arride learning

METHODS OF DIFFERENTIATION

Contents

Syllabus

Derivative of a function, derivative of the sum, difference, product and quotient of two functions, chain rule, derivatives of polynomial, rational, trigonometric, inverse trigonometric, exponential and logarithmic functions. Derivatives of implicit functions, derivatives up to order two,

Name : ____________________________ Contact No. __________________

ARRIDE LEARNING ONLINE E-LEARNING ACADEMY

A-479 indra Vihar, Kota Rajasthan 324005

Contact No. 8033545007

METHODS OF DIFFERENTIATION

KEY CONCEPTS

1. DEFINITION :

If x and x + h belong to the domain of a function f defined by y = f (x), then $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ if it exists, is called the **DERIVATIVE** of f at x & is denoted by f' (x) or $\frac{dy}{dx}$. we have therefore

$$D[f(x)] = f'(x) = \frac{dy}{dx} = y_1(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

2. The derivative of a given function f at a point x = a of its domain is defined as $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$, provided the limit exists & is denoted by f' (a) . Note that alternatively, we can define

$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$, provided the limit exists

3. DERIVATIVE OF f (x) FROM THE FIRST PRINCIPLE / ab INITIO METHOD / DELTA METHOD:

If f (x) is a derivable function then, $\lim_{\delta x\to 0} \frac{\delta y}{\delta x} = \lim_{\delta x\to 0} \frac{f(x+\delta x)-f(x)}{\delta x} = f'(x) = \frac{dy}{dx}$

4. THEOREMS ON DERIVATIVES :

If f and g are derivable function of x, then,

(i) $\frac{d}{dx}(f \pm g) = \frac{df}{dx} \pm \frac{dg}{dx}$

(ii) $\frac{d}{dx}(cf) = c\frac{df}{dx}$, where c is any constant

(iii) $\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$ known as " **PRODUCT RULE**"

(iv) $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\left(\frac{df}{dx}\right) - f\left(\frac{dg}{dx}\right)}{g^2}$ where $g \neq 0$ known as " **QUOTIENT RULE**"

(v) If y = f(u) & u = g (x) then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ known as **"CHAIN RULE"**

5. DERIVATIVE OF STANDARDS FUNCTIONS :

(i) $D(x^n) = n.x^{n-1}$; $x \in R$, $n \in R$, $x > 0$

(ii) $D(e^x) = e^x$

(iii) $D(a^x) = a^x . \ln a \quad a > 0$

(iv) $D(\ln x) = \frac{1}{x}$

(v) $D(\log_a x) = \frac{1}{x} \log_a e$

(vi) D (sinx) = cosx

(vii) D (cosx) = - sinx

(viii) D (tanx) = $\sec^2 x$

(ix) D (secx) = secx . tanx

(x) D (cosecx) = - cosecx . cotx

(xi) D (cotx) = - $\text{cosec}^2 x$

(xii) D (constant) = 0 where $D = \frac{d}{dx}$

6. INVERSE FUNCTIONS AND THEIR DERIVATIVES :

(a) Theorem : If the inverse functions f & g are defined by y = f (x) & x = g (y) & if $f'(x)$ exists & $f'(x) \neq 0$ then $g'(y) = \frac{1}{f'(x)}$. This result can also be written as , if $\frac{dy}{dx}$ exists & $\frac{dy}{dx} \neq 0$, then $\frac{dx}{dy} = 1/\left(\frac{dy}{dx}\right)$ or $\frac{dy}{dx}.\frac{dx}{dy} = 1$ or $\frac{dy}{dx} = 1/\left(\frac{dy}{dx}\right)\left[\frac{dx}{dy} \neq 0\right]$

(b) Results :

(i) $D(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, \ -1 < x < 1$

(ii) $D(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}, \ -1 < x < 1$

(iii) $D(\tan^{-1} x) = \frac{1}{1+x^2}, \quad x \in R$

(iv) $D(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}, \quad |x| > 1$

(v) $D(\operatorname{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}, \quad |x| > 1$

(vi) $D(\cot^{-1} x) = \frac{-1}{1+x^2}, \quad x \in R$

Note : In general if $y = f(u)$ then $\frac{dy}{dx} = f'(u).\frac{du}{dx}$.

