan (f(x)) \) is continuous but \( \frac{1}{f(x)} \) is not.
53. Let \( f(x) \) and \( g(x) \) be defined by \( f(x) = \lfloor x \rfloor \) and \( g(x) = \begin{cases} 0 & \text{, } x \in I \\ x^2 & \text{, } x \in \mathbb{R} - I \end{cases} \) (where \( \lfloor . \rfloor \) denotes the greatest integer function), then

(A) \( \lim_{x \to 1^-} g(x) \) exists, but \( g \) is not continuous at \( x = 1 \)

(B) \( \lim_{x \to 1^+} f(x) \) does not exist and \( f \) is not continuous at \( x = 1 \)

(C) \( \text{g} \circ \text{f} \) is continuous for all \( x \)

(D) \( \text{f} \circ \text{g} \) is continuous for all \( x \)

54. Given \( f(x) = \begin{cases} 3 - \cot^{-1} \left( \frac{2x^3 - 3}{x^2} \right) & , \ x > 0 \\ \{x\} & , \ x < 0 \end{cases} \), where \( \{ . \} & \lfloor . \rfloor \) denotes the fractional part and the integral part functions respectively, then which of the following statement does not hold good.

(A) \( f(0^-) = 0 \)

(B) \( f(0^+) = 3 \)

(C) \( f(0) = 0 \Rightarrow \text{continuity of } f \text{ at } x = 0 \)

(D) \( \text{irremovable discontinuity of } f \text{ at } x = 0 \)

55. Let \( f(x) = \lfloor x \rfloor + \sqrt{x - \lfloor x \rfloor} \), where \( \lfloor . \rfloor \) denotes the greatest integer function. Then

(A) \( f(x) \) is continuous on \( \mathbb{R}^+ \)

(B) \( f(x) \) is continuous on \( \mathbb{R} \)

(C) \( f(x) \) is continuous on \( \mathbb{R} - I \)

(D) discontinuous at \( x = 1 \)

**PART - II : SUBJECTIVE QUESTIONS**

1. State the type of discontinuity of the following functions at \( x = 0 \).

   (i) \( f(x) = \frac{1}{1 + 2 \cos x} \)

   (ii) \( f(x) = \cos \left( \frac{|\sin x|}{x} \right) \)

   (iii) \( f(x) = x \sin \frac{\pi}{x} \)

   (iv) \( f(x) = \frac{1}{|n| x} \)

2. Determine the values of \( a, b \) & \( c \) for which the function \( f(x) = \begin{cases} \frac{\sin (a + 1)x + \sin x}{x} & \text{for } x < 0 \\ c & \text{for } x = 0 \text{ is continuous} \\ \frac{(x + bx^2)^{3/2} - x^{3/2}}{bx^{3/2}} & \text{for } x > 0 \end{cases} \)

   at \( x = 0 \).

3. Let \( f(x) = \begin{cases} 1 + x & , 0 \leq x \leq 2 \\ 3 - x & , 2 < x \leq 3 \end{cases} \). Determine the composite function \( g(x) = f(f(x)) \) & hence find the point of discontinuity of \( g \), if any.

4. Find the point of discontinuity of \( y = f(u) \), where \( f(u) = \frac{3}{2u^2 + 5u - 3} \) and \( u = \frac{1}{x + 2} \).

5. Test the continuity & differentiability of the function defined as under at \( x = 1 \) & \( x = 2 \).

   \( f(x) = \begin{cases} x & , x < 1 \\ 2 - x & , 1 \leq x \leq 2 \\ -2 + 3x - x^2 & , x > 2 \end{cases} \)

6. Examine the differentiability of \( f(x) = \sqrt{1 - e^{-x^2}} \) at \( x = 0 \).

7. If \( f(x) = \begin{cases} ax^2 - b & \text{if } |x| < 1 \\ -\frac{1}{|x|} & \text{if } |x| \geq 1 \end{cases} \) is derivable at \( x = 1 \). Find the values of \( a \) & \( b \).
8. A function $f$ is defined as follows: 
\[ f(x) = \begin{cases} 
1 & \text{for } -\infty < x < 0 \\
1 + \sin x & \text{for } 0 \leq x < \frac{\pi}{2} \\
2 + \left(x - \frac{\pi}{2}\right)^2 & \text{for } \frac{\pi}{2} \leq x < \infty 
\end{cases} \]
Discuss the continuity & differentiability at $x = 0$ & $x = \pi/2$.

