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QUADRATIC EQUATION

Contents

Syllabus

Quadratic equations with real coefficients, relations between roots and coefficients, formation of quadratic equations with given roots, symmetric functions of roots.

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QUADRATIC EQUATION

1. Equation v/s Identity:

A quadratic equation is satisfied by exactly two values of ' x ' which may be real or imaginary. The equation, $a x^2 + b x + c = 0$ is:

*	a quadratic equation if $a \neq 0$		Two Roots
*	a linear equation if	$a = 0,\ b \neq 0$	One Root
*	a contradiction if	$a = b = 0,\ c \neq 0$	No Root
*	an identity if	$a = b = c = 0$	Infinite Roots

If a quadratic equation is satisfied by three distinct values of ' x ', then it is an identity.

2. Relation Between Roots & Co-efficients:

(i) The solutions of quadratic equation, $a x^2 + b x + c = 0$, $(a \neq 0)$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression, $b^2 - 4ac \equiv D$ is called discriminant of quadratic equation.

(ii) If α, β are the roots of quadratic equation, $a x^2 + b x + c = 0$, $a \neq 0$. Then:

(a) $\alpha + \beta = -\frac{b}{a}$ (b) $\alpha \beta = \frac{c}{a}$ (c) $|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$

(iii) A quadratic equation whose roots are α & β, is $(x - \alpha)(x - \beta) = 0$ i.e.

x^2 – (sum of roots) x + (product of roots) = 0

3. Nature of Roots:

Consider the quadratic equation, $a x^2 + b x + c = 0$ having α, β as its roots; $D \equiv b^2 - 4ac$

- $D = 0$
  Roots are equal
  $\alpha = \beta = -\frac{b}{2a}$
- $D \neq 0$
  Roots are unequal
  - $a, b, c \in R$ & $D > 0$
    Roots are real
    - $a, b, c \in Q$ &
      D is a perfect square
      $\Rightarrow$ Roots are rational
      $\downarrow$
      $a = 1,\ b, c \in I$ & D is perfect square
      $\Rightarrow$ Roots are integral.
    - $a, b, c \in Q$ &
      D is not a perfect square
      $\Rightarrow$ Roots are irrational
      i.e. $\alpha = p + \sqrt{q}$, $\beta = p - \sqrt{q}$
  - $a, b, c \in R$ & $D < 0$
    Roots are imaginary
    $\alpha = p + iq$, $\beta = p - iq$

4. Common Roots:

Consider two quadratic equations, $a_1x^2 + b_1x + c_1 = 0$ & $a_2x^2 + b_2x + c_2 = 0$.

(i) If two quadratic equations have both roots common, then the equation are identical and their co-efficient are in proportion. i.e.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

(ii) If only one root is common, then the common root 'α' will be:

$$\alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} = \frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}$$

Hence the condition for one common root is:

$$a_1\left[\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right]^2 + b_1\left[\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right] + c_1 = 0$$

$$\equiv \quad (c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)\,(b_1c_2 - b_2c_1)$$

Note : If f(x) = 0 & g(x) = 0 are two polynomial equation having some common root(s) then those common root(s) is/are also the root(s) of h(x) = a f(x) + bg (x) = 0.

5. Graph of Quadratic Expression:

$y = f(x) = ax^2 + bx + c$

or

$$\left(y + \frac{D}{4a}\right) = a\left(x + \frac{b}{2a}\right)^2$$

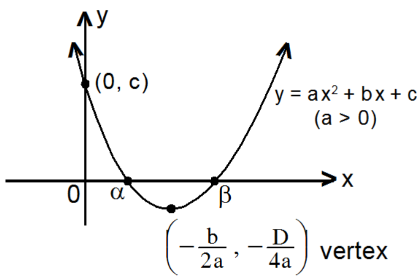
* the graph between x, y is always a parabola.
* the co–ordinate of vertex are $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$
* If a > 0 then the shape of the parabola is concave upwards & if a < 0 then the shape of the parabola is concave downwards.
* the parabola intersect the y–axis at point (0, c).
* the x–co–ordinate of point of intersection of parabola with x–axis are the real roots of the quadratic equation f (x) = 0. Hence the parabola may or may not intersect the x–axis at real points.

6. Range of Quadratic Expression f (x) = $ax^2 + bx + c$.

(i) **Absolute Range:**

If $a > 0 \Rightarrow f(x) \in \left[-\frac{D}{4a}, \infty\right)$

$a < 0 \Rightarrow f(x) \in \left(-\infty, -\frac{D}{4a}\right]$



Hence maximum and minimum values of the expression f (x) is $-\frac{D}{4a}$ in respective cases and it occurs at $x = -\frac{b}{2a}$ (at vertex).

(ii) **Range in restricted domain:**

Given $x \in [x_1, x_2]$

(a) If $-\frac{b}{2a} \notin [x_1, x_2]$ then,

$$f(x) \in \left[\min\left\{f(x_1), f(x_2)\right\},\quad \max\left\{f(x_1), f(x_2)\right\}\right]$$