7. LOGARITHMIC DIFFERENTIATION : To find the derivative of :

(i) A function which is the product or quotient of a number of function **OR**

(ii) A function of the form $[f(x)]^{g(x)}$ where f & g are both derivable, it will be found convenient to take the logarithm of the function first & then differentiate.This is called **LOGARITHMIC DIFFERENTIATION**

8. IMPLICIT DIFFERENTIATION : $\phi(x, y) = 0$

(i) In order to find dy /dx, in the case of implicit functions, we differentiate each term w.r.t. x regarding y as a functions of x & then collect terms in dy/dx together on one side to finally find dy / dx

(ii) In answers of dy/dx in the case of implicit functions, both x & y are present.

9. PARAMETRIC DIFFERENTIATION :

If $y = f(x)$ & $x = g(\theta)$ where θ is a parameter , then $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

10. DERIVATIVE OF A FUNCTION W.R.T. ANOTHER FUNCTION :

Let y= f (x) ; z = g (x) then $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{f'(x)}{g'(x)}$

11. DERIVATIVE OF ORDER TWO & THREE : Let a function y = f (x) be defined on an open interval (a, b) . It's derivative, if it exists on (a, b) is a certain function $f'(x)$ [or (dy / dx) or y '] & is called the first derivative of y w. r. t. x. If it happens that the first derivative has a derivative on (a , b) then this derivative is called second derivative of y w.r.t. x & is denoted by $f''(x)$ or (d^2y/dx^2) or y''.

Similarly, the 3rd order derivative of y w.r.t x , if it exists , is defined by $\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$ It is also denoted by $f'''(x)$ or y'''

12. If $F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$, where f, g, h. l, m, n, u, v, w are differentiable functions of x

then $F'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l'(x) & m'(x) & n'(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix}$

OBJECTIVE

SINGLE CORRECT QUESTION

1. If y = $\frac{x^4+4}{x^2-2x+2}$ then $\left.\frac{dy}{dx}\right]_{x=1/2}$ is :

(A) 3 (B) –1 (C) 4 (D) none

2. If f′(x) = $\sqrt{2x^2-1}$ and y = f(x^2), then $\frac{dy}{dx}$ at x = 1 is equal to

(A) 2 (B) 1 (C) – 2 (D) –1

3. A function f, defined for all positive real numbers, satisfies the equation f(x^2) = x^3 for every x >0. Then the value of f’ (4) =

(A) 12 (B) 3 (C) 3/2 (D) cannot be determined

4. If $y = f\left(\frac{3x+4}{5x+6}\right)$ & $f'(x) = \tan x^2$ then $\frac{dy}{dx} =$

(A) $\tan x^3$ (B) $-2\tan\left[\frac{3x+4}{5x+6}\right]^2 \cdot \frac{1}{(5x+6)^2}$

(C) $f\left(\frac{3\tan x^2+4}{5\tan x^2+6}\right)\tan x^2$ (D) None

5. Let h(x) = f(x) – f(2x) and g(x) = f(x) – f(4x), if h'(1) = 5 and h'(2) = 7, then g'(1) is :

(A) 19 (B) 9 (C) 17 (D) 14

6. Let f(x) = x^n, n being a non - negative integer. The number of values of n for which f’ (p +q) = f’ (p) + f’ (q) is valid for all p,q > 0 is :

(A) 0 (B) 1 (C) 2 (D) none of these

7. If $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right)$ = ax + b, then the value of 'a' and 'b' are respectively

(A) 2 and 1 (B) – 2 and 1 (C) 2 and – 1 (D) None of these

8. If y = f(x) is an odd differentiable function defined on (–∞, ∞) such that f′(3) = – 2, then f′(– 3) is equal to

(A) 4 (B) 2 (C) – 2 (D) 0

9. If $y = xe^{x^2}$, find $\frac{dy}{dx}$

*(A) $e^{x^2}(2x^2+1)$ (B) $e^{x^2}(2x^2-1)$ (C) $e^{x^2}(x^2+1)$ (D) None of these

10. If $f(x) = e^{e^x}$. Let g (x) be it's inverse then g' (x) at x = 2 is–

(A) $\frac{\ln 2}{2}$ (B) $\frac{1}{2\ln 2}$ (C) $2\ln 2$ (D) e^2

11. If y = x^{x^2} then $\frac{dy}{dx}$ is equal to

(A) 2 ℓn x. x^{x^2} (B) (2 ℓn x + 1). x^{x^2} (C) (2 ℓn x + 1). x^{x^2+1} (D) none of these