9. Show that the function 
\[ f(x) = \begin{cases} 
\sin \left(\frac{1}{x}\right) & \text{if } x > 0 \\
n & \text{if } x = 0
\end{cases} \]
is,
(i) differentiable at $x = 0$, if $m > 1$.
(ii) continuous but not differentiable at $x = 0$, if $0 < m \leq 1$.
(iii) neither continuous nor differentiable, if $m \leq 0$.

10. If $f(x)$ is derivable at $x = 3$ & $f'(3) = 2$, then 
\[ \lim_{h \to 0} \frac{f(3 + h^2) - f(3 - h^2)}{2h^2} = \]

11. If $f(x) = |\sin x|$ & $g(x) = x^3$ then $f[g(x)]$ is _____ & _____at $x = 0$. (State continuity and derivability)

12. Let $f(x)$ be a function satisfying the condition $f(-x) = f(x)$ for all real $x$. If $f(0)$ exists, then its value is ________.

13. Draw a graph of the function, $y = [x] + |1 - x|$, $-1 \leq x \leq 3$. Determine the points, if any, where this function is not differentiable, where $[ ]$ denotes the greatest integer function.

14. Discuss the continuity & derivability of 
\[ f(x) = \begin{cases} 
x - \frac{1}{2} & \text{if } 0 \leq x < 1 \\
x \cdot [x] & \text{if } 1 \leq x \leq 2
\end{cases} \]
where $[x]$ indicates the greatest integer not greater than $x$.

15. If $f'(2) = 4$ then, evaluate 
\[ \lim_{x \to 0} \frac{f(1 + \cos x) - f(2)}{\tan^2 x} \]

16. Discuss the continuity on $0 \leq x \leq 1$ & differentiability at $x = 0$ for the function.
\[ f(x) = x \sin \frac{1}{x} \sin \frac{1}{x \sin \frac{1}{x}} \] where $x \neq 0$, $x \neq \frac{1}{r\pi}$ & $f(0) = f \left(\frac{1}{r\pi}\right) = 0$, $r = 1, 2, 3, \ldots$

17. Discuss the continuity & differentiability of the function $f(x) = |\sin x| + \sin |x|$, $x \in \mathbb{R}$. Draw a rough sketch of the graph of $f(x)$. Also comment on periodicity of function $f(x)$.

18. The function $f$ is defined by $y = f(x)$. Where $x = 2t - |t|$, $y = t^2 + t \cdot |t|$, $t \in \mathbb{R}$. Draw the graph of $f$ for the interval $-1 \leq x \leq 1$. Also discuss its continuity & differentiability at $x = 0$.

19. Let $R$ be the set of real numbers and $f: \mathbb{R} \to \mathbb{R}$ be such that for all $x$ & $y$ in $\mathbb{R}$ $|f(x) - f(y)| \leq |x - y|^3$. Prove that $f(x)$ is constant.

20. If $f(x) = \left\{ \begin{array}{ll} 
\frac{\sin^2 x}{x} + ax + b & \text{if } 0 \leq x \leq 1 \\
2\cos x + x^3 & \text{if } 1 \leq x \leq 2 
\end{array} \right.$ is differentiable in $[0, 2]$, find "a" and "b". Here $[ \cdot ]$ stands for the greatest integer function.
PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

1. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be any function. Define \( g : \mathbb{R} \rightarrow \mathbb{R} \) by \( g(x) = |f(x)| \) for all \( x \). Then \( g \) is
(A) onto if \( f \) is onto
(B) one one if \( f \) is one one
(C) continuous if \( f \) is continuous
(D) differentiable if \( f \) is differentiable.

[JEE 2000, Screening, 1 out of 35]

2. Discuss the continuity and differentiability of the function, \( f(x) = \frac{x}{1+|x|}, |x| \geq 1 \) and \( f(x) = \frac{x}{1-|x|}, |x| < 1 \).