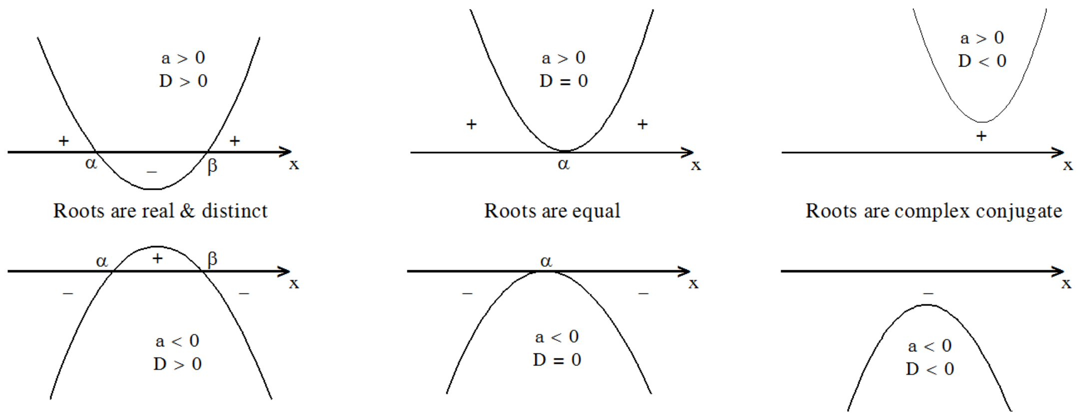
(b) If $-\frac{b}{2a} \in [x_1, x_2]$ then,

$$f(x) \in \left[\min\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\},\quad \max\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}\right]$$

7. Sign of Quadratic Expressions:

The value of expression, $f(x) = ax^2 + bx + c$ at $x = x_0$ is equal to y–co–ordinate of a point on parabola $y = ax^2 + bx + c$ whose x–co–ordinate is x_0. Hence if the point lies above the x–axis for some $x = x_0$, then $f(x_0) > 0$ and vice–versa.

We get six different positions of the graph with respect to x–axis as shown.



NOTE:

(i) $\forall x \in R$, $y > 0$ only if $a > 0$ & $D \equiv b^2 - 4ac < 0$ (figure 3).

(ii) $\forall x \in R$, $y < 0$ only if $a < 0$ & $D \equiv b^2 - 4ac < 0$ (figure 6).

8. Solution of Quadratic Inequalities:

The values of 'x' satisfying the inequality, $ax^2 + bx + c > 0$ $(a \neq 0)$ are:

(i) If $D > 0$, i.e. the equation $ax^2 + bx + c = 0$ has two different roots $\alpha < \beta$.

Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\beta, \infty)$

$a < 0 \Rightarrow x \in (\alpha, \beta)$

(ii) If $D = 0$, i.e. roots are equal, i.e. $\alpha = \beta$.

Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\alpha, \infty)$

$a < 0 \Rightarrow x \in \phi$

(iii) If $D < 0$, i.e. the equation $ax^2 + bx + c = 0$ has no real root.

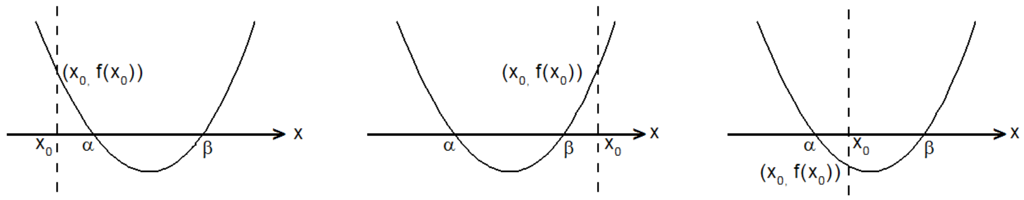
Then $a > 0 \Rightarrow x \in R$

$a < 0 \Rightarrow x \in \phi$

(iv) Inequalities of the form $\frac{P(x)\ Q(x)\ R(x)\ldots\ldots\ldots}{A(x)\ B(x)\ C(x)\ldots\ldots\ldots} \begin{matrix} < \\ = \\ > \end{matrix} 0$ can be quickly solved using the method of intervals, where A, B, C........, P, Q, R......... are linear functions of 'x'.

9. Location Of Roots:

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

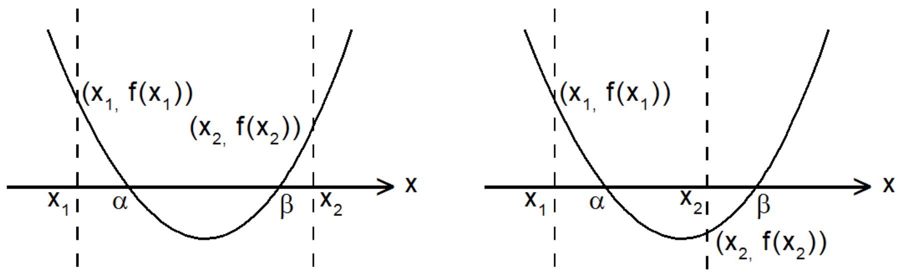


(i) (ii) (iii)

(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'x_0' are $b^2 - 4ac \ge 0$; $f(x_0) > 0$ & $(-b/2a) > x_0$.

(ii) Conditions for both the roots of $f(x) = 0$ to be smaller than a specified number 'x_0' are $b^2 - 4ac \ge 0$; $f(x_0) > 0$ & $(-b/2a) < x_0$.

(iii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'x_0' (in other words the number 'x_0' lies between the roots of $f(x) = 0$), is $f(x_0) < 0$.



(iv) (v)

(iv) Conditions that both roots of $f(x) = 0$ to be confined between the numbers x_1 and x_2, $(x_1 < x_2)$ are $b^2 - 4ac \ge 0$; $f(x_1) > 0$; $f(x_2) > 0$ & $x_1 < (-b/2a) < x_2$.

(v) Conditions for exactly one root of $f(x) = 0$ to lie in the interval (x_1, x_2) i.e. $x_1 < x < x_2$ is $f(x_1) . f(x_2) < 0$.

