arride learning

STRAIGHT LINES

Contents

Syllabus

Cartesian coordinates, distance between two points, section formulae, shift of origin. Equation of a straight line in various forms, angle between two lines, distance of a point from a line; Lines through the point of intersection of two given lines, equation of the bisector of the angle between two lines, concurrency of lines; Centroid, orthocentre, incentre and circumcentre of a triangle.

Name : ______________________________ **Contact No.** ____________________

ARRIDE LEARNING ONLINE E-LEARNING ACADEMY

A-479 indra Vihar, Kota Rajasthan 324005

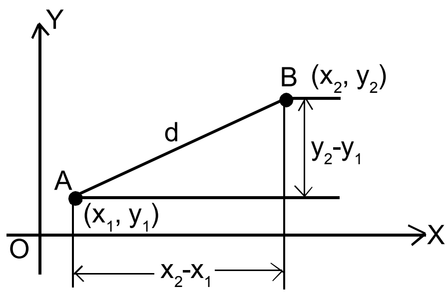
Contact No. 8033545007

Straight Lines

KEY CONCEPTS

1. DISTANCE FORMULA :

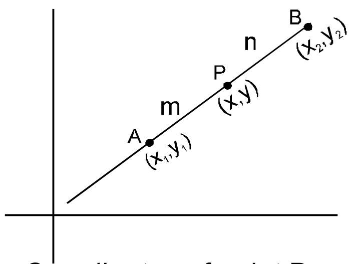
The distance between the two points A (x_1, y_1) and B (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

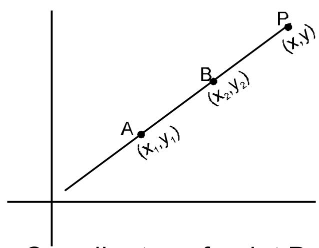


2. SECTION FORMULA :

If P (x, y) divides the line joining A (x_1, y_1) & B (x_2, y_2) in the ratio m : n

(1) Internal division (when P lies between A and B) (2) External division (when P lies outside the line segment AB)





Coordinates of point P.

$x = \dfrac{mx_2 + nx_1}{m+n}$; $y = \dfrac{my_2 + ny_1}{m+n}$.

Coordinates of point P.

$x = \dfrac{mx_2 - nx_1}{m-n}$; $y = \dfrac{my_2 - ny_1}{m-n}$

Note :

1. When co-ordinates of points A, B & P are given and the ratio in which P divides joining of A & B is to be determined then formula for internal division can be used.

If $\dfrac{m}{n}$ is positive, the division is internal, but if $\dfrac{m}{n}$ is negative, the division is external.

2. If P divides, AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be harmonic conjugate of each other w.r.t. AB.

Mathematically ; $\dfrac{2}{AB} = \dfrac{1}{AP} + \dfrac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.

3. CENTROID AND INCENTRE :

If A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are the vertices of triangle ABC,, whose sides BC, CA, AB are of lengths a, b, c respectively, then the coordinates of the centroid are :

$\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$ & the coordinates of the incentre are : $\left(\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$

Note that incentre divides the angle bisectors in the ratio

(b + c) : a ; (c + a) : b & (a + b) : c.

Remember :

(i) Orthocentre, Centroid & circumcentre are always collinear & centroid divides the line joining orthocentre & circumcentre in the ratio 2 : 1

(ii) In an isosceles triangle G, O, I & C lie on the same line.

4. STRAIGHT LINE :

A straight line is a curve such that every point on the line segment joining any two point on it lies on it.

5. SLOPE (GRADIENT) OF A LINE :

If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by m = tan θ. If θ is 90°, m does not exist, but the line is parallel to the y-axis. If θ = 0, then m = 0 & the line is parallel to the x-axis.

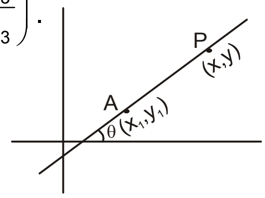
If A (x_1, y_1) & B (x_2, y_2), $x_1 \ne x_2$, are points on a straight line, then the slope m of the line given by : $m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)$.

6. CONDITION OF COLLINEARITY OF THREE POINTS - (SLOPE FROM) :

Points A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are collinear if $\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \left(\frac{y_2 - y_3}{x_2 - x_3}\right)$.

7. EQUATION OF STRAIGHT LINE IN VARIOUS FORMS :

(i) Point-slope form : $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point A (x_1, y_1)



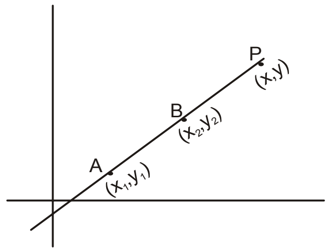
Proof : Point P (x, y) is taken on the line such that slope of PA line $m = \frac{y - y_1}{x - x_1}$

Therefore, $y - y_1 = m(x - x_1)$ is the equation of the reqd. line.

(ii) Two point form : $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of straight line which passes through the point A (x_1, y_1) & B (x_2, y_2).

Proof : Point P (x, y) is taken on the line such that slope of PA line = slope of AB line $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$.

Therefore, $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of the reqd. line.



(iii) Slope - intercept form : y = mx + c is the equation of a straight line whose slope is m & which makes an intercept c on the y-axis.

Proof : From the given fig.

$\angle QRO = \angle PQM = \angle \theta$ (corresponding angles)

and QO = c given (intercept on y-axis).

PM = PN – MN = PN – QO

PM = y – c .... (1)

QM = X .... (2)

So in ΔPQM

$\tan\theta = \frac{y - c}{x}$

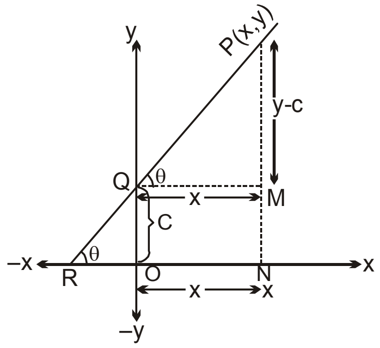
but we know $\tan\theta = m$ is the slope of the line.

$m = \tan\theta = \frac{y - c}{x} \Rightarrow mx = y - c$

$y = mx + c \Rightarrow$ equation of the line in slope-intercept form.

Where m is the slope of the line.

$c \rightarrow$ is the interpcet made by the line on y-axis. It may be negative or positive.



(iv) Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

$$\frac{x}{a} + \frac{y}{b} = 1$$

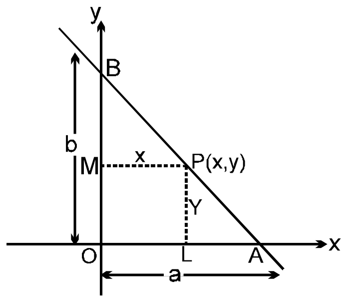
Proof : Take a point P (x, y) on the line AB.

