arride learning

DEFINITE INTEGRATION

Contents

Syllabus

Definite integrals and their properties,
Fundamental Theorem of Integral Calculus,
Application of definite integrals to the determination
of areas involving simple curves.

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ARRIDE LEARNING ONLINE E-LEARNING ACADEMY

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DEFINITE INTEGRATION

KEY CONCEPTS

1. $\int_a^b f(x)dx = F(b) - F(a)$ where $\int f(x)dx = F(x) + c$

VERY IMPORTANT Note : If $\int_a^b f(x)dx = 0 \Rightarrow$ then the equation f(x) = 0 has atleast one root lying in (a,b) provided f is a continuous function in (a,b).

2. PROPERTIES OF DEFINITE INTEGRAL :

P–1 $\int_a^b f(x)dx = \int_a^b f(t)\,dt$ provided f is same

P – 2 $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$

P–3 $\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where c may lie inside or outside the interval [a,b]. This property to be used when f is piecewise continuous in (a, b).

P–4 $\int_{-a}^a f(x)\,dx = \int_0^a [f(x) + f(-x)]\,dx = \begin{cases} 0 & ;\text{ if } f(x) \text{ is an odd function} \\ 2\int_0^a f(x)\,dx & ;\text{ if } f(x) \text{ is an even function} \end{cases}$

P–5 $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$, In particular $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

P–6 $\int_0^{2a} f(x)dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & ; \text{ if } f(2a-x) = f(x) \\ 0 & ; \text{ if } f(2a-x) = -f(x) \end{cases}$

P–7 $\int_0^{na} f(x)dx = n\int_0^a f(x)\,dx$, $(n \in I)$; where 'a' is the period of the function i.e. f(a+x) = f(x)

Note that : $\int_{x}^{a+x} f(t)\,dt$ will be independent of x.

P–8 $\int_{a+nT}^{b+nT} f(x)dx = \int_{a}^{b} f(x)\,dx$ where f(x) is periodic with period T & n $\in$ I.

P–9 $\int_{ma}^{na} f(x)dx = (n-m)\int_{0}^{a} f(x)\,dx$, (n, m $\in$ I) if f(x) is periodic with period 'a'.

P–10 If f(x) $\le \phi(x)$ for a $\le$ x $\le$ b then $\int_{a}^{b} f(x)\,dx \le \int_{a}^{b} \phi(x)\,dx$

P–11 $\left|\int_{a}^{b} f(x)dx\right| \le \int_{a}^{b} |f(x)|\,dx$.

P–12 If $f(x) \ge 0$ on the interval [a,b], then $\int_{a}^{b} f(x)dx \ge 0$.

3. WALLIS' FORMULA :

$$\int_{0}^{\pi/2} \sin^n x.\cos^m x\,dx = \frac{[(n-1)(n-3)(n-5)....1 \text{ or } 2][(m-1)(m-3)....1 \text{ or } 2]}{(m+n)(m+n-2)(m+n-4)....1 \text{ or } 2} K$$

Where $K = \frac{\pi}{2}$ if both m and n are even (m, n $\in$ N) ;

= 1 otherwise

4. DERIVATIVE OF ANTIDERIVATIVE FUNCTION :

If h(x) & g(x) are differentiable functions of x then,

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f[h(x)].h'(x) - f[g(x)].g'(x)$$

5. DEFINITE INTEGRAL AS LIMIT OF A SUM :

$$\int_{a}^{b} f(x)\,dx = \lim_{n\to\infty} h[f(a) + f(a+h) + f(a+2h) + ... + f\left(a + \overline{n+1}\right)]$$

$$\lim_{h\to\infty} h\sum_{r=0}^{n-1} f(a+rh) = \int_{0}^{1} f(x)dx \quad \text{where } b - a = nh$$

If a = 0 & b = 1 then, $\lim_{n\to\infty} h\sum_{r=0}^{n-1} f(rh) = \int_{0}^{1} f(x)dx$; where nh = 1 **OR**

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{r=1}^{n-1} f\left(\frac{r}{n}\right)=\int_0^1 f(x)\,dx\ .$$

6. ESTIMATION OF DEFINITE INTEGRAL :

If f(x) is continuous in [a, b] and it's range in this interval is [m, M], then

$$m(b-a)\le\int_a^b f(x)\,dx\le M(b-a)$$

Also remember that $\int_a^x f(t)\,dt$ will be derivable in [a, b]

7. SOME IMPORTANT EXPANSION :

(i) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}+........\infty=ln2$

(ii) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+........\infty=\frac{\pi^2}{6}$

(iii) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+........\infty=\frac{\pi^2}{12}$

(iv) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+........\infty=\frac{\pi^2}{8}$

(v) $\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\frac{1}{8^2}+........\infty=\frac{\pi^2}{24}$

AREA UNDER CURVE

1. The area bounded by the curve y = f (x) , the x-axis and the ordinates at x = a & x = b is given by,

$$\int_a^b f(x)\,dx=\int_a^b y\,dx\ .$$

2. If the area is below the x-axis then A is negative. The convention is to consider the magnitude only

i.e. $A=\left|\int_a^b y\,dx\right|$ in this case

3. Area between the curves y = f (x) & y = g (x) between the ordinates at x = a & x = b is given by ,

$$A=\int_a^b f(x)\,dx-\int_a^b g(x)\,dx=\int_a^b [f(x)-g(x)]\,dx\ .$$

4. Average value of a function y = f (x) w.r.t. x over an interval $a\le x\le b$ is defined as:

$$y\,(av)=\frac{1}{b-a}\int_a^b f(x)\,dx$$

5. The area function A_a^x satisfies the differential equation $\frac{dA_a^x}{dx}=f(x)$ with initial condition

$A_a^{\,a}=0$.

Note : If F(x) is any integral of f(x) then ,

$A_a{}^x = \int f(x)\,dx = F(x) + c$ $\qquad$ $A_a{}^a = 0$ = F(a) + c $\Rightarrow$ c = – F(a)

Hence $A_a{}^x$ = F(x) – F(a). Finally by taking x = b we get , $A_a{}^b = F(b) - F(a)$.

6. If f(x) is monotonic in [a, b], then area bounded by x = a, x = b, y = f(x) & y = f(c) ; c $\in$ [a, b] is least when $c = \frac{a+b}{2}$

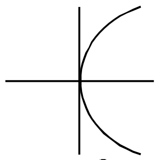
7.Curve-tracing :

To find approximate shape of a curve, the following phrases are suggested :

(a) Symmetry:

(i) Symmetry about x-axis :

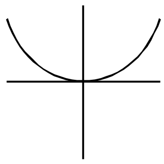
If all the powers of 'y' in the equation are even then the curve (graph) is symmetrical about the x-axis.



E.g. : $y^2 = 4ax$.

(ii) Symmetry about y-axis :

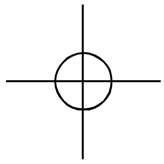
If all the powers of 'x' in the equation are even then the curve (graph) is symmetrical about the y-axis.



E.g. : $x^2 = 4ay$.

(iii) Symmetry about both axis :

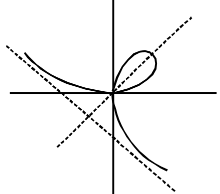
If all the powers of 'x' and 'y' in the equation are even, then the curve (graph) is symmetrical about the axis of 'x' as well as 'y' .



E.g. : $x^2 + y^2 = a^2$.

(iv) Symmetry about the line y = x :

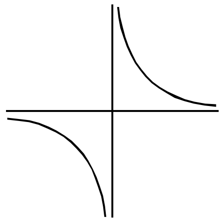
If the equation of the curve remain unchanged on interchanging 'x' and 'y', then the curve (graph) is symmetrical about the line y = x.



E.g. : $x^3 + y^3 = 3axy$.

(v) Symmetry in opposite quadrants :

If the equation of the curve (graph) remain unaltered when 'x' and 'y' are replaced by '–x' and '–y' respectively, then there is symmetry in opposite quadrants.



E.g. : $xy = c^2$

(b) Find the points where the curve crosses the x-axis and the y-axis.

(c) Find $\frac{dy}{dx}$ and equate it to zero to find the points on the curve where you have horizontal tangents.

(d) Examine intervals when f(x) is increasing or decreasing

(e) Examine what happens to 'y' when $x \to \infty$ or $x \to -\infty$

(f) **Asymptotes :**

Asymptote(s) is (are) line (s) whose distance from the curve tends to zero as point on curve moves towards infinity along branch of curve.

(i) If $\lim\limits_{x\to a} f(x) = \infty$ or $\mathop{\text{Lt}}\limits_{x\to a} f(x) = -\infty$, then x = a is asymptote of y = f(x)

(ii) If $\lim\limits_{x\to\infty} f(x) = k$ or $\mathop{\text{Lt}}\limits_{x\to-\infty} f(x) = k$ then y = k is asymptote of y = f(x)

(iii) If $\lim\limits_{x\to\infty} \frac{f(x)}{x} = m_1$, $\mathop{\text{Lt}}\limits_{x\to\infty} (f(x) - m_1x) = c$, then $y = m_1x + c_1$ is an asymptote (inclined to right).

(iv) If $\lim\limits_{x\to-\infty} \frac{f(x)}{x} = m_2$, $\lim\limits_{x\to-\infty} (f(x) - m_2x) = c_2$, then $y = m_2x + c_2$ is an asymptote (inclined to left).

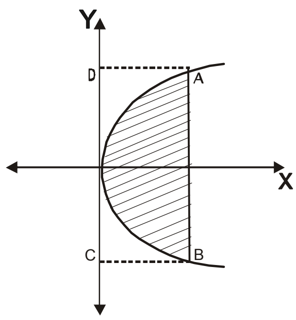
8. USEFUL RESULTS :

(i) Whole the area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is πab.