10. Theory Of Equations:

If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the roots of the equation;

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ where $a_0, a_1, ... a_n$ are all real & $a_0 \neq 0$ then,

$$\sum \alpha_1 = -\frac{a_1}{a_0}, \sum \alpha_1\alpha_2 = +\frac{a_2}{a_0}, \sum \alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}, ..., \alpha_1\alpha_2\alpha_3.......\alpha_n = (-1)^n\frac{a_n}{a_0}$$

NOTE :

(i) If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of $f(x)$ and conversely.

(ii) Every equation of n^{th} degree $(n \ge 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If the coefficients of the equation $f(x) = 0$ are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. imaginary roots occur in conjugate pairs.

(iv) An equation of odd degree will have odd number of real roots and an equation of even degree will have even numbers of real roots.

(v) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not a perfect square.

(vi) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then $f(x) = 0$ must have odd number of real roots (also atleast one real root) between 'a' and 'b'.

(vii) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

* *Marked Questions are having more than one correct option.*

Section (A) : Identity & Relation between the roots and coefficients

A-1. Number of values of 'p' for which the equation $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ possess more than two roots, is:
(A) 0 (B) 1 (C) 2 (D) none

A-2. If α, β are the roots of quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma).(\alpha - \delta)$ is equal to :
(A) q + r (B) q – r (C) – (q + r) (D) – (p + q + r)

A-3. Two real numbers α & β are such that $\alpha + \beta = 3$ & $|\alpha - \beta| = 4$, then α & β are the roots of the quadratic equation:
(A) $4x^2 - 12x - 7 = 0$ (B) $4x^2 - 12x + 7 = 0$
(C) $4x^2 - 12x + 25 = 0$ (D) none of these

A-4. If α, β are the roots of the equation $a(x^2 - 1) + 2bx = 0$, then the equation whose roots are $2\alpha - \frac{1}{\beta}$ and $2\beta - \frac{1}{\alpha}$ is–
(A) $ax^2 + 6bx + 9a = 0$ (B) $bx^2 + 6ax - 9b = 0$
(C) $ax^2 + 6bx - 9a = 0$ (D) $ax^2 + 2bx - a = 0$

A-5. If $x = \frac{5 - \sqrt{-3}}{2}$ then the value of $x^4 - x^3 - 12x^2 + 23x + 12$ is equal to–
(A) 1 (B) 3 (C) 5 (D) 0

A-6. If $4^x - 4^{x-1} = 24$ then $(2x)^{5/2}$ has the value equal to
(A) $5\sqrt{5}$ (B) 25 (C) $25\sqrt{5}$ (D) 125

A-7. The value of $\sqrt{8 + 2\sqrt{8 + 2\sqrt{8 + 2\sqrt{8.....}}}}$ is
(A) 10 (B) 6 (C) 8 (D) none of these

Section (B) : Nature of Roots and Common Roots

B-1. If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the equation $ax^2 + bx + c = 0$ are
(A) Irrational (B) Rational & different
(C) Complex conjugate (D) Rational & equal

B-2. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. Total number of different values of 'n' so that the given equation has integral roots, is
(A) 4 (B) 6 (C) 8 (D) 3

B-3. If $P(x) = ax^2 + bx + c$ & $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x).\ Q(x) = 0$ has
(A) exactly one real root (B) atleast two real roots
(C) exactly three real roots (D) all four are real roots

B-4. If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have exactly one root in common then the equation containing their other root is–

(A) $x^2 - x + pq = 0$ (B) $x^2 + x + pq = 0$

(C) $x^2 - x - pq = 0$ (D) $x^2 + x - pq = 0$

B-5. $x^2 + x + 1 = 0$ and $ax^2 + bx + c = 0$ has common root, $(a, b, c \in R)$ then

(A) a = K, b = K, c = K, $K \neq 0$ (B) a = K, b = 2 K, c = 3K, $K \neq 0$

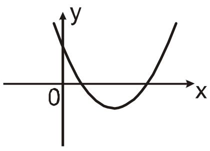
(C) a = 2K, b = K, c = K, $K \neq 0$ (D) a = K, b = 2K, c = K, $K \neq 0$

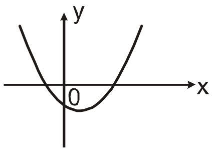
Section (C) : Graph and Range

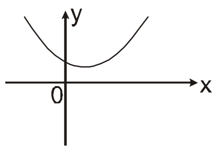
C-1. The entire graph of the expression $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if

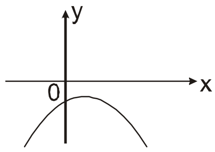
(A) $k < 7$ (B) $-5 < k < 7$ (C) $k > -5$ (D) none

C-2. Which of the following graph represents the expression $f(x) = a x^2 + b x + c$ $(a \neq 0)$ when $a > 0$, $b < 0$ & $c < 0$?