[REE, 2000 (3)]

3. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be a function defined by , \( f(x) = \max \{x, x^2\} \). The set of all points where \( f(x) \) is NOT differentiable is :
(A) \( [-1, 1] \)  
(B) \( (-1, 0) \)  
(C) \( [0, 1] \)  
(D) \( (-1, 0, 1) \)

[JEE 2001 (Screening)]

4. The left hand derivative of \( f(x) = [x] \sin (\pi x) \) at \( x = k \) an integer is :
(A) \( (-1)^k (k-1) \pi \)  
(B) \( (-1)^{k-1} (k-1) \pi \)  
(C) \( (-1)^k k \pi \)  
(D) \( (-1)^{k-1} k \pi \)

5. Which of the following functions is differentiable at \( x = 0 \) ?
(A) \( \cos ([x]+|x|) \)  
(B) \( \cos |x| - |x| \)  
(C) \( \sin ([x]+|x|) \)  
(D) \( \sin |x| - |x| \)

6. Let \( \alpha \in \mathbb{R} \). Prove that a function \( f : \mathbb{R} \rightarrow \mathbb{R} \) is differentiable at \( \alpha \) if and only if there is a function \( g : \mathbb{R} \rightarrow \mathbb{R} \) which is continuous at \( \alpha \) and satisfies \( f(x) - f(\alpha) = g(x)(x-\alpha) \) for all \( x \in \mathbb{R} \).

[JEE 2001, (mains)5 out of 100]

7. The domain of the derivative of the function \( f(x) = \left\{ \begin{array}{ll} \tan^{-1} x & \text{if } |x| \leq 1 \\ \frac{1}{2}|x|-1 & \text{if } |x| > 1 \end{array} \right. \) is
(A) \( \mathbb{R} - \{0\} \)  
(B) \( \mathbb{R} - \{1\} \)  
(C) \( \mathbb{R} - \{-1\} \)  
(D) \( \mathbb{R} - (-1, 1) \)

[JEE 2002, (Screening), 3]

8. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be such that \( f(1)=3 \) and \( f'(1) = 6 \). The Limit \( \lim_{x \to 0} \left( \frac{f(x)}{1!} \right)^{1/x} \) equals
(A) 1  
(B) \( e^{1/2} \)  
(C) \( e^2 \)  
(D) \( e^3 \)

[JEE 2002, (Screening),3]

9. Let \( f(x) = \left\{ \begin{array}{ll} x+a & \text{if } x < 0 \\ x-1 & \text{if } x \geq 0 \end{array} \right. \) and \( g(x) = \left\{ \begin{array}{ll} x+1 & \text{if } x < 0 \\ (x-1)^2 + b & \text{if } x \geq 0 \end{array} \right. \)

Where \( a \) and \( b \) are non negative real numbers. Determine the composite function \( g \circ f \). If \( (g \circ f)(x) \) is continuous for all real \( x \), determine the values of \( a \) and \( b \). Further, for these values of \( a \) and \( b \), is \( g \circ f \) differentiable at \( x = 0 \)? Justify your answer.

[JEE 2002 (Mains), 5 out of 60]
10. Given \( f'(2) = 6 \) and \( f'(1) = 4 \), \( \lim_{{h \to 0}} \frac{f(2h + 2 + h^2) - f(2)}{f(h^2 + 1) - f(1)} \) is equal to -  

(JEE 2003 (Screening), 3)

- (A) \( \frac{3}{2} \)
- (B) \( 3 \)
- (C) \( \frac{5}{2} \)
- (D) \( -3 \)

11. If a function \( f: [-2a, 2a] \to \mathbb{R} \) is an odd function such that \( f(x) = \frac{a}{2} - x \), for \( x \in [a, 2a] \) and the left hand derivative at \( x = a \) is 0 then find the left hand derivative at \( x = -a \)  

(JEE 2003 (Mains), 2 out of 60)

12. If the function \( f \) is differentiable and strictly increasing in a neighbourhood of 0, then \( \lim_{{x \to 0}} \frac{f(x^2) - f(x)}{f(x) - f(0)} \) is equal to-  

(JEE 2004 (Screening), 3)

- (A) \(-1\)
- (B) \(0\)
- (C) \(1\)
- (D) \(\frac{3}{2}\)

13. The function given by \( y = |x| - 1 \) is differentiable for all real numbers except the points  

(JEE 2005 (Screening), 3)

- (A) \(\{0, 1, -1\}\)
- (B) \(\pm1\)
- (C) \(1\)
- (D) \(-1\)