12. If $f(x) = \log_x(\ell n\, x)$, then $f'(x)$ at $x = e$ is equal to

(A) 1/e (B) e (C) 1 (D) zero

13. If $y = x \log(x^2 - 3)$, find $\frac{dy}{dx}$

(A) $\frac{2x^2}{x^2-3} + \log(x^2-3)$ (B) $\frac{x^2}{x^2-3} + \log(x^2-3)$ (C) $\frac{2x}{x^2-3} + \log(x^2-3)$ (D) None of these

14. If $y = \log \frac{\sqrt{x^2+1}}{\sqrt[3]{2+x}}$, find $\frac{dy}{dx}$

(A) $\frac{x}{x^2+1} - \frac{1}{3(2+x)}$ (B) $\frac{-x}{x^2+1} + \frac{1}{3(2+x)}$ (C) $\frac{x}{x^2+1} + \frac{1}{3(2+x)}$ (D) None of these

15. If $x^y \cdot y^x = 1$, then $\frac{dy}{dx}$ equals to

(A) $\frac{y(x\ell n\, y - 1)}{x(y\ell n\, x - y)}$ (B) $\frac{y(x\ell n\, y - y)}{x(y\ell n\, x + x)}$ (C) $\frac{y(x\ell n\, y + y)}{x(y\ell n\, x - x)}$ (D) $\frac{-y(x\,\ell n\, y + y)}{x(y\ell n\, x + x)}$

16. If $y = \frac{1}{x}$, then $\frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}}$ equals to

(A) 0 (B) $-\frac{1}{x^2}$ (C) $\sqrt{\frac{1+y^4}{1+x^4}}$ (D) None of these

17. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{(1+x)^2}$ (B) $-\frac{1}{(1+x)^2}$ (C) $\frac{1}{(1+x^2)}$ (D) $\frac{1}{(1+x)}$

18. If $ax^2 + 2hxy + by^2 = 0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{y}{x}$ (B) $\frac{x}{y}$ (D) $-\frac{x}{y}$ (D) None of these

19. If $\sin(xy) + \cos(xy) = 0$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{y}{x}$ (B) $-\frac{y}{x}$ (C) $-\frac{x}{y}$ (D) $\frac{x}{y}$

20. If $\sin xy + \frac{x}{y} = x^2 - y$, find $\frac{dy}{dx}$

(A) $\frac{y\,(2xy - y^2\cos xy - 1)}{xy^2\cos xy - x + y^2}$ (B) $\frac{x\,(2xy - y^2\cos xy + 1)}{xy^2\cos xy + x - y^2}$

(C) $\frac{y\,(2xy - y^2\cos xy + 1)}{xy^2\cos xy + x - y^2}$ (D) None of these

21. If $y = \tan^{-1}(\sin\sqrt{x})$, find $\frac{dy}{dx}$

(A) $\frac{1}{2\sqrt{x}}\frac{\cos\sqrt{x}}{1+\sin^2\sqrt{x}}$ (B) $\frac{1}{\sqrt{x}}\frac{\cos\sqrt{x}}{1+\sin^2\sqrt{x}}$ (C) $-\frac{1}{2\sqrt{x}}\frac{\cos\sqrt{x}}{1+\sin^2\sqrt{x}}$ (D) None of these

22. If $y = \cos^{-1}\left(\frac{a+b\cos x}{b+a\cos x}\right)$, find $\frac{dy}{dx}$

(A) $\frac{\sqrt{b^2-a^2}}{b+a\cos x}$ (B) $\frac{\sqrt{b^2-a^2}}{-b+a\cos x}$ (C) $\frac{\sqrt{a^2-b^2}}{b-a\cos x}$ (D) None of these

23. If $y = \tan^{-1}(\sec x + \tan x)$ find $\frac{dy}{dx}$

(A) 1 (B) – 1/2 (C) 2 (D) 1/2

24. If $y = \sin^{-1}\left(x\sqrt{1-x} + \sqrt{x}\,\sqrt{1-x^2}\right)$ and $\frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}}$ + p, then p is equal to

(A) 0 (B) $\frac{1}{\sqrt{1-x}}$ (C) $\sin^{-1}\sqrt{x}$ (D) $\frac{1}{\sqrt{1-x^2}}$

25. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is-

(A) $\frac{-8}{\pi^2+4}$ (B) $\frac{4}{\pi^2+4}$ (C) $\frac{8}{\pi^2+4}$ (D) Does not exists

26. If $x = a\cos^3 t$, $y = a\sin^3 t$, find $\frac{dy}{dx}$

(A) $\tan t$ (B) $-\tan t$ (C) $\cot t$ (D) None of these

27. Diff. $\sin x^3$ w.r.t. $\sec^2 x^2$

(A) $\frac{3x\cos x^3}{4\sec^2 x^2\tan x^2}$ (B) $\frac{3\cos x^3}{4\sec^2 x^2\tan x^2}$ (C) $\frac{3x\cos^3 x}{4\sec^2 x^2\tan^2 x}$ (D) None of these

28. If $y = x - x^2$, then the derivative of y^2 w.r.t. x^2 is

(A) $2x^2 + 3x - 1$ (B) $2x^2 - 3x + 1$ (C) $2x^2 + 3x + 1$ (D) none of these

29. Derivative of $\left(\frac{\tan^{-1}x}{1+\tan^{-1}x}\right)$ w.r.t. $\tan^{-1}x$ is:

(A) $\left(\frac{1}{1+\tan^{-1}x}\right)$ (B) – 1 (C) $\frac{1}{\left(1+\tan^{-1}x\right)^2}$ (D) $\frac{-1}{(1+\tan^{-1}x)^2}$

30. Let f (x) be defined for all x > 0 & be continuous. Let f (x) satisfy $f\left(\frac{x}{y}\right)=f(x)-f(y)$ for all x, y & f (e) = 1. Then:

(A) f (x) is bounded (B) $f\left(\frac{1}{x}\right)\to$ as $x\to 0$ (C) $x.f(x)\to 1$ as $x\to 0$ (D) $f(x)=\ell nx$

31. The funcion f(x) = e^x+x, being differentiable and one to one to one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point f(ℓ n2) is

(A) $\frac{1}{\ell n2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) none

32. If x = at^2, y = 2at, then $\frac{d^2y}{dx^2}$ is equal to

(A) $-\frac{1}{t^2}$ (B) $\frac{1}{2at^2}$ (C) $-\frac{1}{t^3}$ (D) $-\frac{1}{2at^3}$

33. Let f(x) be a polynomial in x. Then the second derivative of f(e^x) w.r.t. x is

(A) $f''(e^x).\ e^x + f'(e^x)$ (B) $f''(e^x).\ e^{2x} + f'(e^x).\ e^{2x}$

(C) $f''(e^x)\ e^{2x}$ (D) $f''(e^x).\ e^{2x} + f'(e^x).\ e^x$

34. If y = x + e^x then $\frac{d^2x}{dy^2}$ is :

(A) e^x (B) $-\frac{e^x}{(1+e^x)^3}$ (C) $-\frac{e^x}{(1+e^x)^2}$ (D) $\frac{-1}{(1+e^x)^3}$

35. A curve is parametrically represented by y = R(1 – cosθ) & x = R(θ – sinθ), then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is-

(A) $-\frac{1}{2R}$ (B) $\frac{1}{4R}$ (C) $\frac{1}{2R}$ (D) $-\frac{1}{4R}$

36. If x = ℓnt & y = t^2 – 1 then $y''(1)$ at t = 1 is-

(A) 2 (B) 4 (C) 3 (D) none

37. $x^4 + 3x^2y^2 + 7xy^3 + 4x^3y - 15y^4 = 0$, then $\frac{d^2y}{dx^2}$ at (1, 1) is–

(A) 2 (B) 1 (C*) 7 (D) 0

38. If x=sint and y=sin3t, then the value of 'K' for which $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+Ky=0$ is

(A) 3 (B) 6 (C) 12 (D) 9

39. If f is twice differentiable such that $f''(x) = -f(x)$, $f'(x) = g(x)$

$$h'(x) = [f(x)]^2 + [g(x)]^2 \text{ and}$$
$$h(0) = 2,\ h(1) = 4$$

then the equation $y = h(x)$ represents :

(A) a curve of degree 2
(B) a curve passing through the origin
(C) a straight line with slope 2
(D) a straight line with y intercept equal to –2