(A) 

(B) 

(C) 

(D) 

C-3. If $y = -2x^2 - 6x + 9$, then

(A) maximum value of y is –11 and it occurs at x = 2

(B) minimum value of y is –11 and it occurs at x = 2

(C) maximum value of y is 13.5 and it occurs at x = –1.5

(D) minimum value of y is 13.5 and it occurs at x = –1.5

C-4. If 'x' is real and $k = \frac{x^2 - x + 1}{x^2 + x + 1}$, then :

(A) $\frac{1}{3} \leq k \leq 3$ (B) $k \geq 5$ (C) $k \leq 0$ (D) none

C-5. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has

(A) real roots (B) imaginary roots (C) exactly one root (D) none of these

Section (D) : Location of Roots

D-1. If the inequality $(m - 2)x^2 + 8x + m + 4 > 0$ is satisfied for all $x \in R$ then the least integral m is

(A) 4 (B) 5 (C) 6 (D) none

D-2. For all 'x' $x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is–

(A) $a < 5$ (B) $-5 < a < 2$ (C) $a > 5$ (D) $2 < a < 5$

D-3. The set of values of 'm' for which the equation $x^2 - (m + 1)x + m^2 + m - 8 = 0$ has a root in the interval $(1, \infty)$ and the other in the interval $(-\infty, 1)$ is

(A) $(\sqrt{2}, \infty)$ (B) $(-\infty, 2\sqrt{2})$ (C) $(-2\sqrt{2}, 2\sqrt{2})$ (D) $(2\sqrt{2}, \infty)$

D-4. If both roots of the equation $x^2 - (m + 1)x + m + 4 = 0$ are real and negative, then set of values of 'm' is–

(A) $-3 < m \le -1$ (B) $-4 < m \le -3$ (C) $-3 \le m \le 5$ (D) $-3 \ge m$ or $m \ge 5$

D-5. If both roots of the quadratic equation $(2 - x)(x + 1) = p$ are distinct & positive then p must lie in the interval:

(A) $p > 2$ (B) $2 < p < \frac{9}{4}$ (C) $p < -2$ (D) $-\infty < P < \infty$

D-6. The value of p for which both the roots of the quadratic equation, $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2 lies in :

(A) (4/5, 2) (B) $(2, \infty)$ (C) (–1, 4/5) (D) $(-\infty, -1)$

D-7. The real values of 'a' for which the quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite sign is given by:

(A) $a > 5$ (B) $0 < a < 4$ (C) $a > 0$ (D) $a > 7$

D-8. If α, β are the roots of the quadratic equation $x^2 - 2p(x - 4) - 15 = 0$, then the set of values of p for which one root is less than 1 & the other root is greater than 2 is:

(A) $(7/3, \infty)$ (B) $(-\infty, 7/3)$ (C) $x \in R$ (D) none

Section (E) : Theory of Equation

E-1. The condition that $x^3 - px^2 + qx - r = 0$ may have two of its roots equal to each other but of opposite signs is–

(A) $r = pq$ (B) $r = 2p^3 + pq$ (C) $r = p^2 q$ (D) none of these

E-2. If α, β & γ are the roots of the equation $x^3 - x - 1 = 0$ then, $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ has the value equal to:

(A) zero (B) –1 (C) –7 (D) 1

E-3. Let α, β, γ be the roots of $(x - a)(x - b)(x - c) = d$, $d \ne 0$ then the roots of the equation $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ are :

(A) a + 1, b + 1, c + 1 (B) a, b, c (C) a – 1, b – 1, c – 1 (D) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

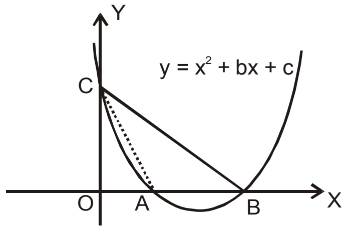
E-4. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation, $x^4 - Kx^3 + Kx^2 + Lx + M = 0$ where K, L & M are real numbers then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is :

(A) 0 (B) -1 (C) 1 (D) 2

PART - II : MISCELLANEOUS OBJECTIVE QUESTIONS

Comprehensions # 1 :

In the given figure ΔOBC is an isosceles right triangle in which AC is a median, then answer the following questions :



1. Roots of y = 0 are

(A) {2, 1} (B) {4, 2} (C) {1, 1/2} (D) {8, 4}

2. The equation whose roots are $(\alpha + \beta)$ & $(\alpha - \beta)$, where α, β $(\alpha > \beta)$ are roots obtained in previous question, is

(A) $x^2 - 4x + 3 = 0$ (B) $x^2 - 8x + 12 = 0$ (C) $4x^2 - 8x + 3 = 0$ (D) $x^2 - 16x + 48 = 0$

3. Minimum value of the quadratic expression correspoinding to the quadratic equation obtained in Q. No. 2 occurs at x =

(A) 8 (B) 1 (C) 4 (D) 2

Comprehensions # 2 :

Consider the equation $x^4 - \lambda x^2 + 9 = 0$.

4. If the equation has four real and distinct roots, then λ lies in the interval

(A) $(-\infty, -6) \cup (6, \infty)$ (B) $(0, \infty)$ (C) $(6, \infty)$ (D) $(-\infty, -6)$

5. If the equation has no real root, then λ lies in the interval

(A) $(-\infty, 0)$ (B) $(-\infty, 6)$ (C) $(6, \infty)$ (D) $(0, \infty)$

6. If the equation has only two real roots, then set of values of λ is

(A) $(-\infty, -6)$ (B) $(-6, 6)$ (C) $\{6\}$ (D) ϕ

Match The Column :

7. For the quadratic equation $x^2 - (k - 3)x + k = 0$, then match the following columns

Column-I	Column-II
(A) Both roots are positive	(P) $(-\infty, 1)$
(B) Both roots are negative	(Q) $(9, \infty)$
(C) Both roots are real	(R) $(0, 1)$
(D) One root < –1, the other > 1	

8. **Match the following Column**

Column-I	Column-II
(A) $(x-1)(x-3) + k(x-2)(x-4) = 0$ ($k \in R$), has real roots for $k \in$	(P) $(-5, -1)$
(B) Range of the function $\frac{x-1}{x^2-k+1}$ does not contain any value in the interval $[-1, 1]$ for $k \in$	(Q) ϕ
(C) The equation, $\sec x + \operatorname{cosec} x = k$ has real roots for $x \in \left(0, \frac{5}{2}\right)$, if $k \in$	(R) $(-\infty, \infty)$
(D) The equation $x^2 + 2(k-1)x + k + 5 = 0$ has positive and distinct roots, if $k \in$	(S) $[2\sqrt{2}, \infty)$

Assertion / Reason :

Direction :

Each question has 5 choices (A), (B), (C), (D) and (E) out of which ONLY ONE is correct.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

(E) Statement-1 and Statement-2 both are False.