(ii) Area enclosed between the parabolas $y^2 = 4ax$ & $x^2 = 4by$ is 16ab/3.

(iii) Area included between the parabola $y^2 = 4ax$ & the line y = mx is $8a^2/3\ m^3$.

(iv) Area included between a double ordinate of a parabola and it is two-third the area of the rectangle formed by the double ordinate , tangent at vertex and the perpendicular on tangent at vertex from point of intersection of the double ordinate and parabola.



Shaded area = 2/3 (area of rectangle ABCD)

Area included between the curve y = f(x), x-axis and the ordinates x = a, x = b

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

** Marked Questions are having more than one correct option.*

Section (A) : Definite Integration in terms of Indefinite Intigration, using substitution and by parts

A-1. If $\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{6}$, then x can be equal to :

(A) $\frac{2}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) 2 (D) none of these

A-2. The value of the integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$, where $0 < \alpha < \frac{\pi}{2}$, is equal to :

(A) $\sin\alpha$ (B) $\alpha\sin\alpha$ (C) $\frac{\alpha}{2\sin\alpha}$ (D) $\frac{\alpha}{2}\sin\alpha$

A-3. If f(x) = $\begin{cases} x & x<1 \\ x-1 & x\ge 1 \end{cases}$, then $\int_0^2 x^2 f(x)\, dx$ is equal to :

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{5}{2}$

A-4. If f(0) = 1, f(2) = 3, f′(2) = 5 and f′(0) is finite, then $\int_0^1 x \,.\, f''(2x)\, dx$ is equal to :

(A) zero (B) 1 (C) 2 (D) none of these

A-5. $\int_0^{\pi} |1+2\cos x|\, dx$ is equal to :

(A) $\frac{2\pi}{3}$ (B) π (C) 2 (D) $\frac{\pi}{3}+2\sqrt{3}$

A-6. The value of $\int_{-1}^{3} (|x-2|+[x])\, dx$ is : ([x] stands for greatest integer less than or equal to x)

(A) 7 (B) 5 (C) 4 (D) 3

A-7. $\int_{\ell n\pi-\ell n2}^{\ell n\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}\, dx$ is equal to :

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

A-8. If $I_1 = \int_e^{e^2} \frac{dx}{\ell n x}$ and $I_2 = \int_1^2 \frac{e^x}{x}\, dx$, then :

(A) $I_1 = I_2$ (B) $2I_1 = I_2$ (C) $I_1 = 2I_2$ (D) none of these

A-9. $\int_0^{\pi/4} \frac{x.\sin x}{\cos^3 x} dx$ equals to :

(A) $\frac{\pi}{4}+\frac{1}{2}$ (B) $\frac{\pi}{4}-\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) none of these

A-10. Let $I_1 = \int_0^{3\pi} f(\cos^2 x)dx$, $I_2 = \int_0^{2\pi} f(\cos^2 x)dx$ and $I_3 = \int_0^{\pi} f(\cos^2 x)dx$, then

(A) $I_1 + 2I_3 + 3I_2 = 0$ (B) $I_1 = 2I_2 + I_3$ (C) $I_2 + I_3 = I_1$ (D) $I_1 = 2I_3$

A-11*. $\int_0^{\infty} \frac{dx}{(1+x)(1+x^2)} =$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) is same as $\int_0^{\infty} \frac{dx}{(1+x)(1+x^2)}$ (D) cannot be evaluated

A-12*. The value of integral $\int_a^b \frac{|x|}{x} dx$, a < b is :

(A) b – a if a > 0 (B) a – b if b < 0 (C) b + a if a < o < b (D) |b| – |a|

Section (B) : Definite Integration using Properties

B-1. Suppose for every integer n, $\int_n^{n+1} f(x)dx = n^2$. The value of $\int_{-2}^{4} f(x)dx$ is :

(A) 16 (B) 14 (C) 19 (D) 21

B-2. Let f: R $\to$ R, g : R $\to$ R be continuous functions. Then the value of integeral

$$\int_{\ell n\lambda}^{\ell n 1/\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x)-f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x)+g(-x)]} dx \text{ is :}$$

(A) depend on λ (B) a non-zero constant (C) zero (D) none of these

B-3. If $\int_{-1}^{3/2} |x \sin \pi x| \, dx = \frac{k}{\pi^2}$, then the value of k is :

(A) $3\pi + 1$ (B) $2\pi + 1$ (C) 1 (D) 4

B-4. The value of $\int_0^{\pi/2} \ell n |\tan x + \cot x| \, dx$ is equal to :

(A) $\pi \, \ell n 2$ (B) $-\pi \, \ell n 2$ (C) $\frac{\pi}{2} \, \ell n 2$ (D) $-\frac{\pi}{2} \, \ell n 2$

B-5. $\int_{2-\ell n 3}^{3+\ell n 3} \frac{\ell n (4+x)}{\ell n(4+x)+\ell n(9-x)} dx$ is equal to :

(A) cannot be evaluated (B) is equal to $\frac{5}{2}$

(C) is equal to $1 + 2\,\ell n 3$ (D) is equal to $\frac{1}{2} + \ell n 3$

B-6*. The value of integral $\int_0^{\pi} x f(\sin x)\, dx$ is :

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$ (B) $\pi\int_0^{\pi/2} f(\sin x)\, dx$ (C) 0 (D) none of these

B-7. If $\int_0^{11} \frac{11^x}{11^{[x]}}\, dx = \frac{k}{\log 11}$, (where [ ] denotes greatest integer function), then value of k is :

(A) 11 (B) 101 (C) 110 (D) none of these

B-8*. If $I = \int_0^{2\pi} \sin^2 x dx$, then

(A) $I = 2\int_0^{\pi} \sin^2 x dx$ (B) $I = 4\int_0^{\pi/2} \sin^2 x dx$ (C) $I = \int_0^{2\pi} \cos^2 x dx$ (D) $I = 8\int_0^{\pi/4} \sin^2 x dx$

B-9*. If $f(x) = \int_0^{x} (\cos^4 t + \sin^4 t)\, dt$, then $f(x + \pi)$ is equal to :

(A) $f(x) + f(\pi)$ (B) $f(x) + 2 f(\pi)$ (C) $f(x) + f\left(\frac{\pi}{2}\right)$ (D) $f(x) + 2f\left(\frac{\pi}{2}\right)$

Section (C) : Differentiation with Leibnitz formula and walli's formula

C-1. The slope of the tangent of the curve $y = \int_x^{x^2} \cos^{-1} t^2$ at $x = \frac{1}{\sqrt[4]{2}}$ is

(A) $\left(\frac{\sqrt[4]{8}}{2} - \frac{3}{4}\right)\pi$ (B) $\left(\frac{\sqrt[4]{8}}{3} - \frac{1}{4}\right)\pi$ (C) $\left(\frac{\sqrt[5]{8}}{4} - \frac{1}{3}\right)\pi$ (D) None of these

C-2. $\lim_{h\to 0} \frac{\int_a^{x+h} \ell n^2 t\ dt - \int_a^{x} \ell n^2 t\ dt}{h} =$

(A) 0 (B) $\ell n^2 x$ (C) $\frac{2\ell nx}{x}$ (D) does not exist

C-3. The value of the function $f(x) = 1 + x + \int_1^{x} (\ell n^2 t + 2\,\ell nt)\, dt$, where $f'(x)$ vanishes is:

(A) e^{-1} (B) 0 (C) $2e^{-1}$ (D) $1 + 2e^{-1}$

C-4. If $\int_a^{y} \cos t^2 dt = \int_1^{x^2} \frac{\sin t}{t} dt$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{2\sin^2 x}{x\cos^2 y}$ (B) $\frac{2\sin x^2}{x\cos y^2}$ (C) $\frac{2\sin x^2}{x\left(1 - 2\sin\frac{y^2}{2}\right)}$ (D) none of these

Section (D) : Integration as a limit of sum and reduction formula

D-1. $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r^3}{r^4+n^4}\right) =$

(A) $\ell n\, 2$ (B) $\frac{1}{2}\ell n\, 2$ (C) $\frac{1}{3}\ell n\, 2$ (D) $\frac{1}{4}\ell n\, 2$

D-2. $\underset{n\to\infty}{\text{Lt}}\sum_{r=2n+1}^{3n}\frac{n}{r^2-n^2} =$

(A) $\log\sqrt{\frac{2}{3}}$ (B) $\log\sqrt{\frac{3}{2}}$ (C) $\log\frac{2}{3}$ (D) $\log\frac{3}{2}$

D-3. $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{1/n} =$

(A) $\frac{e^{\pi/2}}{2e^2}$ (B) $2e^2e^{\pi/2}$ (C) $\frac{2}{e^2}e^{\pi/2}$ (D) none of these

D-4. $\lim_{n\to\infty}\frac{\pi}{n}\left[\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+.....+\sin\frac{(n-1)\pi}{n}\right] =$

(A) 0 (B) π (C) 2 (D) none of these

Section (E) : Area Under Curve

E-1 The area bounded by the x-axis and the curve $y = 4x - x^2 - 3$ is :

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$

E-2. The area of the figure bounded by right of the line $y = x + 1$, $y = \cos x$ and x - axis is :

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{3}{2}$

E-3. Area bounded by curve $y^3 - 9y + x = 0$ and y-axis is :

(A) $\frac{9}{2}$ (B) 9 (C) $\frac{81}{2}$ (D) 81

E-4. The area bounded by the curve $y = e^x$ and the lines $y = |x - 1|$, $x = 2$ is given by

(A) $e^2 + 1$ (B) $e^2 - 1$ (C) $e^2 - 2$ (D) none of these

E-5. The area bounded by $y = 2 - |2 - x|$ & $y = \frac{3}{|x|}$ is :

(A) $\frac{4+3\ln 3}{2}$ (B) $\frac{4-3\ln 3}{2}$ (C) $\frac{3}{2}+\ln 3$ (D) $\frac{1}{2}+\ln 3$

E-6*. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals to 9/2 ?