Draw PL $\perp$ OX

then OL = x & PL = Y

clearly

Area of Δ OAB = Area of Δ OPA + Area of Δ OPB.



$\Rightarrow \quad \frac{1}{2}$ OA x OB = $\frac{1}{2}$ OA . PL + $\frac{1}{2}$ OB . PM

$\Rightarrow \quad \frac{1}{2}ab = \frac{1}{2}ay + \frac{1}{2}bx \Rightarrow ab = ay + bx$

Divide both the side by ab $\Rightarrow$ we get $\frac{x}{a} + \frac{y}{b} = 1$.

$\Rightarrow$ is the equation of straight line where a is the intercept on x-axis, b is the intercept on y-axis.

(v) PERPENDICULAR FORM :

xcos α + ysin α = p is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes angle α with positive side of x-axis.

Proof : Let P (x, y) be any point on the line. Draw PL $\perp$ OX. LM $\perp$ OQ, PN $\perp$ LM Then OL = x & LP = y

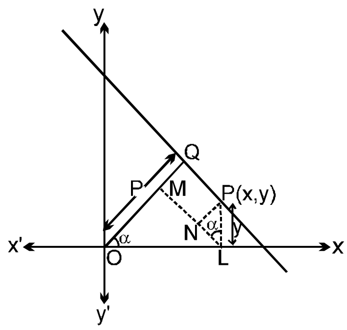
In Δ OLM $\cos\alpha = \frac{OM}{PL}$

$\Rightarrow \quad OM = OL \cos\alpha = x \cos\alpha$ .... (1)

IN ΔPNL $\sin\alpha \ \frac{PN}{PL}$

$\Rightarrow \quad PN = PL \sin\alpha = y \sin\alpha$ .... (2)

$MQ = PN = y \sin\alpha$ .... (3)



Now OQ = OM + MQ = x cos α + y sin α.

So the equation of the required line is

$$x \cos\alpha + y \sin\alpha = p$$

(vi) PARAMETRIC FORM :

The equation of the straight line passing through given point (x_1, y_1) and making an angle θ with the positive direction of x-axis is given by

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$ This equation is called parametric form of a line.

Where r is the distance of the point (x, y) on the line from the point (x_1, y_1)

Given : Point (x, y) through which line passes and point (x_1, y_1) at a distance r from (x, y) and the angle θ which line make make with +ve direction of x-axis.s

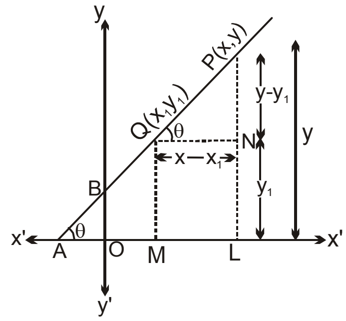
Proof : Let the given line meet x-axis at A, y-axis at B and passes through the point Q (x_1, y_1).
Let P (x, y) be any point on the line at a distance r from Q (x_1, y_1) i.e. PQ = r.

Draw PL $\perp$ OX, QM $\perp$ OX, QN $\perp$ PL.

Then QN = ML = OL – OM = $x - x$,

& PN = PL – NL = PL – QM = $y - y_1$

from Δ PQN



$$\Rightarrow \cos\theta = \frac{QN}{PQ} \Rightarrow \frac{x - x_1}{r} = \cos\theta \qquad .... (1)$$

$$\text{Similarly, } \sin\theta = \frac{QN}{PQ} \Rightarrow \frac{y - y_1}{r} = \sin\theta \qquad .... (2)$$

$\Rightarrow$ From (1) and (2)

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$ This the equation of line in parametric form.

Thus the coordinate of any point on the line at a distance 'r' from a given point (x_1, y_1) are $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$. If P is on right hand side of (x_1, y_1) then r is positive and if P is on left hand side of (x_1, y_1) then r is negative.

(vii) General from : ax + by + c = 0 is the equation of a straight line m the general from.

8. POSITION OF THE POINT (X_1, Y_1) RELATIVE TO THE LINE ax + by + c = 0 :

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of ax + by + c = 0. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie onthe non-origin side of ax + by + c = 0.

9. THE RATIO IN WHICH A GIVEN LINE DIVIDES THE LINE SEGMENT JOINING TWO POINTS

Let the given line ax + by + c = 0 divide the line segment joining A (x_1, y_1) & B (x_2, y_2) in the ratio m : n, then $\frac{m}{n} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$. If A & B are on the same side of the given line then $\frac{m}{n}$ is negative but if A & B are on opposite side of the given line, then $\frac{m}{n}$ is positive.

10. LENGTH OF PERPENDICULAR FROM A POINT ON A LINE :

The length of perpendicular from P (x_1, y_1) on ax + by + c = 0 is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a_2 + b_2}}\right|$.

11. ANGLE BETWEEN TWO STRAIGHT LINES IN TERMS OF THEIR SLOPES :

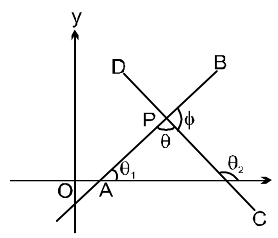
If m_1 & m_2 are the slopes of two intersecting straight lines ($m_1 m_2 \neq -1$) & θ is the acute angle between them, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Proof :

Let m_1 & m_2 be the slope of two line AB & CD which intersect at a point P and make angles θ_1 & θ_2 respectively with positive direction of x-axis. Then $m_1 = \tan\theta_1$ & $m_2 = \tan\theta_2$.

Let $\angle$ APC = θ be the angle between the given lines. Then

$\theta_2 = \theta + \theta_1 \Rightarrow \theta = \theta_2 \Rightarrow \tan\theta = \tan(\theta_2 - \theta_1)$



$$\tan\theta = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1} \Rightarrow \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Since $\angle$ APD = $\pi - \theta$ is also the angle AB & CD therefore ...(i)

$$\tan\angle APO = \tan(\pi - \theta) = -\tan\theta = -\frac{m_2 - m_1}{1 + m_1 m_2} \qquad ...(ii)$$

From (i) & (ii) we find the angle between two lines of spheres m_1 & m_2 is given by

$$\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right) \text{ or } \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

12. CONDITION FOR PERPENDICULAR AND PARALLEL LINES

(I) Condition for two lines which are parallel.

Two lines are parallel then the angle between them is zero.

$$\tan\theta = \frac{m_2 - m_1}{1+m_1 m_2} = 0$$

$\Rightarrow \quad m_2 - m_1 = 0 \quad \Rightarrow \quad m_1 = m_2$

This is the required condition for the two lines are parallel.