9. **STATEMENT - 1** : Maximum value of $\log_{1/3}(x^2 - 4x + 5)$ is '0'.
STATEMENT - 2 : $\log_a x \le 0$ for $x \ge 1$ and $0 < a < 1$.

10. Let α, β be the roots of $f(x) = 3x^2 - 4x + 5 = 0$.
STATEMENT-1 : The equation whose roots are 2α, 2β is given by $3x^2 + 8x - 20 = 0$.
STATEMENT-2 : To obtain, from the equation $f(x) = 0$, having roots α and β, the equation having roots 2α, 2β one needs to change x to $\frac{x}{2}$ in $f(x) = 0$.

EXERCISE # 2

PART - I : MIXED OBJECTIVE

* ***Marked Questions are having more than one correct option.***

1. If the roots of the equations $ax^2 + bx + c = 0$ are real and of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then the value of $(a + b + c)^2$ is-
(A) $b^2 - 4ac$ (B) $b^2 - 2ac$ (C) $2b^2 - ac$ (D) None of these

2. The equation, $\pi^x = -2x^2 + 6x - 9$ has:
(A) no solution (B) one solution (C) two solutions (D) infinite solutions

3. If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is:
(A) positive (B) negative
(C) zero (D) depends on the sign of b

4. If α, β be the roots of $4x^2 - 16x + \lambda = 0$, where $\lambda \in R$, such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral solutions of λ is

(A) 5 (B) 6 (C) 2 (D) 3

5. If both roots of the quadratic equation $(2 - x)(x + 1) = p$ are distinct & positive, then p must lie in the interval:

(A) $(2, \infty)$ (B) $(2, 9/4)$ (C) $(-\infty, -2)$ (D) $(-\infty, \infty)$

6. The value of 'a' for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is:

(A) 0 (B) 1 (C) 2 (D) 3

7. The values of k, for which the equation $x^2 + 2(k - 1)x + k + 5 = 0$ possess atleast one positive root, are:

(A) $[4, \infty)$ (B) $(-\infty, -1] \cup [4, \infty)$ (C) $[-1, 4]$ (D) $(-\infty, -1]$

8. If $b > a$, then the equation $(x - a)(x - b) + 1 = 0$, has:

(A) both roots in (a, b)
(B) both roots in $(-\infty, a)$
(C) both roots in (b, ∞)
(D) one root in $(-\infty, a)$ & other in (b, ∞)

9. If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1$ for all $x \in R$, then λ belongs to the interval

(A) $(-2, 1)$ (B) $\left[-2, \frac{2}{5}\right)$ (C) $\left(\frac{2}{5}, 1\right)$ (D) none of these

10. If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by at most 2m, then b lies in the interval

(A) $(a^2 - m^2, a^2)$ (B) $[a^2 - m^2, a^2)$ (C) $(a^2, a^2 + m^2)$ (D) none of these

11. If $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \alpha_6$, then the equation $(x - \alpha_1)(x - \alpha_3)(x - \alpha_5) + 3(x - \alpha_2)(x - \alpha_4)(x - \alpha_6) = 0$ has

(A) three real roots
(B) a root is $(-\infty, \alpha_1)$
(C) no real root in (α_1, α_2)
(D) no real root in (α_5, α_6)

12. For every $x \in R$, the polynomial $x^8 - x^5 + x^2 - x + 1$ is :

(A) Positive
(B) never positive
(C) positive as well as negative
(D) negative

13. The greatest value of least value of the quadratic trinomial, $x^2 + 2ax + (a + 2)$, is–

(A) $\frac{9}{4}$ (B) $\frac{7}{4}$ (C) 0 (D) 3

14. If $(x - 1)^2$ is a factor of $ax^3 + bx^2 + c$, then roots of the equation $cx^3 + bx + a = 0$ are-

(A) $1, 1, -\frac{1}{2}$ (B) $1, 1, \frac{1}{2}$ (C) $1, 1, -2$ (D) $1, -1, 2$

15. Roots of the equation $x^3 + ax^2 + bx + c = 0$ are 3 consecutive positive integer, the value of $\frac{a^2}{b+1}$ is-

(A) 5 (B) 7 (C) 9 (D) 3

16. If a, b, c are real numbers satisfying the condition $a + b + c = 0$ then the roots of the quadratic equtions, $3ax^2 + 5bx + 7c = 0$ are :

(A) positive (B) negative (C) real & distinct (D) imaginary

17. $ax^2 + bx + c = 0$ has real and distinct roots α and $\beta(\beta > \alpha)$. Further $a > 0$, $b > 0$ and $c < 0$, then

(A) $0 < \beta < |\alpha|$ (B) $0 < |\alpha| < \beta$ (C) $\alpha + \beta > 0$ (D) $|\alpha| + |\beta| = \left|\frac{b}{a}\right|$

18. If l, m, n are real, $l \neq m$, then the roots of the equation : $(l - m)x^2 - 5(l + m)x - 2(l - m) = 0$ are

(A) real and equal (B) Complex

(C) real and unequal (D) none of these

19. If the roots of equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ equals

(A) –2 (B) 3 (C) 2 (D) 1

20. Which of the following statements is true about a quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$, $a \neq 0$

(A) If $ac < 0$ then roots are imaginary (B) If $a + b + c = 0$ then roots are real

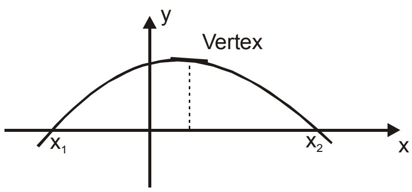
(C) If a, b, c are equal, roots are equal (D) If $abc < 0$ roots are essentially real.