(A) – 4 (B) – 2 (C) 2 (D) 4

E-7. The area bounded by the curve $y^2 = 4x$ and the line $2x - 3y + 4 = 0$ is :

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$

E-8. The area bounded in the first quadrant between the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and the line $3x + 4y = 12$ is :

(A) $6(\pi - 1)$ (B) $3(\pi - 2)$ (C) $3(\pi - 1)$ (D) none

E-9. The area of the region bounded by x = 0, y = 0 , x = 2 , $y \le e^x$ and $y \ge \ln x$, is

(A) $6 - 4\ \ell n\ 2$ (B) $4\ \ell n\ 2 - 2$ (C) $2\ \ell n\ 2 - 4$ (D) $6 - 2\ \ell n\ 2$

E-10. The area bounded by the curve $y = \frac{1}{x^2}$ and its asymptote from x = 1 to x = 3 is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

E-11. The area between two arms of the curve $|y| = x^3$ from x = 0 to x = 2 is :

(A) 2 (B) 4 (C) 8 (D) 16

PART - II : SUBJECTIVE QUESTIONS

Section (A) : Definite Integration in terms of Indefinite Integration, using substitution and by parts

A-1. Evaluate:

(i) $\int_0^4 (x + x^{3/2})dx$ (ii) $\int_4^1 \frac{1}{x}dx$ (iii) $\int_0^1 \frac{\sqrt[3]{x^2} - \sqrt[4]{x}}{\sqrt{x}}dx$

A-2. Evaluate:

(i) $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$ (ii) $\int_{\sqrt{2}}^{\infty} \frac{dx}{x\sqrt{x^2 - 1}}$ (iii) $\int_0^4 \frac{x^2}{1 + x}dx$ (iv) $\int_0^{\pi/2} \left(\sqrt{\cos x}\right)\sin^3 x dx$

A-3. Evaluate:

(i) $\int_0^1 \sin^{-1} x dx$ (ii) $\int_1^2 \frac{\ell nx}{x^2}dx$ (iii) $\int_0^1 xe^x dx$ (iv) $\int_0^1 x^2 \sin^{-1} x\, dx$

A-4. Evaluate:

(i) $\int_0^1 \sin^{-1}\left(\frac{2x}{1 + x^2}\right)dx$ (ii) $\int_0^1 \frac{x \tan^{-1}}{(1 + x^2)^{3/2}}dx$ (iii) $\int_a^b \sqrt{(x - a)(b - x)}$ dx, a > b

(iv) $\int_0^{\sqrt{3}} \tan^{-1}\left(\frac{2x}{1 - x^2}\right)dx$

A-5. Evaluate:

(i) $\int_0^{\infty} \frac{dx}{e^x + e^{-x}}$ (ii) $\int_0^1 \frac{x}{1 + \sqrt{x}}dx$ (iii) $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2}dx$

(iv) $\int_0^{\pi/2} \frac{\sin 2\theta\, d\theta}{\sin^4 \theta + \cos^4 \theta}dx$ (v) $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x}dx$

A-6. The tangent to the graph of the function y = f(x) at the point with abscissa x = a forms with the x-axis an angle of $\pi/3$ and at the point with abscissa x = b at an angle of $\pi/4$, then find the value of the integral.

$\int_a^b f'(x)$. f" dx [assume f"(x) to be continuous]

Section (B) : Definite Integration using Properties

B-1. Let $f(x) = \ell n\left(\frac{1 - \sin x}{1 + \sin x}\right)$, then show that $\int_a^b f(x)dx = \int_b^a \ell n\left(\frac{1 + \sin x}{1 - \sin x}\right)dx$

B-2. Evaluate:

(i) $\int_0^2 [x^2]\, dx$ (ii) $\int_{-1}^1 [\cos^{-1} x]\, dx$

B-3. Evaluate:

(i) $\int_{-1}^1 e^{|x|}\, dx$ (ii) $\int_{-\pi/4}^{\pi/4} |\sin x|\, dx$ (iii) $\int_{-5}^5 |x+2|\, dx$ (iv) $\int_{-\pi/4}^{\pi/4} \frac{x+\pi/4}{2-\cos 2x} dx$

B-4. Evaluate:

(i) $\int_{-1}^1 \sin^5 x \cos^4 x dx$ (ii) $\int_{-\pi/2}^{\pi/2} \frac{g(x)-g(-x)}{f(-x)+f(x)} dx$

B-5. Evaluate:

(i) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ (ii) $\int_0^{\pi/2} \frac{e^{\sin x}}{e^{\sin x}+e^{\cos x}} dx$ (iii) $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ (iv) $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x} dx$

B-6. Evaluate:

(i) $\int_{-1}^2 \{2x\}\, dx$ (where function {.} denotes fractional part function)

(ii) $\int_0^{10\pi} (|\sin x| + |\cos x|)\, dx$

B-7. If f(x) is an odd function defined on $\left[-\frac{T}{2}, \frac{T}{2}\right]$ and has period T, then prove that $\phi(x) = \int_a^x f(t) dt$ is also periodic with period T.

B-8. Prove ther following inequalities:-

(i) $\frac{\sqrt{3}}{8} < \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx < \frac{\sqrt{2}}{6}$ (ii) $4 \le \int_1^3 \sqrt{(3+x^3)}\, dx \le 2\sqrt{30}$

Section (C) : Differentiation with Leibnitz formula and walli's formula

C-1. If $f(x) = 5^{g(x)}$ and $g(x) = \int_2^{x^2} \ell n(1+t^2)$ dt, then find the value of $f'\left(\sqrt{2}\right)$.

C-2. If $f(x) = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}$ dt, then prove that $f'(x) = 0 \ \forall x \in R$.

C-3. If $f(x) = 2x^3 - 15x^2 - 24x$ and $g(x) = \int_0^x f(t)\, dt + \int_0^{5-x} f(t) dt$ $(0 < x < 5)$. Find the interval in which g(x) is increasing.

C-4. Evaluate:

(i) $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x)\, dx$ (ii) $\int_0^{\pi} x \sin^5 x\, dx$ (iii) $\int_0^2 x^{3/2}\sqrt{2-x}\, dx$

Section (D) : Integration as a limit of sum and reduction formula

D-1. Evaluate:

(i) $\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$ (ii) $\lim_{n\to\infty}\frac{3}{n}\left[1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+.....+\sqrt{\frac{n}{n+3(n-1)}}\right]$

D-2. Evaluate:

If $I_n = \int_{r=0}^{\pi/4} \tan^n x\, dx$, then show that $I_n + I_{n-2} = \frac{1}{n-1}$

Section (E) : Area Under Curve

E.1. Find the area enclosed between the curve $y = x^3 + 3$, $y = 0$, $x = -1$, $x = 2$.

E.2. Let f(x) = Maximum $\{x^2, (1-x)^2, 2x(1-x)\}$ where $0 \le x \le 1$. Determine the area of the region bounded by the curves y = f(x), x –axis, x = 0 and x = 1.

E.3. (i) Find the area cut off between x = 0 and $x = 4 - y^2$.

(ii) Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y-axis.

E.4. Find the area bounded by the y-axis and the curve $x = e^y \sin \pi y$, y = 0 , y = 1.

E.5. Compute the area of the figure bounded by straight lines x = 0, x = 2 and the curves $y = 2^x$ and $y = 2x - x^2$

E.6. Let f(x) = $\sqrt{\tan x}$. Show that area bounded by y = f(x), y = f(c), x = 0 and x = a, $0 < c < a < \frac{\pi}{2}$ is minimum when $c = \frac{a}{2}$

E.7. Find the area included between the parabolas $y^2 = x$ and $x = 3 - 2y^2$.

E.8. A tangent is drawn to the curve $x^2 + 2x - 4ky + 3 = 0$ at a point whose abscissa is 3. This tangent is perpendicular to x + 3= 2y. find the area bounded by the cure, this tangent and ordinate x = –1.

E.9. (i) Draw graph of $y = (\tan x)^n$, $n \in N$, $x \in \left[0, \frac{\pi}{4}\right]$. Hence show $0 < (\tan x)^{n+1} < (\tan x)^n$, $x \in \left(0, \frac{\pi}{4}\right)$

(ii) Let A_n be the area bounded by the cunrve $y = (\tan x)^n$ and the lines x = 0,y = 0 and x = π/4. Prove that for n > 2, $A_n + A_{n-2} = 1/(n-1)$ and deduce that $1/(2n+2) < A_n < 1/(2n-2)$.

E.10. Show that the curve $a^2y^2 = x^2(a^2 - x^2)$ consists of two loops and find the area of each loop.

PART - III : MISCELLANEOUS OBJECTIVE QUESTIONS

MATCH THE COLUMN

1.

Column – I	Column – II
(A) $\int_0^{\pi/2} \ell n(\tan x + \cot x)\, dx =$	(p) $\frac{\pi^2}{4}$
(B) $\int_0^{\pi/2} \frac{\sin x - \cos x}{(\sin x + \cos x)^2}\, dx =$	(q) $\pi\, \ell n\, 2$
(C) $\int_0^{2\pi} x(\sin^2 x \cos^2 x)\, dx =$	(r) 0
(D) $\int_0^{\pi/2} (2\ell n \sin x - \ell n \sin 2x)\, dx =$	(s) $-\frac{\pi}{2}\ell n\, 2$

2. **Column-I** **Column-II**

	Column-I		Column-II
(A)	Area bounded by region $0 \le y \le 4x - x^2 - 3$ is :	(p)	32/3
(B)	Area of the region enclosed by $y^2 = 8x$ and $y = 2x$ is :	(q)	1/2
(C)	The area bounded by $\|x\| + \|y\| \le 1$ and $\|x\| \ge 1/2$ is :	(r)	8/3
(D)	Area bounded by $x \le 4 - y^2$ and $x \ge 0$ is :	(s)	4/3

COMPREHENSION

COMPREHENSION # 1

If $y = \int_{u(x)}^{v(x)} f(t)\,dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x)\, f^2(v(x)) - u'(x)\, f^2(u(x))$ and the equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx}\right)_{(a,\,b)} (x - a)$.