40. If $x = \operatorname{cosec}\theta - \sin\theta$; $y = \operatorname{cosec}^n\theta - \sin^n\theta$, then $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 - n^2y^2$ equals to

(A) n^2 (B) $2n^2$ (C) $3n^2$ (D) $4n^2$

41. If $2x = y^{1/5} + y^{-1/5}$, then $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ equals to

(A) 5y (B) 25 y (C) $25y^2$ (D) y + 25

42. If $y = \sin(m \sin^{-1} x)$, then $(1 - x^2) y'' - xy'$ is equal to

(A) m^2y (B) my (C) $-m^2y$ (D) None of these

43. If $y = \ell n\left(\frac{x}{a+bx}\right)^x$, then $x^3 \frac{d^2y}{dx^2}$ is equals to

(A) $\left(x\frac{dy}{dx} - y\right)^2$ (B) $\left(y\frac{dy}{dx} - x\right)^2$ (C) $\left(x\frac{dy}{dx} + y\right)^2$ (D) None of these

44. If $y^2 = P(x)$, is a polynomial of degree 3, then $2\left(\frac{d}{dx}\right)\left(y^3.\frac{d^2y}{dx^2}\right)$ equals :

(A) $P'''(x) + P'(x)$ (B) $P''(x) . P'''(x)$ (C) $P(x) . P'''(x)$ (D) a constant

45. If $y = \tan^{-1}\left(\frac{1}{x^2+x+1}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right) + ...$ to n terms,then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{x^2+n^2} - \frac{1}{x^2+1}$
(B) $\frac{1}{(x+n)^2+1} - \frac{1}{x^2+1}$
(C) $\frac{1}{x^2+(n+1)^2} - \frac{1}{x^2+1}$
(D) None of these

46. If $x^2 + y^2 = 1$, then ;

(A) $yy'' - 2(y')^2 + 1 = 0$ (B) $yy'' + (y')^2 + 1 = 0$ (C) $yy'' - (y')^2 - 1 = 0$ (D) $yy'' + 2(y')^2 + 1 = 0$

[JEE 2000, Screening, 1 out of 35]

47. If $ln(x + y) - 2xy = 0$, then $y'(0)$ is equal to **[JEE 2004, Screening]**

(A) 0 (B) 1 (C) –1 (D) $\frac{1}{3}$

48. If $y = y(x)$ and it follows the relation $x\cos y + y\cos x = \pi$ then $y''(0)$ **[JEE 2005, Screening]**

(A) 1 (B) –1 (C) π (D) $-\pi$

49. If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5$, then $F(10)$ is equal to-

(A) 5 (B) 10 (C) 0 (D) 15

[JEE 2006, (3,–1) out of 184]

50. $\frac{d^2x}{dy^2}$ equals **[JEE 2007, (3,–1) out of 81]**

(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

AIEEE

51. If $y = (x + \sqrt{1+x^2})^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is - **[AIEEE 2002]**

(A) n^2y (B) $-n^2y$ (C) $-y$ (D) $2x^2y$

52. If $\sin y = x \sin(a + y)$, then $\frac{dy}{dx}$ is - **[AIEEE 2002]**

(A) $\frac{\sin a}{\sin^2(a+y)}$ (B) $\frac{\sin^2(a+y)}{\sin a}$ (C) $\sin a \sin^2(a + y)$ (D) $\frac{\sin^2(a-y)}{\sin a}$

53. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is- **[AIEEE 2002]**

(A) $\frac{1+x}{1+\log x}$ (B) $\frac{1-\log x}{1+\log x}$ (C) not defined (D) $\frac{\log x}{(1+\log x)^2}$

54. If $x = e^{y+e^{y+\ldots\infty}}$, $x > 0$ then $\frac{dy}{dx}$ is- **[AIEEE 2004]**

(A) $\frac{x}{1+x}$ (B) $\frac{1}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1+x}{x}$

55. If $x^m y^n = (x + y)^{m+n}$, then $\frac{dy}{dx}$ is - **[AIEEE 2006]**

(A) $\frac{x+y}{xy}$ (B) xy (C) $\frac{x}{y}$ (D) $\frac{y}{x}$

56. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then y′ (A) equals : **[AIEEE 2009]**

(A) 1 (B) log 2 (C) – log 2 (D) – 1

57. Let $f : (-1, 1) \to \mathbf{R}$ be a differentiable function with f(0) = – 1 and f′(0) = 1. Let g(x) = [f(2f(x) + 2)]^2. Then g′(0).