21.* If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal is magnitude and opposite in sign, then

(A) $p + q = r$ (B) $p + q = 2r$

(C) product of roots = $-\frac{1}{2}(p^2 + q^2)$ (D) sum of roots = 1

22.* The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then



(A) $a > 0$ (B) $b > 0$ (C) $c > 0$ (D) $b^2 < 4ac$

23.* Let $Q_1(x) = x^2 + ax + 1 = 0$, $Q_2(x) = x^2 + x + a = 0$ be two quadratic equations, then

(A) they have a common root if a = –2

(B) they have a common root if a = 1

(C) they have at least one common root for a = 1 and a = –2

(D) they have a complex common root if a = 1

24.* If the difference of the roots of the equation $x^2 + hx + 7 = 0$ is 6, then possible value(s) of h are

(A) –4 (B) 4 (C) –8 (D) 8

25.* For the equation $|x|^2 + |x| - 6 = 0$, the correct statement (s) is (are) :

(A) sum of roots is 0 (B) product of roots is – 4

(C) there are 4 roots (D) there are only 2 roots

26.* If α, β are the roots of $ax^2 + bx + c = 0$, and $\alpha + h$, $\beta + h$ are the roots of $px^2 + qx + r = 0$, (where $h \neq 0$), then

(A) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ (B) $h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

(C) $h = \frac{1}{2}\left(\frac{b}{a} + \frac{q}{p}\right)$ (D) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

27.* If a, b are non-zero real numbers and α, β the roots of $x^2 + ax + b = 0$, then

(A) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$

(B) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$

(C) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$

(D) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

28. If the roots of the equation $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equaton $x^3 - Ax^2 + Bx - C = 0$, where A, B, C, P & Q are constants, then the value of A + B + C is equal to :

(A) 18 (B) 19 (C) 20 (D) none

PART - II : SUBJECTIVE QUESTIONS

1. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then find the equation whose roots are given by :

(i) $\alpha + \frac{1}{\beta}, \beta + \frac{1}{\alpha}$ (ii) $\alpha^2 + 2, \beta^2 + 2$

2. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then find the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

3. In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as – 10 in place of – 11 and the roots were found to be 4 and 6. Find the roots of the correct equation.

4. If one root of the equation $ax^2 + bx + c = 0$ is equal to n^{th} power of the other root, show that $(ac^n)^{1/(n+1)} + (a^nc)^{1/(n+1)} + b = 0$.

5. For what values of k the expression $kx^2 + (k + 1)x + 2$ will be a perfect square of a linear polynomial.

6. If a, b, c $\in$ R, then prove that the roots of the equation $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$ are always real and cannot have roots if a = b = c.

7. If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Show that $q^2 - p^2 = (a - c)(b - c)(a + d)(b + d)$.

8. Find all values of the parameter ' a ' such that the roots α, β of the equation

$2x^2 + 6x + a = 0$ satisfy the inequality $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$.

9. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of

$\left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right)$.

10. If α, β and γ are roots of $2x^3 + x^2 - 7 = 0$, then find the value of $\sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$.

11. Find the value of 'a' so that $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$ have a common root.

12. If $x^2 + px + q = 0$ and $x^2 + qx + p = 0$, $(p \neq q)$ have a common root, show that $1 + p + q = 0$; show that their other roots are the roots of the equation $x^2 + x + pq = 0$.

13. The equations $x^2 - ax + b = 0$ & $x^3 - px^2 + qx = 0$, where $b \neq 0$, $q \neq 0$ have one common root & the second equation has two equal roots. Prove that $2(q + b) = ap$.

14. If the equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ & $x^2 + (a + b)x + 36 = 0$ have a common positive root, then find a, b and the roots of the equations.

15. Draw the graph of the following expressions :

(i) $y = x^2 + 4x + 3$ (ii) $y = 9x^2 + 6x + 1$ (iii) $y = -2x^2 + x - 1$

16. If x be real, then find the range of the following rational expressions :

(i) $y = \frac{x^2+x+1}{x^2+1}$ (ii) $y = \frac{x^2-2x+9}{x^2+2x+9}$

17. Solve for real values of 'x' :

(i) $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$ (ii) $x^2 - 2a|x - a| - 3a^2 = 0$, $a \leq 0$

18. If a, b, c are non–zero, unequal rational numbers then prove that the roots of the equation $(abc^2)x^2 + 3a^2cx + b^2cx - 6a^2 - ab + 2b^2 = 0$ are rational.

19. Find all the values of 'K' for which one root of the equation $x^2 - (K + 1)x + K^2 + K - 8 = 0$, exceeds 2 & the other root is smaller than 2.

20. If α & β are the two distinct roots of $x^2 + 2(K - 3)x + 9 = 0$, then find the values of K such that $\alpha, \beta \in (-6, 1)$.