3. If $y = \int_{x}^{x^2} t^2\,dt$, then equation of tangent at x = 1 is :

(A) y = x + 1 (B) x + y = 1 (C) y = x – 1 (D) y = x

4. If $F(x) = \int_1^x e^{t^2/2}(1 - t^2)\,dt$, then $\frac{d}{dx}F(x)$ at x = 1 is :

(A) 0 (B) 1 (C) 2 (D) – 1

5. If $y = \int_{x^3}^{x^4} \ell nt\,dt$, then $\lim_{x \to 0^+} \frac{dy}{dx}$ is :

(A) 0 (B) 1 (C) 2 (D) – 1

COMPREHENSION # 2

Let $g(t) = \int_{x_1}^{x_2} f(t, x)\,dx$. Then $g'(t) = \int_{x_1}^{x_2} \frac{\partial}{\partial t}(f(t, x))\,dx$. Consider $f(x) = \int_0^{\pi} \frac{\ell n(1 + x\cos\theta)}{\cos\theta}\,d\theta$.

6. Range of f(x) is :

(A) $(0, \pi)$ (B) $(0, \pi^2)$ (C) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (D) $\left(\frac{-\pi^2}{2}, \frac{\pi^2}{2}\right)$

7. The number of critical points of f(x), in the interior of its domain, is :

(A) 0 (B) 1 (C) 2 (D) infinitely many

8. f(x) is :

(A) discontinuous at x = 0. (B) continuous but not differentiable at x = 1.

(C) continuous at x = 0. (D) differentiable at x = 1.

COMPREHENSION # 3

Let f(x) be a differentiable function, satisfying f (0) = 2, f′ (0) = 3 and f″ (x) = f(x).

9. Graph of y = f(x) cuts x -axis at :

(A) $x = -\frac{1}{2}\ell n5$ (B) $x = \frac{1}{2}\ell n5$ (C) $x = -\ell n5$ (D) $x = \ell n5$

10. Area enclosed by y = f(x) in the second quadrant is :

(A) $3+\frac{1}{2}\ell n\sqrt{5}$ (B) $2+\frac{1}{2}\ell n5$ (C) $3-\sqrt{5}$ (D) 3

11. Area enclosed by y = f(x), y = f^{-1} (x), x + y = 2 and x + y = $-\frac{1}{2}\ell n5$ is:

(A) $8+\frac{1}{8}(\ell n5)^2$ (B) $8-2\sqrt{5}+\frac{1}{8}(\ell n5)^2$ (C) $2\sqrt{5}-\frac{1}{8}(\ell n5)^2$ (D) $8+2\sqrt{5}-\frac{1}{8}(\ell n5)^2$

Assertion/Reasoning

12. **Statement-1 :** If {.} represents fractional part function, then $\int_0^{5.5}\{x\}\,dx = \frac{21}{8}$.

Statement-2: If [.] and {.} represent greatest integer and frational part function respectively then

$\int_0^{t}\{x\}\,dx = \frac{[t]}{2}+\frac{\{t\}^2}{2}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

13. **STATEMENT-1 :** $\int_0^{10\pi}|\cos x|\,dx = 20$.

STATEMENT-2 : $\int_a^{b} f(x)dx \ge 0$, then $f(x) \ge 0,\ \forall\ x \in (a, b)$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

14. **Statement-1:** Area bounded by y = tan x, y = $\tan^2 x$ in between $x \in \left(0, \frac{\pi}{4}\right)$ is equal to $\left(\frac{\pi}{4}+\ell n\sqrt{2}-1\right)$.

Statement-2: Area bounded by y = f(x) and y = g(x) {f(x) > g(x)} between x = a_1 x = b is $\int_a^{b}(f(x)-g(x))dx$ (b > a).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

Single choice type

1. If f(x) is a function satisfying $f\left(\frac{1}{x}\right)+x^2 f(x)=0$ for all non-zero x, then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)dx$ equals to :

(A) $\sin\theta + \operatorname{cosec}\theta$ (B) $\sin^2\theta$ (C) $\operatorname{cosec}^2\theta$ (D) none of these

2. $\int_0^{(\pi/2)^{1/3}} x^5 \sin x^3 \, dx =$

(A) 1 (B) 1/2 (C) 2 (D) 1/3

3. $\int_0^{\infty}\left[2e^{-x}\right]dx$, where [.] denotes the greatest integer function, is equal to :

(A) 0 (B) $\ell n\, 2$ (C) e^2 (D) $2e^{-1}$

4. If $\int_0^{100} f(x)dx = a$, then $\sum_{r=1}^{100}\left(\int_0^1 f(r-1+x)dx\right) =$

(A) 100 a (B) a (C) 0 (D) 10 a

5. If A = $\int_0^{\pi}\frac{\cos x}{(x+2)^2}dx$, then $\int_0^{\frac{\pi}{2}}\frac{\sin 2x}{x+1}dx$ =

(A) $\frac{1}{2}+\frac{1}{\pi+2}-A$ (B) $\frac{1}{\pi+2}-A$ (C) $1+\frac{1}{\pi+2}-A$ (D) $A-\frac{1}{2}-\frac{1}{\pi+2}$

6. If f(x) = $\begin{cases} 0 & ,\text{where } x = \frac{n}{n+1}, n = 1,2,3..... \\ 1 & ,\text{else where} \end{cases}$, then the value of $\int_0^2 f(x)dx$.

(A) 1 (B) 0 (C) 2 (D) ∞

7. If $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^{\infty} e^{-ax^2}\, dx$ where a > 0 is :

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\frac{\sqrt{\pi}}{2a}$ (C) $2\frac{\sqrt{\pi}}{a}$ (D) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

8. The expression $\frac{\int_0^n [x]\,dx}{\int_0^n \{x\}\,dx}$, where [x] and {x} are integral and fractional parts of x and $n \in N$, is equal to :

(A) $\frac{1}{n-1}$ (B) $\frac{1}{2}$ (C) n (D) n – 1

9. Let A = $\int_0^1 \frac{e^t}{1+t}dt$, then $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1}$ dt has the value:

(A) Ae^{-a} (B) $-Ae^{-a}$ (C) $-ae^{-a}$ (D) Ae^{a}

10. $\int_0^{2n\pi}\left(|\sin x|-\left(\left[\left|\frac{\sin x}{2}\right|\right]\right)\right)dx$ (where [ ] denotes the greatest integer function and n $\in$I) is equal to:

(A) 0 (B) 2n (C) $2n\pi$ (D) 4n

11. f(x) = Minimum {tan x, cot x} $\forall$ x $\in$ $\left(0,\frac{\pi}{2}\right)$. Then $\int_0^{\pi/3} f(x)$ dx is equal to:

(A) $\ell n\left(\frac{\sqrt{3}}{2}\right)$ (B) $\ell n\left(\sqrt{\frac{3}{2}}\right)$ (C) $\ell n\left(\sqrt{2}\right)$ (D) $\ell n\left(\sqrt{3}\right)$

12. If f(π) = 2 and $\int_0^{\pi}(f(x)+f''(x))$ sin x dx = 5, then f(0) is equal to:

(it is given that f(x) is continuous in [0, π])

(A) 7 (B) 3 (C) 5 (D) 1

13. If $u_n = \int_0^{\pi/2} x^n \sin x\,dx$, then the value of $u_{10}+90u_8$ is :

(A) $9\left(\frac{\pi}{2}\right)^8$ (B) $\left(\frac{\pi}{2}\right)^9$ (C) $10\left(\frac{\pi}{2}\right)^9$ (D) $9\left(\frac{\pi}{2}\right)^9$

14. The value of $\int_{1/e}^{\tan x}\frac{t}{1+t^2}dt+\int_{1/e}^{\cot x}\frac{1}{t(1+t^2)}dt$, where x $\in$ (π/6, π/3), is equal to:

(A) 0 (B) 2 (C) 1 (D) cannot be determined

15. $\lim_{n\to\infty}\left(\sin\frac{\pi}{2n}.\sin\frac{2\pi}{2n}.\sin\frac{3\pi}{2n}......\sin\frac{(n-1)\pi}{n}\right)^{1/n}$ =

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) none of these

16*. Given f is an odd function defined everywhere, periodic with period 2 and integrable on every interval Let

g(x) = $\int_0^x f(t)dt$. Then :

(A) g(2n) = 0 for every integer n (B) g(x) is an even function
(C) g(x) and f(x) have the same period (D) none of these

17*. If $I_n=\int_0^1\frac{dx}{(1+x^2)^n};n\in N$, then which of the following statements hold good?