14. \( f(x) \) is continuous and differentiable function and \( f\left(\frac{1}{n}\right) = 0 \) \( \forall n \geq 1, n \in \mathbb{N} \), then  

(JEE 2005 (Screening), 3)

- (A) \( f(x) = 0, \forall x \in (0, 1] \)
- (B) \( f(0) = 0, f'(0) = 0 \)
- (C) \( f'(0) = 0 = f''(0), \forall x \in (0, 1] \)
- (D) \( f(0) = 0 \) and \( f'(0) \) need not to be zero

15. If \( |f(x_1) - f(x_2)| < (x_1 - x_2)^2 \), for all \( x_1, x_2 \in \mathbb{R} \). Find the equation of tangent to the curve \( y = f(x) \) at the point \((1, 2)\).  

(JEE 2005 (Mains), 2 out of 60)

16. If \( f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x) \) and \( g(x) \) and \( g(x) \) are \( g(x) \) and \( g(x + y) = g(x) \cdot g(y) + f(x) \cdot f(y) \) for all \( x, y \in \mathbb{R} \). If right hand derivative at \( x = 0 \) exists for \( f(x) \). Find derivative of \( g(x) \) at \( x = 0 \)  

(JEE 2005 (Mains), 4 out of 60)

17. If \( f(x) = \min \{1, x^2, x^3\} \), then  

(JEE 2006 (5, -1) out of 184)

- (A) \( f(x) \) is continuous \( \forall x \in \mathbb{R} \)
- (B) \( f'(x) > 0, \forall x > 1 \)
- (C) \( f(x) \) is not differentiable but continuous \( \forall x \in \mathbb{R} \)
- (D) \( f(x) \) is not differentiable but for two values of \( x \)

18. Match the Column  
Here \([x]\) denotes the greatest integer less than or equal to \( x \).  

(JEE 2007 (6, 0) out of 81)

<table>
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<tr>
<th>Column I</th>
<th>Column II</th>
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<td>(D) (</td>
<td>x - 1</td>
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</tbody>
</table>
19. Let $f$ and $g$ be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0$, $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.

IIT-JEE 2008, P-2, (3, -1), 82

**STATEMENT - 1:** \[
\lim_{x \to 0} \left[ g(x) \cot x - g(0) \cosec x \right] = f''(0)
\]

and

**STATEMENT-2:** $f(0) = g(0)$

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

20. Let $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}; 0 < x < 2$, $m$ and $n$ are integers, $m \neq 0$, $n > 0$, and let $p$ be the left hand derivative of $|x - 1|$ at $x = 1$. If $\lim_{x \to 1^-} g(x) = p$, then

IIT-JEE 2008, P-2, (3, -1), 82

(A) $n = 1, m = 1$
(B) $n = 1, m = -1$
(C) $n = 2, m = 2$
(D) $n > 2, m = n$

21. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that

$$f(x + y) = f(x) + f(y), \quad \forall \ x, y \in \mathbb{R}$$

If $f(x)$ is differentiable at $x = 0$, then

(A) $f(x)$ is differentiable only in a finite interval containing zero
(B) $f(x)$ is continuous $\forall \ x \in \mathbb{R}$
(C) $f'(x)$ is constant $\forall \ x \in \mathbb{R}$
(D) $f(x)$ is differentiable except at finitely many points

IIT-JEE 2011, P-1, 55

22. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If

$$\int_{1}^{x} f'(t)dt = 3x f(x) - x^3$$

for all $x \geq 1$, then the value of $f(2)$ is

IIT-JEE 2011, P-1, 69

23. If

$$f(x) = \begin{cases} 
-x - \frac{\pi}{2} & x \leq -\frac{\pi}{2} \\
-\cos x & -\frac{\pi}{2} < x \leq 0 \\
x - 1 & 0 < x \leq 1 \\
\ln x & x > 1 
\end{cases}$$

IIT-JEE 2011, P-2, 51

(A) $f(x)$ is continuous at $x = -\frac{\pi}{2}$
(B) $f(x)$ is not differentiable at $x = 0$
(C) $f(x)$ is differentiable at $x = 1$
(D) $f(x)$ is differentiable at $x = -\frac{3}{2}$
PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. If \( f(x) = \begin{cases} \frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases} \), then \( f(x) \) is:
   (1) continuous as well as differentiable for all \( x \)
   (2) continuous for all \( x \) but not differentiable at \( x = 0 \)
   (3) neither differentiable nor continuous at \( x = 0 \)
   (4) discontinuous everywhere