(A) – 4 (B) 0 (C) – 2 (D) 4 **[AIEEE 2009]**

MULTIPLE CORRECT QUESTION

---

58. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$ then $\frac{dy}{dx}$

(A) $\frac{1}{2y-1}$ (B) $\frac{x}{x-2y}$ (C) $\frac{1}{\sqrt{1+4x}}$ (D) $\frac{y}{2x+y}$

59. If $2^x + 2^y = 2^{x+y}$ then $\frac{dy}{dx}$ has the value equal to :

(A) $-\frac{2^y}{2^x}$ (B) $\frac{1}{1-2^x}$ (C) $1-2^y$ (D) $\frac{2^x(1-2^y)}{2^y(2^x-1)}$

60. If x = cos t, y = $\log_e t$ then

(A) $\frac{dy}{dx} = -\frac{2}{\pi}$ at t = $\frac{\pi}{2}$ (B) $\frac{dy}{dx} = \frac{4}{\pi^2}$ at t = $\frac{\pi}{2}$

(C) $\frac{dy}{dx} = \frac{144}{\pi^2}$ at t = $\frac{\pi}{6}$ (D) $\frac{dy}{dx} = -\frac{12}{\pi}$ at t = $\frac{\pi}{6}$

61. If f(x) = | (x – 4) (x – 5) |, then f′(x) is equal to

(A) –2x + 9, for all $x \in R$ (B) 2x – 9 if x > 5

(C) –2x + 9 if 4 < x < 5 (D) not defined for x = 4, 5

SUBJECTIVE

1. The value of *x* at which the first derivative of the function $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ w.r.t. x is 3/4, are

2. If $y = \log(\log(\sin\sqrt{x^2+1}))$, find $\frac{dy}{dx}$

3. If $y=\frac{u-1}{u+1}$ and $u=\sqrt{x}$, find $\frac{dy}{dx}$

4. If $x^3 + 8xy + y^3 = 64$, find $\frac{dy}{dx}$

5. If $y=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$, find $\frac{dy}{dx}$

6. If $x=\frac{a(1-t^2)}{1+t^2}$ & $y=\frac{2bt}{1+t^2}$, find $\frac{dy}{dx}$

7. If $x=ae^{\theta}(\sin\theta-\cos\theta), y=ae^{\theta}(\sin\theta+\cos\theta)$, find the value of $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$

8. If $x^y + y^x = (x + y)^{x+y}$, find $\frac{dy}{dx}$

9. Diff. $x^{\sin x}$ w.r.t. $(\sin x)^x$

10. If $y = x^2 + x^{\log x}$, find $\frac{dy}{dx}$

11. If $x^y = e^{x-y}$, then find $\frac{dy}{dx}$ in terms of x only.

12. If $y=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$, find $\frac{dy}{dx}$

13. If $y = \sin(\sin x)$, and $\frac{d^2y}{dx^2}+\frac{dy}{dx}\tan x+f(x)=0$, then $f(x)$ equal

14. If $\sqrt{x^2+y^2}=e^{\tan^{-1}(y/x)}$ Prove that $\frac{d^2y}{dx^2}=\frac{2(x^2+y^2)}{(x-y)^3}$, $x > 0$.

15. If $x=\sec\theta-\cos\theta$; $y=\sec^n\theta-\cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2-n^2(y^2+4)=0$

16. If $x \sin xy + 2x^2 = 0$. Find the value of K if $x\cos xy\left(y+x\frac{dy}{dx}\right)+\sin xy+Kx=0$

17. If $\sin y = x \sin(a + y)$, show that $\frac{dy}{dx}=\frac{\sin a}{1-2x\cos a+x^2}$ = $\frac{\sin^2(a+y)}{\sin a}$

18. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a^3.(x^3-y^3)$, prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

NCERT BOARD QUESTIONS

1. The derivative of log |x| is

2. If y = $\sqrt{\frac{1+x}{1-x}}$, then $\frac{dy}{dx}$ equals-

3. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then $\frac{dy}{dx}$ equals-