(A) $2nI_{n+1}=2^{-n}+(2n-1)I_n$ (B) $I_2=\frac{\pi}{8}+\frac{1}{4}$

(C) $I_2=\frac{\pi}{8}-\frac{1}{4}$ (D) $I_3=\frac{\pi}{16}-\frac{5}{48}$

18*. If f(x) is integrable over [1,2], then $\int_1^2 f(x)dx$ is equal to :

(A) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (B) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$ (C) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (D) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

19. The area bounded by $x^2 + y^2 - 2x = 0$ and $y = \sin\frac{\pi x}{2}$ in the upper half of the circle is :

(A) $\frac{\pi}{2}-\frac{4}{\pi}$ (B) $\frac{\pi}{4}-\frac{2}{\pi}$ (C) $\pi-\frac{8}{\pi}$ (D) none of these

20. The area enclosed between the curves $y = \log_e(x + e)$, $x = \log_e\left(\frac{1}{y}\right)$ and the x- axis is

(A) 2 (B) 1 (C) 4 (D) none of these

21. The area bounded by the curve $x = a\cos^3 t$, $y = a\sin^3 t$ is:

(A) $\frac{3\pi a^2}{8}$ (B) $\frac{3\pi a^2}{16}$ (C) $\frac{3\pi a^2}{32}$ (D) $3\pi a^2$

22. The area bounded by the curve $f(x) = x + \sin x$ and its inverse function between the ordinates $x = 0$ & $x = 2\pi$ is:

(A) 4π (B) 8π (C) 4 (D) 8

23. The area bounded by the curve $y = 2x^4 - x^2$, x-axis and the two ordinates corresponding to the minima of the function is :

(A) $\frac{3}{120}$ (B) $\frac{5}{120}$ (C) $\frac{1}{20}$ (D) $\frac{7}{120}$

24. The ratio in which the curve $y = x^2$ divides the region bounded by the curve ; $y = \sin\left(\frac{\pi x}{2}\right)$ & the x-axis as x varies from 0 to 1 , is :

(A) $2 : \pi$ (B) $1 : 3$ (C) $3 : \pi$ (D) $(6-\pi) : \pi$

25. The area bounded by the curves $y = x e^x$, $y = x e^{-x}$ and the line x = 1

(A) $\frac{2}{e}$ (B) $1-\frac{2}{e}$ (C) $\frac{1}{e}$ (D) $1-\frac{1}{e}$

26. Area of the curve $y^2 = (7 - x)(5 + x)$ above x–axis and between the ordinates $x = -5$ and $x = 1$ is :

(A) 9π (B) 18π (C) 15π (D) none

27. The area included between the curve $xy^2 = a^2(a - x)$ and its asymptote is

(A) $\frac{\pi a^2}{2}$ (B) $2\pi a^2$ (C) πa^2 (D) none

28. The area bounded by the curve y =f(x), x-axis and the ordinates x = 1 and x = b is $(b - 1)\sin(3b + 4)$, $\forall b \in R$, then f(x) =

(A) $(x - 1)\cos(3x + 4)$ (B) $\sin(3x + 4)$
(C) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$ (D) none of these

PART - II : SUBJECTIVE QUESTIONS

1. $\int_0^{\pi} e^{\cos^2} \cos^3(2n+1)\, x\, dx, n \in I$

2. Evaluate : $\int_{1/2}^{2} \frac{1}{x} \sin\left(x - \frac{1}{x}\right) dx$

3. If f, g, h be continuous function on [0, a] such that f (a – x) = f(x), g(a –x) = –g(x) and 3h(x) – 4h (a–x) = 5, then prove that, $\int_0^a f(x)\, g(x)\, h(x) = 0$.

4. Let f(x) = $\begin{bmatrix} 1-x & \text{if} & 0 \le x \le 1 \\ 0 & \text{if} & 1 < x \le 2 \\ (2-x)^2 & \text{if} & 2 < x \le 3 \end{bmatrix}$. Define the function F(x) = $\int_0^x f(t)$ dt and show that F is continuous in [0,3] and differentiable in (0, 3).

5. Find f(x) if it satisfies the relation f(x) = $e^x + \int_0^1 (x + ye^x) f(y)\, dy$

6. Find the value of c for which the area of the figure bounded by the curves y = sin 2x, the straight lines x = $\pi/6$, x = c and the abscissa axis is equal to 1/2.

7. Find the area of the figure bounded by the parabolas, x = $-2y^2$, x = $1 - 3y^2$ and y – axis.

8. Compute the area of the figure bounded by the curve y = ln x and y = $\ln^2 x$.

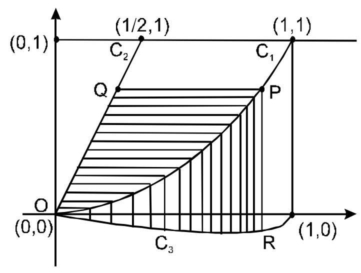
9. Find the area of the region bounded in the first quadrant by the curve C : y = tan x, tangent drawn to C at x = $\frac{\pi}{4}$ and the x – axis.

10. Find the values of m (m > 0) for which the area bounded by the line y = mx + 2 and x = $2y - y^2$ is, (i) 9/2 square units and (ii) minimum. Also find the minimum area.

11. Find the area between the curve $y^2(2a - x) = x^3$ and its asymptotes.

12. Find the area of the loop of the curve, $ay^2 = x^2(a - x)$.

13. Let C_1 & C_2 be the graphs of the function y = x^2 & y = 2x, $0 \le x \le 1$ respectively . Let C_3 be the graph of a function y = f(x), $0 \le x \le 1$, f(0) = 0. for a point P on C_1 , let the lines through P, parallel to the axes, meet C_2 & C_3 at Q & R respectively (see figure). If for every position of P (on C_1), the areas of the shadded regions OPQ & ORP are equal, determine the function f (x).



EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

1. If l (m, n) = $\int_0^1 t^m(1+t)^n dt$, then the expression for l (m, n) in terms of l (m + 1, n – 1) is : **[IIT-JEE 2003, 3]**

(A) $\frac{m}{n+1} l(m + 1, n - 1)$ (B) $\frac{n}{m+1} l(m + 1, n - 1)$

(C) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m + 1, n - 1)$ (D) $\frac{2^n}{m+1} - \frac{n}{m+1} l(m + 1, n - 1)$

2. If f(x) = $\int_{x^2}^{x^2+1} e^{-t^2}$ dt, then the function f(x) decreases in **[IIT-JEE-2003]**

(A) (–2, 2) (B) (0, ∞) (C) no value of x (D) (– ∞, 0)

3. The area bounded by the curves y = $\sqrt{x}, 2y + 3 = x$ and x-axis in the first quadrant is **[IIT-JEE-2003]**

(A) 9 (B) $\frac{27}{4}$ (C) 36 (D) 18

4. If f(x) is differentiable and $\int_0^{t^2} x \ f(x) \ dx = \frac{2}{5} t^5$, then $f\left(\frac{4}{25}\right)$ equals : **[IIT-JEE 2004, 3]**

(A) $\frac{2}{5}$ (B) $-\frac{5}{2}$ (C) 1 (D) $\frac{5}{2}$

5. The definite integral $\int_0^4 \sqrt{\frac{1-x}{1+x}}$ dx is equal to **[IIT-JEE 2004]**

(A) 1 (B) π (C) $\frac{\pi}{2} - 1$ (D) $\frac{\pi}{2} + \frac{1}{2}$

6. If $y(x) = \int_{\pi^2/16}^{x^2} \frac{\cos x. \ \cos\sqrt{\theta}}{1+\sin^2 \ \sqrt{\theta}} d\theta$, then find $\frac{dy}{dx}$ at x = π. **[IIT-JEE 2004, 2 Out of 60]**

7. Evaluate : $\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}$ dx . **[IIT-JEE 2004, 4 Out of 60]**

8. If area enclosed between the curves y = ax^2 and x = ay^2 in the 1st quadrant is 1. Then the possible value of a is **[IIT-JEE 2004]**

(A) $\frac{1}{\sqrt{3}}$ (B) 1 (C) $\sqrt{3}$ (D) $\frac{1}{2}$

9. If $\int_{\sin x}^{1} t^2(f(t))$ dt = (1 – sinx) then $f\left(\frac{1}{\sqrt{3}}\right)$ is : **[IIT-JEE 2005, 3]**

(A) 1/3 (B) $1/\sqrt{3}$ (C) 3 (D) $\sqrt{3}$

10. $\int_{-2}^{0}(x^3+3x^2+3x+3+(x+1)\cos(x+1))\,dx$ is equal to : **[IIT-JEE 2005,3]**

(A) -4 (B) 0 (C) 4 (D) 6

11. Evaluate $\int_{0}^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\,dx$. **[IIT-JEE 2005, 2 Out of 60]**

12. The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = \frac{1}{4}$ is.

(A) 4 sq. units (B) $\frac{1}{6}$ sq. units (C) $\frac{4}{3}$ sq. units (D) $\frac{1}{3}$ sq. units

[JEE '2005' (Screening), 3]

13. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. **[JEE '2005, (Mains) 4]**

14. If $\begin{bmatrix}4a^2 & 4a & 1\\ 4b^2 & 4b & 1\\ 4c^2 & 4c & 1\end{bmatrix}\begin{bmatrix}f(-1)\\ f(1)\\ f(2)\end{bmatrix}=\begin{bmatrix}3a^2+3a\\ 3b^2+3b\\ 3c^2+3c\end{bmatrix}$, f(x) is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of y = f(x) with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by f(x) and chord AB. **[JEE2005,(Mains)6 out of 60]**

15*. $f(x) = \begin{cases} x &, 0 \le x \le 1 \\ 2-e^{x-1} &, 1 < x \le 2 \\ x-e &, 2 < x \le 3\end{cases}$ and $g(x) = \int_0^x f(t)\,dt, x \in [1, 3]$ then **[IIT-JEE 2006]**

(A) g(x) has no local maxima (B) g(x) has no local minima

(C) g(x) has a local maxima at x = 1 + ℓn2 (D) g(x) has a local minima at x = e

16. The value of $5050\dfrac{\int_0^1 (1-x^{50})^{100}\,dx}{\int_0^1 (1-x^{50})^{101}\,dx}$ is. **[IIT-JEE 2006, (6, 0) out of 184]**