2. Let \( f(x) = \frac{1 - \tan x}{4x - \pi}, \quad x \neq \frac{\pi}{4}, \quad x \in \left(0, \frac{\pi}{2}\right) \) and \( f(x) \) is continuous in \( \left(0, \frac{\pi}{2}\right) \), then \( f\left(\frac{\pi}{4}\right) \) is equal to
   (1) \(-\frac{1}{2}\)  
   (2) \(\frac{1}{2}\)  
   (3) 1  
   (4) \(-1\)

3. If \( f \) is a real-valued differentiable function satisfying \( |f(x) - f(y)| \leq (x - y)^2, \quad x, y \in \mathbb{R} \) and \( f(0) = 0 \), then \( f(1) \) equals:
   (1) 1  
   (2) 2  
   (3) 0  
   (4) \(-1\)

4. Suppose \( f(x) \) is differentiable at \( x = 1 \) and \( \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 5 \), then \( f'(1) \) equals:
   (1) 6  
   (2) 5  
   (3) 4  
   (4) 3

5. The set of points, where \( f(x) = \frac{x}{1+|x|} \) is differentiable, is:
   (1) \((-\infty, -1) \cup (-1, \infty)\)  
   (2) \((-\infty, \infty)\)  
   (3) \((0, \infty)\)  
   (4) \((-\infty, 0) \cup (0, \infty)\)

6. The function \( f : \mathbb{R} - \{0\} \to \mathbb{R} \) given by \( f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1} \) can be made continuous at \( x = 0 \) by defining \( f(0) \) as
   (1) 2  
   (2) \(-1\)  
   (3) 0  
   (4) 1

7. Let \( f(x) = \min \{x+1, |x|+1\} \). Then which of the following is true?
   (1) \( f(x) \geq 1 \) for all \( x \in \mathbb{R} \)  
   (2) \( f(x) \) is not differentiable at \( x = 1 \)  
   (3) \( f(x) \) is differentiable everywhere  
   (4) \( f(x) \) is not differentiable at \( x = 0 \)

8. Let \( f(x) = \begin{cases} \sin \frac{1}{x-1}, & x \neq 1 \\ 0, & x = 1 \end{cases} \) Then which one of the following is true?
   (1) \( f \) is differentiable at \( x = 0 \) and at \( x = 1 \)  
   (2) \( f \) is differentiable at \( x = 0 \) but not at \( x = 1 \)  
   (3) \( f \) is differentiable at \( x = 1 \) but not at \( x = 0 \)  
   (4) \( f \) is neither differentiable at \( x = 0 \) nor at \( x = 1 \)

9. Let \( f(x) = x|x| \) and \( g(x) = \sin x \)
   \( \text{Statement-1:} \) \( g\circ f \) is differentiable at \( x = 0 \) and its derivative is continuous at that point.
   \( \text{Statement-2:} \) \( g\circ f \) is twice differentiable at \( x = 0 \).
   (1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
   (2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
   (3) Statement-1 is True, Statement-2 is False.
   (4) Statement-1 is False, Statement-2 is True.
NCERT BOARD QUESTIONS

1. Examine the continuity of the function
   \[ f(x) = x^2 + 2x^2 - 1 \] at \( x = 1 \)

   Find the which of the functions in Exercises 2 to 10 is continuous or discontinuous at the indicated points:

2. \[ f(x) = \begin{cases} 3x + 5, & \text{if } x \geq 2 \\ x^2, & \text{if } x < 2 \end{cases} \] at \( x = 2 \)

3. \[ f(x) = \begin{cases} 1 - \cos 2x, & \text{if } x \neq 0 \\ 5, & \text{if } x = 0 \end{cases} \] at \( x = 0 \)

4. \[ f(x) = \begin{cases} 2x^2 - 3x - 2, & \text{if } x \neq 2 \\ 5, & \text{if } x = 2 \end{cases} \] at \( x = 2 \)

5. \[ f(x) = \begin{cases} |x - 4|, & \text{if } x \neq 4 \\ 2(x - 4), & \text{if } x = 4 \end{cases} \] at \( x = 4 \)