4. If y = $\cot^{-1}\sqrt{x^2-1}$ + $\sec^{-1}$ x, then $\frac{dy}{dx}$ equals-

5. If $x^2 + y^2 = t - \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx}$ equals-

6. If $x^2 e^y + 2xye^x + 13 = 0$, then $\frac{dy}{dx}$ equals-

7. The derivation of

$\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

8. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right]$ equals-

9. If $y=\frac{\sec x-\tan x}{\sec x+\tan x}$ then $\frac{dy}{dx}$ equals-

10. If y = $\sin^{-1}\sqrt{\sin x}$ then $\frac{dy}{dx}$ equals-

11. If $y=\frac{1}{(t+2)(t+1)}$, then $\frac{dy}{dt}$ equals-

12. If x = $\theta - \frac{1}{\theta}$ and y = $\theta + \frac{1}{\theta}$, then $\frac{dy}{dx}$ =

13. Derivative of $\sin^{-1}x$ w.r.t. $\cos^{-1}\sqrt{1-x^2}$ is-

14. If $\sqrt{\frac{\nu}{\mu}}+\sqrt{\frac{\mu}{\nu}}$ = 6 then $\frac{d\nu}{d\mu}$ =

15. If x = exp. $\tan^{-1}\left(\frac{y-x^2}{x^2}\right)$, then $\frac{dy}{dx}$ equals-

16. $\frac{d}{dx}\cos^{-1}\left(\frac{4x^3}{27}-x\right)=$

17. Derivative of $\sec^{-1}\left(\frac{1}{2x^2+1}\right)$ w.r.t $\sqrt{1+3x}$ at $x = \frac{-1}{3}$ is-

18. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)$ = log a then $\frac{dy}{dx}$ =

19. $\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\}$ =

20. If $y = \sqrt{(a-x)(x-b)} - (a-b)\tan^{-1}\sqrt{\left(\frac{a-x}{x-b}\right)}$, then $\frac{dy}{dx}$ equals

21. If sin $(x + y)$ + cos $(2x + 2y)$ = log $(3x + 3y)$, then $\frac{dy}{dx}$ is

22. Find the value of the expression $y^3\frac{d^2y}{dx^2}$ on the ellipse $3x^2 + 4y^2 = 12$. [Ans. $-\frac{9}{4}$]

23. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = xf(x)$, then find $\frac{dy}{dx}$ at $x = 1$.

24. $y = (\sin x + \cos x)^x$, then $\frac{dy}{dx}$ is

25. $y = \log_{\sin^{-1}x}\cos^{-1}x$, then $\frac{dy}{dx}$ is

26. $y = \sin^{-1}\left(\frac{e^{ax}-e^{-ax}}{e^{ax}+e^{-ax}}\right)$, then $\frac{dy}{dx}$ is

27. $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$, then $\frac{dy}{dx}$ is

28. If $y = (1+x)(1+x^2)(1+x^4).......(1+x^{2^n})$ then $\frac{dy}{dx}$ at x = 0 is

29. If $y = \log\left(x+\sqrt{1+x^2}\right)$ then the value of $y_2(0)$ is :

30. If $y = \left(x+\sqrt{1+x^2}\right)^m$ then $\left(1+x^2\right)y_2 + xy_1 - m^2y =$

31. If $\sqrt{x+y}+\sqrt{y-x} = c$ then $\frac{d^2y}{dx^2}$ is :

32. If y = sin (sinx), and $\frac{d^2y}{dx^2}+\frac{dy}{dx}\tan x + f(x) = 0$ then f(x) is :

33. If x = 2 cos t – cos 2t and y = 2 sin t – sin 2t then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$ is :

34. If $x^2 + y^2 + xy = 2$, find $\frac{dy}{dx}$.

ANSWERS

OBJECTIVE

1.	(A)	**2.**	(A)	**3.**	(B)	**4.**	(A)	**5.**	(A)	**6.**	(B)	**7.**	(C)
8.	(C)	**9.**	(A)	**10.**	(B)	**11.**	(C)	**12.**	(A)	**13.**	(A)	**14.**	(A)
15.	(D)	**16.**	(A)	**17.**	(B)	**18.**	(A)	**19.**	(B)	**20.**	(A)	**21.**	(A)
22.	(A)	**23.**	(D)	**24.**	(D)	**25.**	(A)	**26.**	(B)	**27.**	(A)	**28.**	(B)
29.	(C)	**30.**	(D)	**31.**	(B)	**32.**	(D)	**33.**	(D)	**34.**	(B)	**35.**	(D)
36.	(B)	**37.**	(D)	**38.**	(D)	**39.**	(C)	**40.**	(D)	**41.**	(B)	**42.**	(C)
43.	(A)	**44.**	(C)	**45.**	(A)	**46.**	(B)	**47.**	(B)	**48.**	(C)	**49.**	(A)
50.	(D)	**51.**	n^2y	**52.**	(B)	**53.**	(D)	**54.**	(C)	**55.**	(D)	**56.**	(D)
57.	(A)	**58.**	(A,C, D)			**59.**	(A,B,C,D)			**60.**	(A, D)	**61.**	(B, C, D)