17. Match the following : **[IIT-JEE 2006, (1.5, +1.5) out of 184]**

(A) $\int_0^{\pi/2}(\sin x)^{\cos x}\{\cos x\cot x - \sin x\,.\ln(\sin x)\}\,dx$: (p) 0

(B) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ (q) 1

(C) cosine of the angle of intersection of curves
$y = 3^{x-1}\log x$ and $y = x^x - 1$ is (r) 4/3

(D) A continuous function f: $[1, 6] \to [0, \infty]$ is such that
$f'(x) = \frac{2}{x+f'(x)}$ and f(1) = 0. Then maximum value of f cannot exceed. (s) 2 ln 6

18. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\int_2^{\sec^2 x} f(t)\,dt}{x^2-\frac{\pi^2}{16}}$ equals : **[IIT-JEE 2007, (3, –1) out of 81]**

(A) $\frac{8}{\pi}f(2)$ (B) $\frac{2}{\pi}f(2)$ (C) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (D) $4f(2)$

19. Match the Integrals in **Column I** with the values in **Column II**. **[IIT-JEE 2007, (6, 0) out of 81]**

Column I		**Column II**	
(A)	$\int_{-1}^{1} \frac{dx}{1+x^2} =$	(p)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B)	$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}} =$	(q)	$2\log\left(\frac{2}{3}\right)$
(C)	$\int_{2}^{3} \frac{dx}{1-x^2} =$	(r)	$\frac{\pi}{3}$
(D)	$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}} =$	(s)	$\frac{\pi}{2}$

20*. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$, for n = 1, 2, 3, ... Then :

[IIT-JEE 2008, (4, 0) out of 82]

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ (B) $S_n > \frac{\pi}{3\sqrt{3}}$ (C) $T_n < \frac{\pi}{3\sqrt{3}}$ (D) $T_n > \frac{\pi}{3\sqrt{3}}$

Comprehension#2 (Q.21 to Q.23)

Consider the function f: $(-\infty, \infty) \to (-\infty, \infty)$ defined by $f(x) = \frac{x^2-ax+1}{x^2+ax+1}, 0 < a < 2$.

21. Which of the following is true ? **[IIT-JEE-2008]**

(A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$ (B) $(2 - a)^2 f''(1) - (2 + a)^2 f''(-1) = 0$

(C) $f'(1) f'(-1) = (2 - a)^2$ (D) $f'(1) f'(-1) = -(2 + a)^2$

22. Which of the following is true ?

(A) f(x) is decreasing on (–1, 1) and has a local minimum at x = 1

(B) f(x) is increasing on (–1, 1) and has a local maximum at x = 1

(C) f(x) is increasing on (–1, 1) but has neither a local maximum nor a local minimum at x = 1

(D) f(x) is increasing on (–1, 1) but has neither a local maximum nor a local minimum at x = 1

23. Let $g(x) = \int_{0}^{e^x} \frac{f'(t)}{1+t^2} dt$. Which of the following is true ? **[IIT-JEE-2008]**

(A) g'(x) is positive on $(-\infty, 0)$ and negative on $(0, \infty)$

(B) g'(x) is negative on $(-\infty, 0)$ and positive on $(0, \infty)$

(C) g'(x) change sign on both $(-\infty, 0)$ and $(0, \infty)$

(D) g'(x) does not change sign on $(-\infty, \infty)$

Comprehension # 3 (24 to 26)

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function y = f(x). If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function y = g(x) satisfying g(0) = 0

24. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right) =$ **[IIT-JEE-2008]**

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$ (C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

25. the area of the region bounded by the curve y = f(x) , the x-axis and the lines x = a and x = b, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx + bf(b) - af(a)$ (B) $-\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx - bf(b) + af(a)$ (D) $-\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx - bf(b) + af(a)$

26. $\int_{-1}^{1} g'(x)dx =$ **[IIT-JEE-2008]**

(A) 2g (–1) (B) 0 (C) – 2g(1) (D) 2g (1)

27. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines x = 0 and x $= \frac{\pi}{4}$ is

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$ (B) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ (C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ (D) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

28* If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2, ... , then : **[IIT-JEE 2009, (4, –1) out of 80]**

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

29. Let f : R → R be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of f(ln 5) is. **[IIT-JEE 2009, (4, –1) out of 80]**

30. Area of the region bounded by the curve $y = e^x$ and lines x = 0 and y = e is **[IIT-JEE-2009]**

(A) e – 1 (B) $\int_1^e \ell n(e+1-y)\,dy$ (C) $e - \int_0^1 e^x dx$ (D) $\int_1^e \ell n\,y\,dy$

31. The value of $\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\ln(1+t)}{t^4+4}dt$ is : **[IIT-JEE 2010, (3, –1) out of 84]**

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

32. The value (s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}$ dx of is (are) : **[IIT-JEE 2010,(3, 0) Out of 84]**

(A) $\frac{22}{7} - \pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15} - \frac{3\pi}{2}$

33*. Let f be a real-valued function defined on the interval (0 , ∞) by $f(x) = \ell nx + \int_0^x \sqrt{1+\sin t}$ dt. Then which of the following statement(s) is (are) true? **[IIT-JEE-2010]**

(A) f"(x) exists for all x ∈ (0 , ∞)
(B) f"(x) exists for all x ∈ (0 , ∞) and f' is continuous on (0, ∞), but not differentiable on (0, ∞)
(C) there exists α > 1 such that |f'(x)| < |f(x)| for all x ∈ (α , ∞)
(D) there exists β > 0 such that |f(x)| + |f'(x)| ≤ β for all x ∈ (0, ∞)

34. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [-10, 10] by , **[IIT-JEE 2010,(3, 0) Out of 84]**

$$f(x)=\begin{cases} x-[x] & \text{if } [x] \text{ is odd} \\ 1+[x]-x & \text{if } [x] \text{ is even} \end{cases}.$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is.

35. Let f be a real-valued function defined on the interval (–1, 1) such that $e^{-x}f(x) = 2+\int_0^x \sqrt{t^4+1}\, dt$, for all $x \in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $(f^{-1})'$ (2) is equal to : **[IIT-JEE 2010,(5, -2) Out of 79]**

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{e}$

Comprehension Q.36 to 38

Consider the polynomial : $f(x) = 1 + 2x + 3x^2 + 4x^3$

Let s be the sum of all distinct real roots of f(x) and let t = | s |

36. The real number s lies in the interval :

(A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, -\frac{3}{4}\right)$ (C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

37. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval:

(A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$ (C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

38. The function f′(x) is : **[JEE 2010, (3, –1) × 3 Out of 82]**

(A) increasing in $\left(-t, \frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, \frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in (–t, t)

(D) decreasing in (–t, t)

39. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)}\, dx$ is : **[IIT-JEE 2011, (3, –1) Out of 80]**

(A) $\frac{1}{4}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\ln\frac{3}{2}$ (D) $\frac{1}{6}\ln\frac{3}{2}$

40. Let the straight line x = b divide the area enclosed by $y = (1-x)^2$, y = 0 and x = 0 into two parts R_1 $(0 \le x \le b)$ and R_2 $(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals : **[JEE 2011, (4, –1)]**

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

41. Let f : [–1, 2] [0, ∞) be a continuous function such that f(x) = f(1 – x) for all x ∈ [–1, 2]. Let $R_1 = \int_{-1}^{2} x f(x)\, dx$, and R_2 be the area of the region bounded by y = f(x) , x = –1, x = 2, and the x-axis. Then :

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

42.* Let S be the area of the region enclosed by y = e^{-x^2}, y = 0, x = 0, and x = 1. Then :

[IIT - JEE 2012]

(A) $S \geq \frac{1}{e}$ (B) $S \geq 1 - \frac{1}{e}$ (C) $S \leq \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

43. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2 + \ln\frac{\pi+x}{\pi-x}\right)\cos x\, dx$ is : **[IIT-JEE 2012]**

(A) 0 (B) $\frac{\pi^2}{2} - 4$ (C) $\frac{\pi^2}{2} + 4$ (D) $\frac{\pi^2}{2}$

Comprehension 44 to 45

Let $f(x) = (1 - x)^2 \sin^2 x + x^2$ for all x ∈ IR and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ell nt\right) f(t)\, dt$ for all x ∈ (1, ∞)

44. Which of the following is true?

(A) g is increasing on (1, ∞)
(B) g is decreasing on (1, ∞)
(C) g is increasing on (1, 2) and decreasing on (2, ∞)
(D) g is decreasing on (1, 2) and increasing on (2, ∞)

45. Consider the statements :

P : There exists some x ∈ IR such that $f(x) + 2x = 2(1 + x^2)$
Q : There exists some x ∈ IR such that 2f(x) + 1 = 2(1 + x)
Then

(A) both P and Q are true (B) P is true and Q is false
(C) P is false and Q is true (D) both P and Q are false

46*. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)\, dt$ for all x ∈ (0, ∞) then

(A) f ahs a local maximum at x= 2
(B) f is decreasing on (2, 3)
(C) there exists some c ∈ (0, ∞) such that f''(c) = 0
(D) f has a local minimum at x = 3

47. Let $f : \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that f'(x) < 2 f(x) and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval **[IIT-JEE 2013]**

(A) (2e – 1, 2e) (B) (e –1, 2e –1) (C) $\left(\frac{e-1}{2}, e-1\right)$ (D) $\left(0, \frac{e-1}{2}\right)$

48. The area enclosed by the curves y = sin x + cos x and y = |cos x – sin x| over the interval $\left[0, \frac{\pi}{2}\right]$ is

[IIT-JEE 2013]