6. \[ f(x) = \begin{cases} |x| \cos \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases} \] at \( x = 0 \)

7. \[ f(x) = \begin{cases} |x - a| \sin \frac{1}{x - a}, & \text{if } x \neq a \\ 0, & \text{if } x = a \end{cases} \] at \( x = a \)

8. \[ f(x) = \begin{cases} \frac{1}{e^x}, & \text{if } x \neq 0 \\ 1 + e^x, & \text{if } x = 0 \end{cases} \] at \( x = 0 \)

9. \[ f(x) = \begin{cases} \frac{x^2}{2}, & \text{if } 0 \leq x \leq 1 \\ 2x^2 - 3x + \frac{3}{2}, & \text{if } 1 < x \leq 2 \end{cases} \] at \( x = 1 \)

10. \( f(x) = |x| + |x - 1| \) at \( x = 1 \)

Find the value of \( k \) in each of the Exercises 11 to 14 so that the function \( f \) is continuous at the indicated point:

11. \( f(x) = \begin{cases} 3x - 8, & \text{if } x \leq 5 \\ 2k, & \text{if } x > 5 \end{cases} \) at \( x = 5 \)

12. \( f(x) = \begin{cases} 2^{x-2} - 16, & \text{if } x \neq 2 \\ 4^{x-16}, & \text{if } x = 2 \end{cases} \) at \( x = 2 \)

13. \( f(x) = \begin{cases} \sqrt{1 + kx} - \sqrt{1 - kx}, & \text{if } -1 \leq x < 0 \\ \frac{2x + 1}{x^2 + 1}, & \text{if } 0 \leq x \leq 1 \end{cases} \) at \( x = 0 \)

14. \( f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases} \) at \( x = 0 \)

15. Prove that the function \( f \) defined by

   \[ f(x) = \begin{cases} \frac{x}{|x| + 2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \]

   remains discontinuous at \( x = 0 \), regardless the choice of \( k \).
16. Find the values of $a$ and $b$ such that the function $f$ defined by

$$f(x) = \begin{cases} 
  \frac{x - 4}{|x - 4|} + a, & \text{if } x < 4 \\
  a + b, & \text{if } x = 4 \\
  \frac{x - 4}{|x - 4|}, & \text{if } x > 4
\end{cases}$$

is a continuous function at $x = 4$.

17. Given the function $f(x) = \frac{1}{x + 2}$. Find the points of discontinuity of the composite function $y = f(f(x))$.

18. Find all points of discontinuity of the function $f(t) = \frac{1}{t^2 + t - 2}$, where $t = \frac{1}{x - 1}$.

19. Show that the function $f(x) = |\sin x + \cos x|$ is continuous at $x = \pi$. Examine the differentiability of $f$, where $f$ is defined by

$$f(x) = \begin{cases} 
  [x], & \text{if } 0 \leq x < 2 \\
  (x - 1)x, & \text{if } 2 \leq x < 3
\end{cases} \text{ at } x = 2.$$

20. $f(x) = \begin{cases} 
  x[x], & \text{if } 0 \leq x < 2 \\
  (x - 1)x, & \text{if } 2 \leq x < 3
\end{cases} \text{ at } x = 2.$

21. $f(x) = \begin{cases} 
  x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\
  0, & \text{if } x = 0
\end{cases} \text{ at } x = 0.$

22. $f(x) = \begin{cases} 
  1 + x, & \text{if } x \leq 2 \\
  5 - x, & \text{if } x > 2
\end{cases} \text{ at } x = 2.$

23. Show that $f(x) = |x - 5|$ is continuous but not differentiable at $x = 5$.

24. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies the equation $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$, $f(x) \neq 0$. Suppose that the function is differentiable at $x = 0$ and $f'(0) = 2$. Prove that $f'(x) = 2f(x)$.

25. Find the values of $p$ and $q$ so that

$$f(x) = \begin{cases} 
  x^2 + 3x + p, & \text{if } x \leq 1 \\
  qx + 2, & \text{if } x > 1
\end{cases}$$

is differentiable at $x = 1$.  