SUBJECTIVE

1. +2 **2.** $\dfrac{x\cot\sqrt{x^2+1}}{(\sqrt{x^2+1})\,\log\sin\sqrt{x^2+1}}$ **3.** $\dfrac{1}{\sqrt{x}\,(1+\sqrt{x})^2}$ **4.** $-\dfrac{(3x^2+8y)}{8x+3y^2}$

5. $\dfrac{x\sqrt{1-x^2}+\sin^{-1}x}{(1-x^2)^{3/2}}$ **6.** $-\dfrac{b(1-t^2)}{2at}$ **7.** 1

8. $\dfrac{yx^{y-1}+y^x\log y-(x^y+y^x)\,\{1+\log(x+y)\}}{[(x^y+y^x)\{1+\log(x+y)\}-x^y\log x-xy^{x-1}]}$

9. $\dfrac{x^{\sin x}[(1/x)\sin x+\cos x\log x]}{(\sin x)^x\,[x\cot x+\log\sin x]}$ **10.** $\dfrac{2\,(x^2+\log x.x^{\log x})}{x}$

11. $y' = \dfrac{\ell nx}{(1+\ell nx)^2}$ **12.** $\dfrac{1}{a+b\,\cos x}$ **13.** $\cos^2 x\sin(\sin x)$

NCERT BOARD QUESTIONS

1. $\dfrac{1}{x}$ **2.** $\dfrac{1}{\sqrt{1-x^2}}\left(\dfrac{1}{1-x}\right)$ **3.** $\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}$ **4.** 0

5. $\frac{1}{x^3 y}$ **6.** $-\frac{2xe^{y-x} + 2y\,(x+1)}{x\,(xe^{y-x}+2)}$ **7.** $\frac{1}{2}$ **8.** $-\frac{1}{2}$

9. $-2 \sec x\,(\sec x - \tan x)^2$ **10.** $\frac{1}{2}\sqrt{1+\operatorname{cosec}\ x}$ **11.** $-\frac{1}{(t+1)^2} + \frac{1}{(t+2)^2}$

12. $\frac{x}{y}$ **13.** 1 **14.** $\frac{\mu - 17\nu}{17\mu - \nu}$ **15.** $2x\,[1 + \tan(\log x)] + x \sec^2(\log x)$

16. $\frac{-3}{\sqrt{9-x^2}}$ **17.** 0 **18.** $\frac{y}{x}$ **19.** $-\frac{1}{2}$ **20.** $\sqrt{\left(\frac{a-x}{x-b}\right)}$

21. -1 **22.** $-\frac{9}{4}$ **23.** $\frac{7}{8}$

24. $(\sin x + \cos x)^x \left[\frac{x(\cos x - \sin x}{\sin x + \cos x} + \log(\sin x + \cos x)\right]$

25. $\frac{dy}{dx} = \frac{1}{\log \sin^{-1} x} \times \frac{1}{\cos^{-1} x} \times \frac{-1}{\sqrt{1-x^2}} + \log(\cos^{-1}x) \times -1 \times \frac{1}{(\log \sin^{-1} x)^2} \times \frac{1}{\sin^{-1} x} \times \frac{1}{\sqrt{1-x^2}}$

26. $\frac{2a}{e^{ax} + e^{-ax}}$ **27.** $\frac{y^2}{x^3} + 2x$ **28.** 1 **29.** 0 **30.** $(1+x^2)y_2 + xy_1 = m^2 y.$

31. $\frac{2}{c^2}$ **32.** $\cos^2 x.\ \sin(\sin x)$ **33.** $-\frac{3}{2}$ **34.** $\frac{(2x+y)}{(2y+x)}$