(A) $4\left(\sqrt{2}-1\right)$ (B) $2\sqrt{2}\left(\sqrt{2}-1\right)$ (C) $2\left(\sqrt{2}+1\right)$ (D) $2\sqrt{2}\left(\sqrt{2}+1\right)$

49*. For a $\in$ R (the set of all real numbers), a $\neq -1$, **[IIT-JEE 2013]**

$$\lim_{n\to\infty} \frac{(1^a + 2^a + ... + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+....+(na+n)]} = \frac{1}{60}$$

Then a =

(A) 5 (B) 7 (C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. If f(a + b – x) = f(x), then $\int_a^b$ xf(x) dx is equal to : **[AIEEE 2003]**

(A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}\int_a^b f(x)\,dx$ (C) $\frac{b-a}{2}\int_a^b f(x)dx$ (D) $\frac{a+b}{2}\int_a^b f(a+b+x)\,dx$

2. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x \sin x}$ is : **[AIEEE 2003]**

(A) 3 (B) 2 (C) 1 (D) – 1

3. The value of the integral $I = \int_0^1 x(1-x)^n\,dx$ is : **[AIEEE 2003]**

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n+2}$ (C) $\frac{1}{n+1} - \frac{1}{n+2}$ (D) $\frac{1}{n+1} + \frac{1}{n+2}$

4. Let $\frac{d}{dx}$ F(x) = $\left(\frac{e^{\sin x}}{x}\right)$, x > 0. If $\int_1^4 \frac{3}{x} e^{\sin x^3}\,dx$ = F (k) – F(1), then one of the possible values of k, is :

(A) 15 (B) 16 (C) 63 (D) 64 **[AIEEE 2003]**

5. Let f(x) be a function satisfying f′(x) = f(x) with f(0) = 1 and g(x) be a function that satisfies f(x) + g(x) = x^2. Then the value of the integral $\int_0^1$ f(x)g(x) dx, is : **[AIEEE 2003]**

(A) $e - \frac{e^2}{2} - \frac{5}{2}$ (B) $e + \frac{e^2}{2} - \frac{3}{2}$ (C) $e - \frac{e^2}{2} - \frac{3}{2}$ (D) $e + \frac{e^2}{2} + \frac{5}{2}$

6. The area of the region bounded by the curves y = | x – 1| and y = 3 – | x | is : **[AIEEE 2003]**

(1) 2 sq unit (2) 3 sq unit (3) 4 sq unit (4) 6 sq unit

7. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n}$ = **[AIEEE 2004]**

(A) e (B) e – 1 (C) 1 – e (d) 1 + e

8. The value of $\int_{-2}^{3} |1 - x^2|\, dx$ is : **[AIEEE 2004]**

(A) $\frac{28}{3}$ (B) $\frac{14}{3}$ (C) $\frac{7}{3}$ (D) $\frac{1}{3}$

9. The value of $\int_{0}^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}\, dx$ is : **[AIEEE 2004]**

(A) 0 (B) 1 (C) 2 (D) 3

10. If $\int_0^{\pi} x\, f(\sin x)\, dx = A \int_0^{\pi/2} f(\sin x) dx$, then A is equals to : **[AIEEE 2004]**

(A) 0 (B) π (C) $\frac{\pi}{4}$ (D) 2π

11. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x g\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}\, dx$, then the value of $\frac{I_2}{I_1}$ is : **[AIEEE 2004]**

(A) 2 (B) – 3 (C) – 1 (D) 1

12. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is : **[AIEEE 2004]**

(A) 1 (B) 2 (C) 3 (D) 4

13. Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then, $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals : **[AIEEE 2005]**

(A) 18 (B) 12 (C) 36 (D) 24

14. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then : **[AIEEE 2005]**

(A) $I_3 > I_4$ (B) $I_3 = I_4$ (C) $I_1 > I_2$ (D) $I_2 > I_1$

15. $\lim_{n\to\infty} \left(\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{n}{n^2}\sec^2 1\right)$ equals to : **[AIEEE 2005]**

(A) $\frac{1}{2}$ tan 1 (B) tan 1 (C) $\frac{1}{2}$ cosec 1 (D) $\frac{1}{2}$ sec 1

16. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is : **[AIEEE 2005]**

(A) 4 (B) 3 (C) 2 (D) 1

17. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is : **[AIEEE 2005]**

(A) 1 : 1 : 1 (B) 2 : 1 : 2 (C) 1 : 2 : 3 (D) 1 : 2 : 1

18. Let f(x) be a nonnegative continuous function such that the area bounded by the curve y = f(x), x – axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is : **[AIEEE 2005]**

(A) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ (B) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$ (C) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ (D) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

19. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx$ is : **[AIEEE 2006]**

(A) $\frac{3}{2}$ (B) 2 (C) 1 (D) $\frac{1}{2}$

20. $\int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2 (x + 3\pi)]\, dx$ is equal to : **[AIEEE 2006]**

(A) $\left(\frac{\pi^4}{32}\right) + \left(\frac{\pi}{2}\right)$ (B) $\frac{\pi}{2}$ (C) $\left(\frac{\pi}{4}\right) - 1$ (D) $\frac{\pi^4}{32}$

21. $\int_0^{\pi} x f(\sin x)\, dx$ is equal to : **[AIEEE 2006]**

(A) $\pi\int_0^{\pi} f(\sin x)\, dx$ (B) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)\, dx$ (C) $\pi\int_0^{\pi/2} f(\cos x)\, dx$ (D) $\pi\int_0^{\pi} f(\cos x)\, dx$

22. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, $f(x) = \int_1^x \frac{\log t}{1+t}\, dt$. Then F(e) equals : **[AIEEE 2007]**

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2

23. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is : **[AIEEE 2007]**

(A) $\frac{2}{3}$ sq unit (B) 1 sq unit (C) $\frac{1}{6}$ sq unit (D) $\frac{1}{3}$ sq unit

24. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\, dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\, dx$. Then, which one of the following is true ? **[AIEEE 2008]**

(A) $I > \frac{2}{3}$ and $J > 2$ (B) $I < \frac{2}{3}$ and $J < 2$ (C) $I < \frac{2}{3}$ and $J > 2$ (D) $I > \frac{2}{3}$ and $J < 2$

25. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to : **[AIEEE 2008]**

(1) $\frac{5}{3}$ sq unit (2) $\frac{1}{3}$ sq unit (3) $\frac{2}{3}$ sq unit (4) $\frac{4}{3}$ sq unit

26. $\int_0^{\pi} [\cot x]\, dx$, where $[\cdot]$ denotes the greatest integer function, is equal to : **[AIEEE 2009]**

(A) 1 (B) – 1 (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

27. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is : **[AIEEE 2009]**

(A) 6 sq unit (B) 9 sq unit (C) 12 sq unit (D) 3 sq unit

28. Let p(x) be a function defined on **R** such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(A) = 41$. Then $\int_0^1 p(x)dx$ equals : **[AIEEE 2010]**

(A) 21 (B) 41 (C) 42 (D) $\sqrt{41}$

29. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is :

(A) $4\sqrt{2}+2$ (B) $4\sqrt{2}-1$ (C) $4\sqrt{2}+1$ (D) $4\sqrt{2}-2$ **[AIEEE 2010]**

30. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\ \sin t\ dt$. Then f has: **[AIEEE 2011]**

(A) local maximum at π and 2π

(B) local minimum at π and 2π

(C) local minimum at π and local maximum at 2π

(D) local maximum at π and local minimum at 2π.

31. Let [.] denote the greatest integer function then the value of $\int_0^{1.5} x[x^2]dx$ is : **[AIEEE 2011]**

(A) 0 (B) $\frac{3}{2}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$

32. The area of the region enclosed by the curves y = x, x = e, $y = \frac{1}{x}$ and the positive x-axis is :

(A) 1 square units (B) $\frac{3}{2}$ square units (C) $\frac{5}{2}$ square units (D) $\frac{1}{2}$ square units

[AIEEE 2011]

33. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is :

(A) $\frac{32}{3}$ (B) $\frac{16}{3}$ (C) $\frac{8}{3}$ (D) 0

34. The area bounded between the parabola $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line y = 2 is :

(A) $20\sqrt{2}$ (B) $\frac{10\sqrt{2}}{3}$ (C) $\frac{20\sqrt{2}}{3}$ (D) $10\sqrt{2}$ **[AIEEE 2012]**

35*. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2} dx$ is : **[AIEEE 2011]**

(A) $\frac{\pi}{8}\log 2$ (B) $\frac{\pi}{2}\log 2$ (C) log2 (D) πlog2

36. **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$ **[AIEEE 2013]**

Statement-II : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(A) Statement-I is true; Statement-II is true, Statement-II is a correct explanation for Statement-I.

(B) Statement-I is true; Statement-II is true, Statement-II is not a correct explanation for Statement-I.

(C) Statement-I is true; Statement-II is false.

(D) Statement-I is false; Statement-II is true.

37. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis and lying in the first quadrant is:

(A) 9 (B) 36 (C) 18 (D) $\frac{27}{4}$ **[AIEEE 2013]**

38. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\,dt,\ x \in R$, which are parallel to the line y = 2x, are equal to : **[AIEEE 2013]**

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

EXERCISE # 4

NCERT BOARD QUESTIONS

1. Evaluate $\int_{-1}^{2}(7x-5)\,dx$ as a limit of sums.

2. Evaluate $\int_{0}^{\frac{\pi}{2}}\frac{\tan^7 x}{\cot^7 x+\tan^7 x}dx$

3. Find $\int_{2}^{8}\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$

4. Find $\int_{0}^{\frac{\pi}{4}}\sqrt{1+\sin 2x}\,dx$

5. Show that $\int_{0}^{\frac{\pi}{2}}\frac{\sin^2 x}{\sin x+\cos x}dx$

6. $\int_{-2}^{2}|x\cos\pi x|\,dx =$

Evaluate the following a limit of sums:

7. $\int_{0}^{2}(x^2+3)\,dx =$

8. $\int_{0}^{2}e^x dx =$

Evaluate the following

9. $\int_{0}^{1}\frac{dx}{e^x+e^{-x}}$

10. $\int_{0}^{\frac{\pi}{2}}\frac{\tan x dx}{1+m^2\tan^2 x}$

11. $\int_{1}^{2}\frac{dx}{\sqrt{(x-1)(2-x)}}$

12. $\int_{0}^{1}\frac{xdx}{\sqrt{1+x^2}} =$

13. $\int_{0}^{\pi}x\sin x\cos^2 x dx$

14. $\int_{0}^{\frac{1}{2}}\frac{dx}{(1+x^2)\sqrt{1-x^2}}$

15. $\int_{0}^{\pi}\frac{x}{1+\sin x}$

16. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}}$

17. $\int_{0}^{\frac{\pi}{2}}\frac{dx}{(a^2\cos^2 x+b^2\sin^2 x)^2}$

18. $\int_{0}^{1}x\log(1+2x)\,dx$

19. $\int_{0}^{\pi}x\log\sin x\,dx$

20. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\log(\sin x+\cos x)dx$

ANSWERS

EXERCISE # 1

PART # I

A-1.	(A)	**A-2.**	(C)	**A-3.**	(C)	**A-4.**	(C)	**A-5.**	(D)	**A-6.**	(A)	**A-7.**	(A)
A-8.	(A)	**A-9.**	(B)	**A-10.**	(C)	**A-11*.**	(A,C)	**A-12*.**	(A,B,C,D)			**B-1.**	(C)
B-2.	(C)	**B-3.**	(A)	**B-4.**	(A)	**B-5.**	(D)	**B-6*.**	(A, B)	**B-7.**	(C)	**B-8*.**	(A, B, C)
B-9*.	(A, D)	**C-1.**	(B)	**C-2.**	(B)	**C-3.**	(D)	**C-4.**	(B)	**D-1.**	(D)	**D-2.**	(B)
D-3.	(C)	**D-4.**	(C)	**E-1**	(C)	**E-2.**	(D)	**E-3.**	(C)	**E-4.**	(C)	**E-5.**	(B)
E-6*.	(B, D)	**E-7.**	(A)	**E-8.**	(B)	**E-9.**	(A)	**E-10.**	(B)	**E-11.**	(C)		

PART # II

A-1. (i) $\frac{104}{5}$ (ii) $-\ell n4$ (iii) $-\frac{10}{21}$ **A-2.** (i) π (ii) $\frac{\pi}{4}$ (iii) $4+\ell n5$ (iv) $\frac{8}{21}$

A-3. (i) $\frac{\pi-2}{2}$ (ii) $\frac{1}{2}\ell n\left(\frac{e}{2}\right)$ (iii) 1 (iv) $\frac{\pi}{6}-\frac{2}{9}$

A-4. (i) $\frac{\pi}{2}-\ell n2$ (ii) $\frac{4-\pi}{4\sqrt{2}}$ (iii) $-\frac{\pi}{8}(b-a)^2$ (iv) $\pi\left(1-\frac{1}{\sqrt{3}}\right)-\ell n\,4$

A-5. (i) $\frac{\pi}{4}$ (ii) $2\left(\frac{5}{6}-\ell n\,2\right)$ (iii) $\ell n\left(\frac{9}{8}\right)$ (iv) $\frac{\pi}{2}$ (v) $\frac{1}{20}\ell n3$ **A-6.** –1

B-2. (i) $5-\sqrt{2}-\sqrt{3}$ (ii) cos 1 + cos 2 + cos 3 + 3

B-3. (i) 2e – 2 (ii) $2-\sqrt{2}$ (iii) 29 (iv) $\frac{\pi^2}{6\sqrt{3}}$ **B-4.** (i) 0 (ii) 0

B-5. (i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{a}{2}$ (iv) (a + b) $\frac{\pi}{4}$ **B-6.** (i) $\frac{3}{2}$ (ii) 40 **C-1.** $4\sqrt{2}$ **C-3.** $\left(0,\frac{5}{2}\right]$

C-4. (i) $\frac{4}{15}$ (ii) $\frac{8\pi}{15}$ (iii) $\frac{\pi}{2}$ **D-1.** (i) $\frac{\pi}{2}$ (ii) 2 **E.1.** $\frac{51}{4}$ sq. unit. **E.2.** $\frac{17}{27}$

E.3. (i) 32/3 sq. unit (ii) 4/3 sq. units **E.4.** $\frac{(e+1)\pi}{1+\pi^2}$ **E.5.** $\left(\frac{3}{\log_e 2}-\frac{4}{3}\right)$ sq. units

E.7. 4 sq. units. **E.8.** $\frac{16}{3}$ sq. units **E.10.** $\frac{2}{3}a^2$

PART # III

1. (A-q), (B-r), (C-p), (D-s) **2.** (A-s), (B-s), (C-q), (D-p) **3.** (C) **4.** (A)

5.	(A)	**6.**	(D)	**7.**	(A)	**8.**	(C)	**9.**	(A)	**10.**	(C)	**11.**	(B)
12.	(A)	**13.**	(C)	**14.**	(A)								

EXERCISE # 2

PART # I

1.	(D)	**2.**	(D)	**3.**	(B)	**4.**	(B)	**5.**	(A)	**6.**	(C)	**7.**	(D)
8.	(D)	**9.**	(B)	**10.**	(D)	**11.**	(D)	**12.**	(B)	**13.**	(C)	**14.**	(C)
15.	(C)	**16*.**	(A,B,D)	**17*.**	(A, B)	**18*.**	(B, C)	**19.**	(A)	**20.**	(A)	**21.**	(A)
22.	(D)	**23.**	(D)	**24.**	(D)	**25.**	(A)	**26.**	(A)	**27.**	(C)	**28.**	(C)

PART # II

1. 0 **2.** 0 **4.** $\begin{cases} x-\frac{x^2}{2} & \text{If } 0 \le x \le 1 \\ \frac{1}{2} & \text{If } 1 < x \le 2 \\ \frac{(x-2)^3}{3}+\frac{1}{2} & \text{If } 2 < x \le 3 \end{cases}$ **5.** $\frac{-3e^x}{2(e-1)}-3x$

6. C = $-\frac{\pi}{6}$ or $\frac{\pi}{3}$ **7.** $\frac{4}{3\sqrt{3}}\left(\sqrt{3}-1\right)$ **8.** (3 – 1) sq. units **9.** $\frac{1}{2}\ell n\, 2-\frac{1}{4}$

10. (i) m = 1, (ii) m = ∞ ; A_{min} = 4/3 **11.** $3\pi a^2$ **12.** $\frac{8a^2}{15}$ **13.** $f(x) = x^3 - x^2$

EXERCISE # 3

PART # I

1. (A) **2.** (B) **3.** (A) **4.** (A) **5.** (C) **6.** 2π

7. $\frac{4\pi}{\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\right)$ **8.** (A) **9.** (C) **10.** (C) **11.** $\frac{24}{5}\left[e\cos\left(\frac{1}{2}\right)+\frac{1}{2}e\sin\left(\frac{1}{2}\right)-1\right]$

12. (D) **13.** $\frac{1}{3}$ square units **14.** $\frac{125}{3}$ square units. **15*.** (C, D) **16.** 5051

17. (A-q) (B-r) (C-q) (D-s) **18.** (A) **19.** **(A-s) (B-s) (C-p) (D-r)** **20*.** (A, D) **21.** (A)

22. (A) **23.** (B) **24.** (B) **25.** (A) **26.** (D) **27.** (B) **28*** (B, C)

29. **0** **30.** (B,C,D) **31.** (B) **32.** (A) **33*.** (B, C) **34.** 4 **35.** (B)

36. (C) **37.** (A) **38.** (B) **39.** (A) **40.** (B) **41.** (C) **42.*** (A,B,D)

43. (B) **44.** (B) **45.** (C) **46*.** (A,B,C) **47.** (D) **48.** (B) **49*.** (B, D)

PART # II

1. (B) **2.** (C) **3.** (C) **4.** (D) **5.** (C) **6.** (3) **7.** (B)

8. (A) **9.** (C) **10.** (B) **11.** (A) **12.** (A) **13.** (A) **14.** (C)

15. (A) **16.** (D) **17.** (A) **18.** (D) **19.** (A) **20.** (B) **21.** (C)

22. (A) **23.** (C) **24.** (B) **25.** (4) **26.** (C) **27.** (B) **28.** (A)

29. (D) **30.** (D) **31.** (C) **32.** (B) **33.** (B) **34.** (C) **35*.** (B,C)

36. (D) **37.** (A) **38.** (A)

EXERCISE # 4

1. $\frac{-9}{2}$ **2.** $\frac{\pi}{4}$ **3.** 3 **4.** 1 **5.** $\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$ **6.** $\frac{8}{\pi}$

7. $\frac{26}{3}$ **8.** e^2-1 **9.** $\tan^{-1}e-\frac{\pi}{4}$ **10.** $\frac{\log m}{m^2-1}$ **11.** π

12. $\sqrt{2}-1$ **13.** $\frac{\pi}{3}$ **14.** $\frac{\sqrt{2}}{2}\tan^{-1}\frac{\sqrt{2}}{3}$ **15.** π **16.** $\frac{3}{2}$ **17.** $\frac{\pi}{4}\left(\frac{a^2+b^2}{a^3+b^3}\right)$

18. $\frac{3}{8}\log 3$ **19.** $\frac{\pi^2}{2}\log\frac{1}{2}$ **20.** $\frac{\pi}{4}\log\frac{1}{2}$