(i) When two straight lines are parallel their slopes are equal.

Thus any line parallel to

$ax + by + c = 0$ is of the type $ax + by + k = 0$. Where k is a parameter.

(ii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$

is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Note : The coefficient of x & y in both the equations must be same.

(iii) The area of the parallelogram = $\frac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1x + c_1$, $y = m_1x + c_2$ and $y = m_2x + d_1$, $y = m_2x + d_2$ is given by $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

(II) Condition for two line which are perpendicular.

Two line are $\perp$ then the angle between them is 90°.

$$\tan 90° = \frac{m_2 - m_1}{1+m_1 m_2} = \frac{1}{O} = \frac{m_2 - m_1}{1+m_1 m_2}$$

$\Rightarrow \quad 1 + m_1 m_2 = 0 \Rightarrow m_1 m_2 = -1.$

This is the required condition for the two lines are perpendicular means if the two lines are perpendicular then the product of their slope is – 1.

or $\quad m_1 m_2 = -1.$

If the equation of the lines are in general from. Then the condition for the two lines are parallel is

$$m_1 = m_2 \qquad \Rightarrow + \frac{a_1}{b_1} = + \frac{a_2}{b_2} \quad \Rightarrow a_1b_2 - a_2b_1 = 0$$

The condition for the two lines are perpendicular

(i) Any line perpendicular two $ax + by + c = 0$ is of the form $bx - ay + k = 0$ where k is any parameter.

(ii) Stright lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are at right angle if an only if $aa' + bb' = 0$.

Note : Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0$; $L_2 = 0$; $L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angle of the ΔABC found by these lines are given by,

$$\tan A = \frac{m_1 - m_2}{1+m_1 m_2} \; ; \; \tan B = \frac{m_2 - m_3}{1+m_2 m_3} \; \& \; \tan C = \frac{m_3 - m_1}{1+m_3 m_1}$$

13. EQUATIONS OF STRAIGHT LINE THROUGH (x_1, y_1) MAKING ANGLE α WITH $y = mx + c$ ARE :

$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

Proof : Let P (x_1, y_1) be the given point and let the given line be LMN, making angle θ with the axis of x. Then $m = \tan\theta$

Let PMR & PNS be two required lines which make angle α with the given line. Let these lines meet the axis of x at R & X respectively. Suppose lines PMR & PNS make angles θ_1 & θ_2 with positive direction of x-axis. Then the equation of two required lines are

$$(y - y_1) = \tan\theta_1 (x - x_1)$$

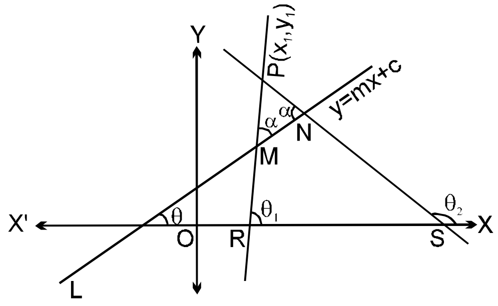
& $(y - y_2) = \tan\theta_2 (x - x_2)$

In Δ LMR we have $\theta_1 = \theta + \alpha$

In Δ LNS we have $\theta_2 = \theta + 180° - \alpha$

$\therefore$ Equation of line PR

$$y - y_1 = \tan(\theta + \alpha)(x - x_1) \quad \ldots (1)$$



and equation of lines PS

$$y - y_1 = \tan(180° + \theta - \alpha) \quad \ldots (2)$$

$$y - y_1 = \tan(\theta - \alpha)$$

(1) and (2) are the equations of the required lines.

14. CONDITION OF CONCURRENCY :

Three lines $a_1x + b_1y + c_2 = 0$, $a_1x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Alternatively : If three constant A, B & C can be found such that $A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) = 0$, then the three straight lines are concurrent.

15. AREA OF A TRIANGLE :

If (x_i, y_i), i = 1, 2, 3 are the vertices of a triangle, then its area is equal to $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$,

provided the vertices are considered in the counter clockwise sense. The above formula will give a (-)ve area if the vertices (x_i, y_i), i = 1, 2, 3 are placed in the clockwise sense.

16. CONDITION OF COLLINEARITY OF THREE POINT - (AREA FROM) :

The points (x_i, y_i), i = 1, 2, 3 are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

17. THE EQUATION OF A FAMILY OF STARLIGHT LINES PASSING THROUGH THE POINTS OF INTERSECTION OF TWO GIVEN LINES.

The equation of a family of lines passing through the point of intersection of $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ is given by $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

Note : If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4\ a'x + b'y + d'$ then $u_1 = 0$; $u_2 = 0$; $u_3 = 0$; $u_4 = 0$; form a parallelogram.

$u_2u_3 - u_1u_4 = 0$ represents the diagonal BD.

Proof : Since it is the first degree equation in x & y it is a straight line. Secondly point B satisfies the equation because the co-ordinates of B satisfy $u_2 = 0$ and $u_1 = 0$.
Similarly for the point D. Hence the result. On the similar lines $u_1u_2 - u_3u_4 = 0$ represents the diagonal AC.

Note : The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some λ and μ.

[For getting the values of λ & μ compare the coefficients of x, y & the constant terms.]

18. BISECTORS OF THE ANGLES BETWEEN TWO LINES :

(i) Equations of the bisectors of angles between the lines ax + by + c = 0 &

a'x + b'y + c' = 0 (ab' ≠ a'b) are : $\frac{ax+by+c}{\sqrt{a^2+b^2}} = \pm \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

(ii) To discriminate between the acute angle bisector & the obtuse angle bisector

If θ be the angle between one o the lines & one of the bisectors, find tan θ. If |tan θ| < 1, then $2\theta < 90°$ so that this bisector is the acute angle bisector. If |tan θ| > 1, then we get the bisector to the obtuse angle bisector.

(iii) To discriminate between the bisector of the angle containing the origin & that of the angle not containing the origin. Rewrite the equations, ax + by + c = 0 & a'x + b'y + c' = 0 such that the constant terms c, c' are positive. Then :

$\frac{ax+by+c}{\sqrt{a^2+b^2}} = + \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ given the equation of the bisector of the angle containing the origin & $\frac{ax+by+c}{\sqrt{a^2+b^2}} = - \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ given the equation of the bisector of the angle not containing the origin.

(iv) To discriminate between acute angle bisector & obtuse angle bisector proceed as follows write ax + by + c = 0 & a'x + b'y + c = 0 such that constant terms are positive.