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EXERCISE # 1

PART # I

B-1. (C) B-2. (D) B-3. (C) B-4. (C) B-5. (D) B-6. (C) B-7. (C)
B-8. (D) B-9. (B) B-10. (B) B-11. (B) B-12. (B) B-13. (B)
C-1. (B) C-2. (D) C-3. (B)

PART # II

1. (C) 2. (C) 3. (C) 4. (B) 5. (C) 6. (B)
7. (A) → (p, r, s), (B) → (p, r, s), (C) → (q, r, s), (D) → (r, s)
10. (A)

EXERCISE # 2

PART# I

1. (D) 2. (D) 3. (B) 4. (A) 5. (A) 6. (B) 7. (A)
8. (B) 9. (D) 10. (C) 11. (B) 12. (C) 13. (B) 14. (C)
15. (D) 16. (B) 17. (B) 18. (B, C, D) 19. (A, C) 20. (B, C)
26. (B, C, D) 27. (A) 28. (C) 29. (C) 30. (D) 31. (B)
32. (D) 33. (C) 34. (C) 35. (A) 36. (D) 37. (B) 38. (D)
39. (A) 40. (C) 41. (C) 42. (C) 43. (D) 44. (B, D) 45. (A, C, D)
46. (A, C) 47. (A, B) 48. (A, B, D) 49. (A, B, C) 50. (B, D) 51. (B, D) 52. (C, D)
53. (A)(B)(C) 54. (B)(D) 55. (A, B, C)

PART# II

1. (i) Non-removable (ii) Removable
   (iii) Removable (iv) Removable
2. \( a = -\frac{3}{2}, b \neq 0, c = \frac{1}{2} \) 3. \( g(x) = 2 + x; 0 \leq x \leq 1, \)
4. \( -\frac{7}{3}, -2, 0 \)
5. continuous at both points but differentiable only at \( x = 2 \)
6. not differentiable at \( x = 0 \)
7. \( a = 1/2, b = 3/2 \)
8. continuous but not differentiable at \( x = 0; \) differentiable & continuous at \( x = \pi/2 \)
10. continuous & differentiable
11. differentiable
12. continuous & differentiable
13. f is not derivable at all integral values in \(-1 < x \leq 3\)
14. f is continuous but not derivable at \(x = 1/2\), f is neither differentiable nor continuous at \(x = 1 \& x = 2\)
15. 
16. continuous in \(0 \leq x \leq 1\) & not differentiable at \(x = 0\)
17. f(x) is continuous but not differentiable at \(x = n\pi, n \in I\), f(x) is not periodic.
18. \(f(x) = 2x^2\) for \(0 \leq x \leq 1\) & \(f(x) = 0\) for \(-1 \leq x < 0\), f is differentiable & hence continuous at \(x = 0\)
20. \(a = \frac{1}{6}\); \(b = \frac{\pi}{4} - \frac{13}{6}\)

**EXERCISE # 3**

**PART# I**

1. (C)
2. Discontinuous hence not deri. at \(x = 1 \& -1\). Continuous \& -1. Continuous \& deri. at \(x = 0\)
3. (D) 4. (A) 5. (D) 7. (D) 8. (C)
9. \(a = 1; b = 0\) \((gof)'(0) = 0\)
10. (B) 11. 0 12. (A) 13. (A)
14. (B) 15. \(y - 2 = 0\) 16. \(g'(0) = 0\) 17. (A)(C)
18. (A)\&(p), (q), (r); (B)\&(p), (s); (C)\&(r), (s); (D)\&(p), (q)
19. (A) 20. (C) 21. (B, C) 22. 6 23. (A, B, C, D)

**PART# II**

1. (2) 2. (1) 3. (3) 4. (2) 5. (2) 6. (4) 7. (3)
8. (3)

**EXERCISE # 4**

1. Continuous at \(x = 1\).
2. Discontinuous
3. Discontinuous
4. Continuous
5. Discontinuous
6. Continuous
7. Continuous
8. Discontinuous
9. Continuous
10. Continuous
11. \(k = \frac{7}{2}\)
12. \(k = \frac{1}{2}\)
13. \(k = -1\)
14. \(k = \pm 1\)
15. \(a = 1, b = -1\)
16. Discontinuous at \(x = -2\) and \(x = \frac{-5}{2}\)
17. Discontinuous at \(x = 1, \frac{1}{2}\) and 2
18. Not differentiable at \(x = 2\)
19. Differentiable at \(x = 0\)
20. Not differentiable at \(x = 2\)
21. \(p = 3, q = 5\)