21. If $p, q, r, s \in R$ and $pr = 2(q + s)$, then show that atleast one of the equations $x^2 + px + q = 0$, $x^2 + rx + s = 0$ has real roots.

22. Find all values of a for which atleast one of the roots of the equation $x^2 - (a - 3)x + a = 0$ is greater than 2.

23. If x_1 is a root of $ax^2 + bx + c = 0$, x_2 is a root of $-ax^2 + bx + c = 0$ where $0 < x_1 < x_2$, show that the equation $ax^2 + 2bx + 2c = 0$ has a root x_3 satisfying $0 < x_1 < x_3 < x_2$.

24. Obtain real solutions of the simultaneous equations $xy + 3y^2 - x + 4y - 7 = 0$, $2xy + y^2 - 2x - 2y + 1 = 0$.

EXERCISE # 3

PART - I : IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions are having more than one correct option.**

1. The set of all real numbers x for which $x^2 - |x+2| + x > 0$, is **[IIT-JEE 2002]**

(A) $(-\infty,-2)\cup(2,\infty)$ (B) $(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

(C) $(-\infty,-1)\cup(1,\infty)$ (D) $(\sqrt{2},\infty)$

2. If $x^2 + (a-b)x + (1-a-b) = 0$ where a, b $\in$ R then find the values of 'a' for which equation has unequal real roots for all values of 'b'. **[IIT-JEE 2003]**

3. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **[IIT-JEE 2005]**

4. In quadratic equation $ax^2 + bx + c = 0$, if α, β are roots of equation, $\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G. P. then **[IIT-JEE 2005]**

(A) $\Delta \neq 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

5. If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d then the value of a + b + c + d is (a, b, c and d are distinct numbers) **[IIT-JEE 2006]**

6. Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in$ R. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real, then **[IIT-JEE 2006]**

(A) $\lambda < \frac{4}{3}$ (B) $\lambda > \frac{5}{3}$ (C) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

7. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is- **[IIT-JEE 2007]**

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

8. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is : **[IIT-JEE 2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ (B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 . q)x^2 . (5p^3 . 2q)x + (p^3 . q) = 0$ (D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

9. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[IIT-JEE 2011]**

(A) 1 (B) 2 (C) 3 (D) 4

10. A value of b for which the equations **[IIT-JEE 2012]**

$x^2 + bx - 1 = 0$; $x^2 + x + b = 0$,

have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

PART - II : AIEEE PROBLEMS (PREVIOUS YEARS)

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having the roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is. **[AIEEE-2002]**

(1) $3x^2 + 19x + 3 = 0$ (2) $3x^2 - 19x + 3 = 0$ (3) $3x^2 - 19x - 3 = 0$ (4) $x^2 - 16x + 1 = 0$

2. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is : **[AIEEE-2003]**

(1) $\frac{2}{3}$ (2) $-\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$.

3. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are : **[AIEEE-2004]**

(1) 0,1 (2) –1,1 (3) 0, –1 (4) –1,2

4. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is : **[AIEEE-2004]**

(1) 49/4 (2) 12 (3) 3 (4) 4

5. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$, then : **[AIEEE2005]**

(1) $b = a + c$. (2) $b = c$. (3) $c = a + b$. (4) $a = b + c$.

6. The value of 'a' for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is - **[AIEEE- 2005]**

(1) 1 (2) 0 (3) 3 (4) 2

7. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then 'k' lies in the interval **[AIEEE- 2005]**

(1) (5, 6) (2) $(6, \infty)$ (3) $(-\infty, 4)$ (4) [4, 5]

8. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan 15° respectively, then the value of $2 + q - p$ is : **[AIEEE-2006]**

(1) 3 (2) 0 (3) 1 (4) 2

9. All the values of 'm' for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than – 2 but less than 4 lie in the interval : **[AIEEE-2006]**

(1) $m > 3$ (2) $-1 < m < 3$ (3) $1 < m < 4$ (4) $-2 < m < 0$

10. If 'x' is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is - **[AIEEE- 2006]**

(1) 41 (2) 1 (3) $\frac{17}{7}$ (4) $\frac{1}{4}$

11. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of 'a' is **[AIEEE-2007]**

(1) (–3, 3) (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

12. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE- 2008]**

(1) 4 (2) 3 (3) 2 (4) 1

13. How many real solution does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ has? **[AIEEE- 2008]**

(1) 1 (2) 3 (3) 5 (4) 7

14. If the equation $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a commons root, then a : b : c is : **[JEE Mains_2013]**

(1) 1 : 2 :3 (2) 3 : 2 : 1 (3) 1 : 3 : 2 (4) 3 : 1 : 2

EXERCISE # 4

BOARD PATTERN QUESTIONS

Solve each of the following questions (1 to 6):

1. $2x^2 + x + 1 = 0$

2. $x^2 + 3x + 5 = 0$

3. $\sqrt{2}\,x^2 + x + \sqrt{2} = 0$

4. $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

5. $27x^2 - 10x + 1 = 0$

6. $21x^2 - 28x + 10 = 0$

7. The roots of the equation
$x^2 - 2\sqrt{2}\,x + 1 = 0$ are-
(A) real and different (B) imaginary and different
(C) real and equal (D) rational and different

8. If the roots of the equation $ax^2 + x + b = 0$ be real and different, then the roots of the equation $x^2 - 4\sqrt{ab}\,x + 1 = 0$ will be-
(A) rational (B) irrational (C) real (D) imaginary