If aa' + bb' < 0, then the angle between the lines that contains the origin is acute and the equation of the bisector of this acute angle is $\frac{ax+by+c}{\sqrt{a^2+b^2}} = + \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$; therefore

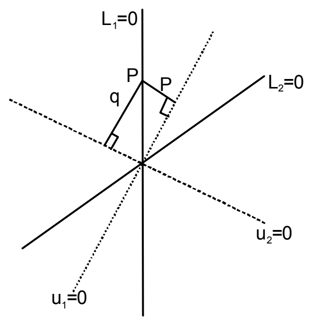
$\frac{ax+by+c}{\sqrt{a^2+b^2}} = - \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ is the equation of other bisector.

If, however, aa' + bb' > 0, then the angle between the lines that contains the origin is obtuse & the equation of the bisector of this obtuse angle is :

$\frac{ax+by+c}{\sqrt{a^2+b^2}} = - \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$; therefore $\frac{ax+by+c}{\sqrt{a^2+b^2}} = - \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

is the equation of other bisector.

(v) Another way of identifying an acute and obtuse angle bisector is as follows : Let L_1 = 0 & L_2 = are the given lines & u_1 = 0 and u_2 = 0 are the isectors between L_1 = 0 & L_2 = 0. Take a point P on any one of the lines L_1 = 0 or L_2 = 0 and drop perpendicular on u_1 = 0 & u_2=0 as shown. If, |p| < |q| ⇒ u_1 is the acute angle bisector. |p| > |q| ⇒ u_1 is the obtuse angle bisector.



|p| = |q| ⇒ the lines L_1 & L_2 are perpendicular.

Note : Equation of straight lines passing through P (x_1, y_1) & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

19. A PAIR OF STRAIGHT LINES THROUGH ORIGIN

(**i**) A homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin & if :

(a) h^2 > ab ⇒ lines are real & distinct.

(b) h^2 = ab ⇒ lines are coincident.

(c) h^2 < ab ⇒ lines are imaginary with real point of intersection i.e. (0, 0).

(ii) If y = m_1x & y = max m_2x be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then:

$$m_1 + m_2 = -\frac{2h}{b} \ \& \ m_1 m_2 = \frac{a}{b}.$$

(iii) If θ is the acute angle between the pair of straight lines represented by,

$$ax^2 + 2hxy + by^2 = 0, \text{ then ; } \tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|.$$

The condition that these lines are :

(a) At right angles to each other is a + b = 0. i.e. co-efficient of x^2 + + co-efficient of y^2 = 0.

(b) Coincident is h^2 = ab.

(c) Equally inclined to the axis of x is h = 0. i.e. coeff. of xy = 0.

Note : A homogeneous equation of degree n represent n straight lines passing through origin.

20. GENERAL EQUATION OF SECOND DEGREE REPRESENTING A PAIR OF STRAIGHT LINES :

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0, \text{ i.e. if } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogenies part only.

21. The joint equation of a pair of straight lines joining origin to the points of intersection of the line given by

$lx + my + n = 0$ ... (i)

& the 2nd degree curve :

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ... (ii)

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{-n}\right) + 2fy\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right) = 0$ .... (iii)

is obtained by homogenizing (ii) with the help of (i), by writting (i) in the form : $\left(\frac{lx+my}{-n}\right) = 1$.

22. The equation to the straight lines bisecting the angle between the straight lines,

$$ax^2 + 2hxy + by^2 = 0 \text{ is } \frac{x^2-y^2}{a-b} = \frac{xy}{h}.$$

23. The product of the perpendicular, dropped from (x_1, y_1) to the pair of lines represented by the equation,

$$ax^2 + 2hxy + by^2 = 0 \text{ is } \frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}.$$

24. Any second degree curve through the four point of intersection of f (xy) = 0 & xy = 0 is given by f (xy) + λ xy = 0 where (xy) = 0 is also a second degree curve.

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

*** Marked Questions are having more than one correct option.***

Section A: Distance formula, section formula, special points, area of triangle, Slope, Condition of collinearity, locus

A-1. The three points (–2, 2), (8, –2) and (– 4, –3,) are the vertices of
(A) An isosceles triangle (B) An equilateral triangle
(C) A right angled triangle (D) none of these

A-2. Find a point on the line joining points (0, 4) and (2, 0) dividing the line segment externally in ratio 3 : 2
(A) (3, –4) (B) (6, – 8) (C) $\left(\frac{3}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{5}, \frac{3}{5}\right)$

A-3*. The points which trisect the line segment joining the point (0, 0) and (9, 12) are
(A) (3, 4) (B) (8, 6) (C) (6, 8) (D) (4, 0)

A-4. The points A(–4, –1), B (–2, –4), C (4, 0) and D(2, 3) are the vertices of
(A) parallelogram (B) rectangle (C) rhombus (D) none of these

A-5. The incentre of the triangle formed by (0, 0), (5, 12), (16, 12) is
(A) (7, 9) (B) (9, 7) (C) (–9, 7) (D) (–7, 9)

A-6. Find the area of the triangle formed by the mid points of sides of the triangle whose vertices are (2, 1), (– 2, 3), (4, – 3)
(A) 1.5 sq. units (B) 3 sq. units (C) 6 sq. units (D) 12 sq. units

A-7*. If the points (k, 2 – 2k), (1 – k, 2k) and (–k –4, 6 – 2k) be collinear, the possible values of k are
(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) –1

A-8. Given the points A (0, 4) and B (0, – 4), the equation of the locus of the point P (x, y) such that $|AP - BP| = 6$ is :
(A) $9x^2 - 7y^2 + 63 = 0$ (B) $9x^2 - 7y^2 - 63 = 0$
(C) $7x^2 - 9y^2 + 63 = 0$ (D) $7x^2 - 9y^2 - 63 = 0$

A-9. Slope of line joining points (5, 3) abd (k^2, k + 1) is 1/2 then k is
(A) 1 (B) $1 + \sqrt{2}$ (C) $\sqrt{2} - 1$ (D) $-1 - \sqrt{2}$

Section B: Equation of straight lines including parametric form, angle between two lines, Position of points, parallel lines, perpendicular lines, angle bisector

B-1. Find the equation to the straight line which passes through the point (– 4, 3) and is such that the portion of it between the axes is divided by the point in the ratio 5 : 3.
(A) 9x – 20y + 96 = 0 (B) 2x – y + 11 = 0 (C) 2x + y + 5 = 0 (D) 3x – 2y + 7 = 0

B-2. The equations of the perpendicular bisector of the sides AB and AC of a ΔABC are x – y + 5 = 0 and x + 2y = 0 respectively. If the point A is (1, –2) then the equation of the line BC is :
(A) 14x + 23y = 40 (B) 14x – 23y = 40 (C) 23x + 14y = 40 (D) 23x – 14y = 40

B-3. The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$ is :

(A) $5\sqrt{3}$ (B) $4\sqrt{2}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

B-4*. The points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$, are

(A) (3, 1) (B) (7, 11) (C) (–7, 11) (D) (1, 3)

B-5*. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Then the equations of other sides are :

(A) $7x - 4y + 25 = 0$ (B) $7x + 4y + 25 = 0$ (C) $7x - 4y - 3 = 0$ (D) $4x + 7y = 11$

B-6. Which pair of points lie on the same side of $3x - 8y - 7 = 0$

(A) (0, –1) and (0, 0) (B) (4, –3) and (0, 1) (C) (– 3, – 4) and (1, 2) (D) (–1, –1) and (3, 7)

B-7. The reflection of the point (4, –13) in the line $5x + y + 6 = 0$ is

(A) (–1, –14) (B) (3, 4) (C) (1, 2) (D) (–4, 13)

B-8. Find the equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$.

(A) $11x - 3y + 9 = 0$ (B) $3x + 11y - 13 = 0$ (C) $3x + 11y - 3 = 0$ (D) $11x - 3y + 2 = 0$

B-9. A ray of light passing through the point A (1,2) is reflected at a point B on the x-axis and then passes through (5,3). Then the equation of AB is :

(A) $5x + 4y = 13$ (B) $5x - 4y = -3$ (C) $4x + 5y = 14$ (D) $4x - 5y = -6$

Section C: Family of lines, condition of concurrency, pair of lines, homogenisation

C-1. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$ are concurrent at the point :

(A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (B) (1, 3) (C) (3, 1) (D) $\left(\frac{3}{4}, \frac{1}{2}\right)$

C-2. If the straight line $x + 2y = 9$, $3x - 5y = 5$ & $ax + by = 1$ are concurrent, then the straight line $5x + 2y = 1$ passes through the point :

(A) (a, –b) (B) (–a, b) (C) (a, b) (D) (–a , –b)

C-3. Given the family of lines, $a(3x + 4y + 6) + b(x + y + 2) = 0$. The line of the family situated at the greatest distance from the point P (2,3) has equation :

(A) $4x + 3y + 8 = 0$ (B) $5x + 3y + 10 = 0$ (C) $15x + 8y + 30 = 0$ (D) None

C-4. If the slope of one line of the pair of lines represented by $ax^2 + 10xy + y^2 = 0$ is four times, the slope of the other line, then a =

(A) 1 (B) 2 (C) 4 (D) 16

C-5. The combined equation of the bisectors of the angle between the lines represented by $(x^2 + y^2)\sqrt{3} = 4xy$ is

(A) $y^2 - x^2 = 0$ (B) $xy = 0$ (C) $x^2 + y^2 = 2xy$ (D) $\frac{x^2 - y^2}{\sqrt{3}} = \frac{xy}{2}$

C-6. The equation of the pair of bisectors of the angles between two straight lines is, $12x^2 - 7xy - 12y^2 = 0$. If the equation of one line is $2y - x = 0$ then the equation of the other line is:

(A) $41x - 38y = 0$ (B) $38x - 41y = 0$ (C) $38x + 41y = 0$ (D) $41x + 38y = 0$

C-7. The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is

(A) 4 (B) $\frac{4}{\sqrt{3}}$ (C) 2 (D) $2\sqrt{3}$

C-8. The straight lines joining the origin to the points of intersection of the line $2x + y = 1$ and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

C-9. Area of the rhombus bounded by the four lines, $ax \pm by \pm c = 0$ is :

(A) $\frac{c^2}{2ab}$ (B) $\frac{2c^2}{ab}$ (C) $\frac{4c^2}{ab}$ (D) $\frac{ab}{4c^2}$

PART - II : SUBJECTIVE QUESTIONS

Section A: Distance formula, section formula, special points, area of triangle, Slope, Condition of collinearity, locus

1. Prove that the points (2a, 4a), (2a, 6a) and $(2a + \sqrt{3}a, 5a)$ are the vertices of an equilateral triangle whose side is 2a.

2. In what ratio does the point $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (– 7, 9) ?

3. Find the third vertex of a triangle if two of its vertices are at (– 2, 4) and (7, – 3) and the centroid at (3, 2).

4. Find circumcentre of triangle whose vertices are (–2, –3) (–1, 0) (7, –6).

5. A and B are the points (3, 4) and (5, – 2). Find the coordinates of a point P such that PA = PB and the area of the triangle PAB = 10.

6. Find the area of the quadrilateral with vertices as the points given in each of the following :

(i) (0, 0) (6, 0) (4, 3) (0, 3) (ii) (0, 0) (a, 0) (a, b), (0, b)

7. Find the equation of the locus of the point whose distance from x-axis is twice that from y-axis.

8. Find the locus of the centroid of a triangle whose vertices are (a cos t, a sin t), (b sin t, –b cos t) and (1, 0) where ‘t’ is the parameter.

9. If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) be collinear, show that $\frac{y_2 - y_3}{x_2x_3} + \frac{y_3 - y_1}{x_3x_1} + \frac{y_1 - y_2}{x_1x_2} = 0$.

Section B: Equation of straight lines including parametric form, angle between two lines, Position of points, parallel lines, perpendicular lines, angle bisector

10. Find the equation to the straight line cutting off an intercept – 5 from the axis of y and being equally inclined to the axes.

11. The coordinates of the mid-points of the sides of a triangle ABC are D(2, 1), E(5, 3) and F(3, 7). Find the lengths and equations of its sides.

12. Find the slope of the lines which make an angle of 45° with the line $x - 2y = 3$.

13. Find the coordinates of the othocentre of the triangle whose sides are $3x - 2y = 6$, $3x + 4y + 12 = 0$ and $3x - 8y + 12 = 0$.

14. Write down parametric equation of line passing through the points (2, 1) and (1, 2)

15. Find the distance between the parallel lines $3x - 4y + 5 = 0$ and $3x - 4y + 7 = 0$.

16. If p and p′ be the perpendiculars from the origin upon the straight lines whose equations are $x \sec \theta + y \operatorname{cosec} \theta = a$ and $x \cos\theta - y \sin \theta = a \cos 2\theta$, prove that $4p^2 + p'^2 = a^2$.

17. Find the area of parallelogram whose two sides are $y = x + 3$ and $2x - y + 1 = 0$ also remaining two sides are passing through (0, 0).

18. Find the equation of the bisector of the angle between the lines $4x + 3y - 7 = 0$ and $24x + 7y - 31 = 0$ which contains the origin.