9. The number of real solutions of
$x - \frac{1}{x^2-4} = 2 - \frac{1}{x^2-4}$ is-
(A) 0 (B) 1 (C) 2 (D) infinite

10. Sum of roots of the equation
$(x + 3)^2 - 4|x + 3| + 3 = 0$ is-
(A) 4 (B) 12 (C) – 12 (D) – 4

11. If α, β are roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 . (2\beta - 35)^3$ is equal to-
(A) 1 (B) 8 (C) 64 (D) None of these

12. If α, β are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is-
(A) $x^2 - 11x + 30 = 0$ (B) $(x - 3)^2 - 5(x - 3) + 6 = 0$
(C) Both (A) and (B) (D) None of these

13. If α, β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is-
(A) $x^2 + 4x + 1 = 0$ (B) $x^2 - 4x + 4 = 0$ (C) $x^2 - 4x - 1 = 0$ (D) $x^2 + 2x + 3 = 0$

14. The minimum value of the expression $4x^2 + 2x + 1$ is-
(A) 1/4 (B) 1/2 (C) 3/4 (D) 1

ANSWERS

Exercise # 1

PART - I

A-1.	(B)	**A-2.**	(C)	**A-3.**	(A)	**A-4.**	(C)	**A-5.**	(C)	**A-6.**	(C)	**A-7.**	(D)
B-1.	(A)	**B-2.**	(C)	**B-3.**	(B)	**B-4.**	(B)	**B-5.**	(A)	**C-1.**	(B)	**C-2.**	(B)
C-3.	(C)	**C-4.**	(A)	**C-5.**	(A)	**D-1.**	(B)	**D-2.**	(B)	**D-3.**	(B)	**D-4.**	(B)
D-5.	(B)	**D-6.**	(D)	**D-7.**	(B)	**D-8.**	(B)	**E-1.**	(A)	**E-2.**	(C)	**E-3.**	(B)
E-4.	(B)												

PART-II

1.	(A)	**2.**	(A)	**3.**	(D)	**4.**	(C)	**5.**	(B)	**6.**	(D)

7. **(A) $\rightarrow$ Q ; (B) $\rightarrow$ R ; (C) $\rightarrow$ PQ ; (D) $\rightarrow$ P**

8. **(A)$\rightarrow$ R ; (B) $\rightarrow$ Q ; (C)$\rightarrow$ S ; (D) $\rightarrow$ P**

9. (A) **10.** (D)

Exercise # 2

PART - I

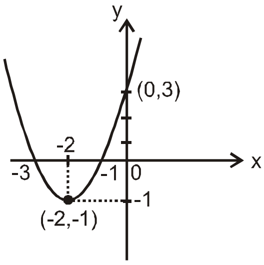
1.	(A)	**2.**	(A)	**3.**	(A)	**4.**	(D)	**5.**	(B)	**6.**	(B)	**7.**	(D)
8.	(A)	**9.**	(B)	**10.**	(B)	**11.**	(A)	**12.**	(A)	**13.**	(B)	**14.**	(C)
15.	(D)	**16.**	(C)	**17.**	(A)	**18.**	(C)	**19.**	(D)	**20.**	(B)	**21.***	(BC)
22.*	(BC)	**23.***	(ABCD)	**24.***	(CD)	**25.***	(ABD)	**26.***	(BD)	**27.***	(BCD)	**28.**	(A)

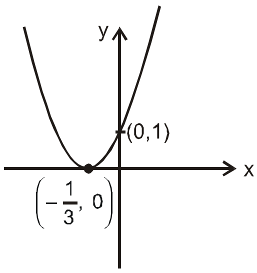
PART - II

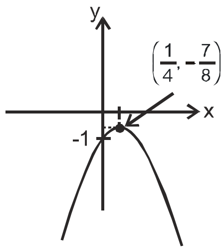
1. (i) $ac\,x^2 + b(a + c)\,x + (a + c)^2 = 0$ (ii) $a^2\,x^2 + (2ac - 4a^2 - b^2)\,x + 2b^2 + (c - 2a)^2 = 0$

2. $3x^2 - 19x + 3 = 0.$ **3.** 8, 3 **5.** $3 \pm 2\sqrt{2}$ **8.** $(-\infty, 0) \cup (9/2, \infty)$ **9.** $-\frac{(r+1)^3}{r^2}$

10. -3 **11.** $a = 0, 24$ **14.** $a = -7, b = -8$; roots (3, 4), (3, 5), (3, 12)

15. (i) 

(ii) 

(iii) 

16. (i) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (ii) $\left[\frac{1}{2}, 2\right]$

17. (i) $x = \pm 2, \pm \sqrt{2}$ (ii) $x = a(1 - \sqrt{2})$, $x = a(\sqrt{6} - 1)$

19. $K \in (-2, 3)$ **20.** $6 < K < 6.75$ **22.** $a \in [9, \infty)$ **24.** $x \in R$ if $y = 1$, $x = 2$ if $y = -3$

Exercise # 3

PART - I

1.	(B)	**2.**	a > 1	**3.**	$\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$	**4.**	(C)	**5.**	210
6.	(A)	**7.**	(D)	**8.**	(B)	**9.**	(C)	**10.**	(B)

PART-II

1.	(2)	**2.**	(1)	**3.**	(3)	**4.**	(1)	**5.**	(3)	**6.**	(1)	**7.**	(3)
8.	(1)	**9.**	(2)	**10.**	(1)	**11.**	(1)	**12.**	(3)	**13.**	(1)	14.	(1)

Exercise # 4

7.	(A)	**8.**	(D)	**9.**	(A)	**10.**	(C)	**11.**	(C)	**12.**	(C)	**13.**	(D)
14.	(C)												