Section C: Family of lines, condition of concurrency, pair of lines, homogenisation

19. Prove that $x(a + 2b) + y (a - 3b) = (a - b)$ passes through a fixed point for all $a, b \in R$

20. Find the value of k so that the following equations may represent pairs of straight lines :
$12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0$

PART - III : MISCELLANEOUS OBJECTIVE QUESTIONS

MATCH THE COLUMN

1. **Match the following**
A(0, 0), B(6, 0), C(0, 8) are vertices of a triangle then

Column-I		Column-II
(A) Incentre of triangle ABC is	(p)	$\left(2, \frac{8}{3}\right)$
(B) Circumcentre of triangle ABC is	(q)	(0, 0)
(C) Orthocentre of triangle ABC is	(r)	(3, 4)
(D) Centroid of triangle ABC is	(s)	(2, 2)

2. **Column – I** / **Column – II**

	Column – I		Column – II
(A)	Slope of line bisecting the angle between co-ordinate axes, is	(p)	3
(B)	Area of Δ formed by line $3x + 4y + 12 = 0$ with co-ordinate axis is	(q)	1
(C)	If the equation $2x^2 - 2xy - y^2 - 6x + 6y + c = 0$ represents a pair of lines, then 'c' is	(r)	6
(D)	If distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is $25\sqrt{2}$, then 'a/5' is equal to	(s)	– 1

(ASSERTION/REASON)

3. **Statement-1** : $3x^2 + 4xy + y^2 = 0$ and $7x^2 + 12xy + y^2 = 0$ are representing lines equally inclined to each other

Statement-2 : If lines represented by $a_1x^2 + 2h_1xy + b_1y^2 = 0$ and $a_2x^2 + 2h_2xy + b_2y^2 = 0$ are inclined to each other then both pairs have same set of angle bisectors.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

4. **Statement-1 :** The diagonal of the quadrilateral whose sides are $3x + 2y + 1 = 0$, $3x + 2y + 2 = 0$, $2x + 3y + 1 = 0$ and $2x + 3y + 2 = 0$ include angle $\pi/2$

Statement-2 : Diagonals of a parallelogram are bisecting each other.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5. Statement-1 : Each point on the line $y - x + 12 = 0$ is at same distance from the lines $3x + 4y - 12 = 0$ and $4x + 3y - 12 = 0$.

Statement-2 : locus of point which is at equal distance from the two given lines is the angle bisectors of the two lines.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

6. **Statement-1** Area of Δ formed by the line which is passing through the point (5, 6) such that segment of the line between axis is bisected at the point, with coordinate axis is 60 sq. units

Statement-2 : Area of Δ formed by line passing through point (α, β), with axis is maximum when point (α, β) is mid point of segment of line b/w axis.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

(COMPREHENSION)

Comprehenssion # 1

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points E(3, 4) and F(1, 2) respectively and centroid of Δ ABC is G(3, 2), then answer the following questions :

7. The equation of side AB is

(A) $2x + y = 4$ (B) $x + y - 3 = 0$ (C) $4x - 2y = 0$ (D) none of these

8. Coordinate of D are

(A) (7, –4) (B) (5, 0) (C) (7, 4) (D) (– 3, 0)

9. Height of altitude drawn from point A is (in units)

(A) $4\sqrt{2}$ (B) $3\sqrt{2}$ (C) $6\sqrt{2}$ (D) $2\sqrt{3}$

Comprehension # 2

Given two straight lines AB and AC whose equations are $3x + 4y = 5$ and $4x - 3y = 15$ respectively. Then the possible equation of line BC through (1, 2), such that ΔABC is isosceles, is $L_1 = 0$ or $L_2 = 0$, then answer the following questions

10. If $L_1 \equiv ax + by + c = 0$ & $L_2 \equiv dx + ey + f = 0$ where a, b, c, d, e, f $\in$ I, and a, d > 0, then c + f =

(A) 1 (B) 2 (C) 3 (D) 4

11. A straight line through P(2, c + f – 1), inclined at an angle of 60° with positive y axis. The coordinates of one of the points on it at a distance (c + f) units from point P is (c, f obtained from previous question)

(A) $(2 + 2\sqrt{3}, 5)$ (B) $(3 + 2\sqrt{3}, 3)$ (C) $(2 + 3\sqrt{2}, 4)$ (D) $(2 + 3\sqrt{2}, 3)$

12. If (a, b) is the coordinate of the point obtained in previous question, then find the equation of line which is at the distance |b – 2a – 1| units from origin and make equal intercept on coordinate axis in first quadrant.

(A) $x + y + 4\sqrt{6} = 0$ (B) $x + y + 2\sqrt{6} = 0$ (C) $x + y - 4\sqrt{6} = 0$ (D) $x + y - 2\sqrt{6} = 0$

EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

Single choice

1. The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin then the co–ordinates of C are :

(A) (2a, 2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2}, 0\right)$ (D) none

2. A triangle ABC with vertices A (– 1, 0), B (– 2, 3/4) & C (– 3, – 7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be :

(A) (– 3, – 2) (B) (1, 3) (C) (– 1, 2) (D) none of these

3. In a triangle ABC, co-ordinates of A are (1, 2) and the equations to the medians through B and C are x + y = 5 and x = 4 respectively. Then the co-ordinates of B and C will be

(A) (– 2, 7), (4, 3) (B) (7, – 2), (4, 3) (C) (2, 7), (– 4, 3) (D) (2, – 7), (3, – 4)

4. Equation of a straight line passing through the origin and making with x – axis an angle twice the size of the angle made by the line y = 0.2 x with the x – axis, is :

(A) y = 0.4 x (B) y = (5/12) x (C) 6y – 5x = 0 (D) none of these

5. If the vertices P and Q of a triangle PQR are given by (2,5) and (4,–11) respectively, and the point R moves along the line N : 9x + 7y + 4 = 0, then the locus of the centroid of the triangle PQR is a straight line parallel to

(A) PQ (B) QR (C) RP (D) N

6. A variable straight line passes through a fixed point (a, b) intersecting the co–ordinates axes at A & B. If 'O' is the origin then the locus of the centroid of the triangle OAB is :

(A) bx + ay – 3xy = 0 (B) bx + ay – 2xy = 0

(C) ax + by – 3xy = 0 (D) ax + by – 2xy = 0

7. Area of the quadrilateral formed by the lines $|x| + |y| = 2$ is :

(A) 8 (B) 6 (C) 4 (D) none

8. Points A & B are in the first quadrant; point 'O' is the origin, If the slope of OA is 1, slope of OB is 7 and OA = OB, then the slope of AB is :

(A) –1/5 (B) –1/4 (C) –1/3 (D) –1/2

9. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + a\,by + 1 = 0\ \forall\ a \in R, b > 0$ are :

(A) $b \in (2, 4)$ (B) $b \in (0, 2)$ (C) $b \in [0, 2]$ (D) $(2, \infty)$

10. The point $(a^2, a + 1)$ is a point in the angle between the lines 3x – y + 1 = 0 and x + 2y – 5 = 0 containing the origin if :

(A) $a \geq 1$ or $a \leq -3$ (B) $a \in (-3, 0) \cup (1/3, 1)$

(C) $a \in (0, 1)$ (D) none of these

11. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line, 2x + y = a. Then the area of the triangle is :

(A) $\frac{a^2}{2}$ (B) $\frac{a^2}{3}$ (C) $\frac{a^2}{5}$ (D) none

12. Let the co-ordintes of the two points A & B be (1,2) and (7,5) respectively. The line AB is rotated through 45° in anti clockwise direction about the point of trisection of AB which is nearer to B. The equation of the line in new position is :

(A) $2x - y - 6 = 0$ (B) $x - y - 1 = 0$
(C) $3x - y - 11 = 0$ (D) None of these

13. The image of the point A (1, 2) by the line mirror y = x is the point B and the image of B by the line mirror y = 0 is the point (α, β) then :

(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$ (C) $\alpha = 2, \beta = -1$ (D) none of these

14. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$. The equation of the other line is :

(A) $3x + 3y - 1 = 0$ (B) $x - 3y + 2 = 0$ (C) $5x + 5y - 3 = 0$ (D) none

15. On the portion of the straight line, $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co–ordinates :

(A) (2, 3) (B) (3, 2) (C) (3, 3) (D) none

16. A light beam emanating from the point A(3, 10) reflects from the straight line $2x + y - 6 = 0$ and then passes through the point B(4, 3). The equation of the reflected beam is :

(A) $3x - y + 1 = 0$ (B) $x + 3y - 13 = 0$ (C) $3x + y - 15 = 0$ (D) $x - 3y + 5 = 0$

17. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains the points (– 1, 4) is :

(A) $21x + 27y - 121 = 0$ (B) $21x - 27y + 121 = 0$
(C) $21x + 27y + 191 = 0$ (D) $\frac{-3x+4y-12}{5} = \frac{12x-5y+7}{13}$

18. The equation of bisectors of two lines L_1 & L_2 are $2x - 16y - 5 = 0$ and $64x + 8y + 35 = 0$. If the line L_1 passes through (– 11, 4), the equation of acute angle bisector of L_1 & L_2 is :

(A) $2x - 16y - 5 = 0$ (B) $64x + 8y + 35 = 0$ (C) data insufficient (D) none of these

19. AB is a variable line sliding between the coordinate axes is such a way that a lies on x-axis and B lies on y-axis. If P is a variable point on AB such that PA = b, PB = a and AB = a + b, then equation of locus of P is

(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $x^2 + y^2 = a^2 + b^2$ (D) none of these

20. Equations of the line pair through the origin and perpendicular to the line pair $xy - 3y^2 + y - 2x + 10 = 0$ is :

(A) $xy - 3y^2 = 0$ (B) $xy + 3x^2 = 0$ (C) $xy + 3y^2 = 0$ (D) $x^2 - y^2 = 0$

21. If the line y = mx bisects the angle between the lines $ax^2 + 2hxy + by^2 = 0$ then m is a root of the quadratic equation :

(A) $hx^2 + (a–b)x - h = 0$ (B) $x^2 + h(a–b)x - 1 = 0$
(C) $(a - b)x^2 + hx - (a - b) = 0$ (C) $(a - b)x^2 - hx - (a - b) = 0$

22. If pairs of straight lines, $x^2 - 2pxy - y^2 = 0$ & $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair then :

(A) pq = – 1/2 (B) pq = – 2 (C) pq = – 1 (D) p/q = – 1

23. If the straight lines joining the origin and the points of intersection of the curve $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis then the value of k is equal to :

(A) 1 (B) – 1 (C) 2 (D) 3

Multiple choice

24. The equation of the bisectors of the angle between the two intersecting lines :

$\frac{x-3}{\cos\theta} = \frac{y+5}{\sin\theta}$ and $\frac{x-3}{\cos\phi} = \frac{y+5}{\sin\phi}$ are $\frac{x-3}{\cos\alpha} = \frac{y+5}{\sin\alpha}$ and $\frac{x-3}{\beta} = \frac{y+5}{\gamma}$ then

(A) $\alpha = \frac{\theta+\phi}{2}$ (B) $\beta = -\sin\alpha$ (C) $\gamma = \cos\alpha$ (D) $\beta = \sin\alpha$

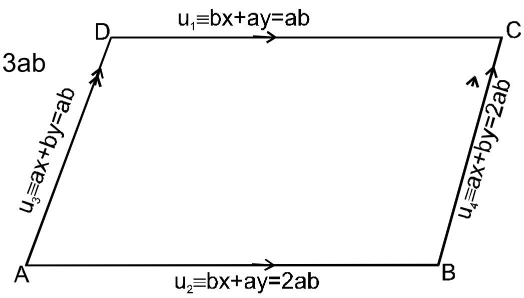
25. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/are always rational point (s) ?

(A) centriod (B) incentre (C) circumcentre (D) orthocentre

26. Equation of a straight line passing through the point (4, 5) and equally inclined to the lines, $3x = 4y + 7$ and $5y = 12x + 6$ is

(A) $9x - 7y = 1$ (B) $9x + 7y = 71$ (C) $7x + 9y = 73$ (D) $7x - 9y + 17 = 0$

27. In a parallelogram as shown in the figure $(a \neq b)$:



(A) Equation of the diagonol AC is $(a + b) x + (a + b) y = 3ab$

(B) Equation of the diagonal BD is $u_1 u_4 - u_2 u_3 = 0$

(C) Co-ordinates of the points of intersection of the two diagonals are $\left(\frac{3ab}{2(a+b)}, \frac{3ab}{2(a+b)}\right)$

(D) The angle between the two diagonols is $\pi/3$.

28. If the equation, $2x^2 + k\,xy - 3y^2 - x - 4y - 1 = 0$ represents a pair of lines then the value of k can be:

(A) 1 (B) 5 (C) – 1 (D) – 5

29. If $a^2 + 9b^2 - 4c^2 = 6\,ab$ then the family of lines $ax + by + c = 0$ are concurrent at :

(A) (1/2, 3/2) (B) (– 1/2, – 3/2) (C) (– 1/2, 3/2) (D) (1/2, – 3/2)

PART - II : SUBJECTIVE QUESTIONS

1. The variable line $x \cos\theta + y \sin\theta = 2$ cuts the x and y axes at A and B respectively. Find the locus of the vertex P of the rectangle OAPB, O being the origin.

2. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ & $\frac{x}{b} + \frac{y}{a} = 1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a + b) = ab(x + y)$.

3. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then show that :

(i) The median through A can be written in the form $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

(ii) the line through A & parallel to BC can be written in the form ; $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

(iii) equation to the angle bisector through A is $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

where b = AC & c = AB.

4. If the straight lines, $ax + by + p = 0$ & $x\cos\alpha + y\sin\alpha - p = 0$ enclose an angle $\pi/4$ between them, and meet the straight line $x\sin\alpha - y\cos\alpha = 0$ in the same point, then find the value of $a^2 + b^2$.

5. Drive the conditions to be imposed on β so that $(0, \beta)$ should lie on or inside the triangle having sides $y + 3x + 2 = 0$, $3y - 2x - 5 = 0$ & $4y + x - 14 = 0$.

6. Let the co–ordinates of the two points A & B be (1, 2) and (7, 5) respectively. The line AB is rotated through 45º in anti clockwise direction about the point of trisection of AB which is nearer to B. Find the equation of the line in new position.

7. Find the equations of the straight lines passing through the point (1, 1) and parallel to the lines represented by the equation, $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$.

8. Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrence & if not, give reasons.

9. Find the coordinates of the vertices of a square inscribed in the triangle with vertices A (0, 0), B (2, 1), C (3, 0); given that two of its vertices are on the side AC.

10. The equations of perpendiculars of the sides AB & AC of Δ ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is (– 2, 3) and point of intersection of perpendiculars bisectors $\left(\frac{3}{2}, \frac{5}{2}\right)$ is, find the equation of medians to the sides AB and AC respectively.

11. Two consecutive side of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation to one diagonal is $11x + 7y = 9$. Find the equation of the other diagonal.

12. Two fixed points A and B are taken on the x & y axes respectively such that OA = a and OB = b. Two variable points C & D are taken on the x & y axes respectively. Find the locus of the point of intersection of AD & CB when, $\frac{1}{OC} - \frac{1}{OD} = \frac{1}{OA} - \frac{1}{OB}$.

13. Two ends A & B of a straight line segment of common length c slide upon the fixed rectangular axes Ox & Oy respectively. If the rectangle OAPB is completed show that the locus of the foot of the perpendicular drawn from P to AB is, $x^{2/3} + y^{2/3} = c^{2/3}$.

14. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

1. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of OP + OQ, as L varies, where O is the origin.
IIT - 2002, 5]

2. A straight line L through the origin meets the lines x + y = 1 and x + y = 3 at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn parallel to 2x – y = 5 and 3x + y = 5 respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **[IIT - 2002, 5]**

3. A straight line through the origin O meets the parallel lines 4x + 2y = 9 and 2x + y + 6 = 0 at points P and Q respectively. Then the point O divides the segement PQ in the ratio **[IIT - 2002 S, 3]**
(A) 1 : 2 (B) 3 : 4 (C) 2 : 1 (D) 4 : 3

4. Let P = (–1, 0), Q = (0, 0) and R = (3, $3\sqrt{3}$) be three points. Then the equation of the bisector of the angle PQR is **[IIT - 2002 S, 3]**
(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

5. Let $0 < \alpha < \frac{\pi}{2}$ be fixed angle. If P = (cosθ, sin θ) and Q = (cos(α – θ), sin (α – θ)), then Q is obtained from P by
[IIT - 2002 S, 3]
(A) clockwise rotation around origin through an angle α
(B) anticlockwise rotation around origin through an angle α
(C) reflection in the line through origin with slope tan α
(D) reflection in the line through origin with slope tan (α/2)

6. The area bounded by the curves y = | x | – 1 and y = – | x | + 1 is **[IIT - 2002, 3]**
(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

7. The centre of circle inscribed in a square formed by lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is
[IIT- 2003]
(A) (4, 7) (B) (7, 4) (C) (9, 4) (D) (4, 9)

8. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is **[IIT - 2003, 3]**
(A) $\left(3, \frac{5}{4}\right)$ (B) (3, 12) (C) $\left(3, \frac{3}{4}\right)$ (D) (3, 9)

9. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is **[IIT - 2003 S, 3]**
(A) 133 (B) 190 (C) 233 (D) 105

10. Area of the triangle formed by the line x + y = 3 and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is **[IIT - 2004 S,]**
(A) 2 sq units (B) 4 sq. units (C) 6 sq. units (D) 8 sq. units

11. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines y = x and x + y = 2 is $4h^2$. Find the locus of the point P. **[IIT - 2005, 2]**

12. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[IIT - JEE 2007]**

STATEMENT - 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

because

STATEMENT - 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is** a correct explanation for Statement - 1
(B) Statement - 1 is True, Statement - 2 is True; Statement - 2 is **NOT** a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement - 2 is False
(D) Statement - 1 is False, Statement - 2 is True

13. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are **[IIT - JEE 2007]**

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$ (C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

14. A straight line L through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$ **[JEE 2011]**

15. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$, and $y = 0$, where $p \neq q$, is :

(A) a hyperbola (B) a parabola (C) an ellipse (D) a straight line

[JEE 2009 (3, –1) out of 80]

16. Consider three points $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then, **[JEE 2008 (3, –1) out of 81]**

(A) P lies on the line segment RQ (B) Q lies on the line segment PR
(C) R lies on the line segment QP (D) P,Q, R are non-collinear

17. Consider the lines given by :

$L_1 = x + 3y - 5 = 0$; $L_2 = 3x - ky - 1 = 0$; $L_3 = 5x + 2y - 12 = 0$

Match the statement / Expression in **Column–I** with the statement / Expressions in **Column–II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in OMR.

Column–I	Column–II
(A) L_1, L_2, L_3 are concurrent, if :	(p) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if :	(q) $k = -\frac{6}{5}$
(C) L_1, L_2, L_3 form a triangle, if :	(r) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if :	(s) $k = 5$

[JEE 2008, 6]

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. The locus of the mid point of the intercept of the variable line x cos a + y sin a = p (p constant) between the coordinate axes is : **[AIEEE 2002]**

(1) $x^2 + y^{-2} = p^{-2}$ (2) $x^{-2} + y^{-2} = 2p^{-2}$ (3) $x^{-2} + y^{-2} = 4p^{-2}$ (4) none of these

2. Three straight lines 2x + 11y – 5 = 0, 4x – 3y –2 = 0 and 24x + 7y – 20 = 0 : **[AIEEE 2002]**

(1) form a triangle.

(2) are only concurrent.

(3) are concurrent with one line bisecting the angle between the other two.

(4) none of these.

3. A straight line through the point (2,2) intersects the lines $\sqrt{3}\,x + y = 0$ and $\sqrt{3}\,x - y = 0$ at the points A and B.The equation to the line AB so that the triangle OAB is equilateral is : **[AIEEE 2002]**

(1) x – 2 = 0 (2) y – 2 = 0 (3) x + y – 4 = 0 (4) none of these

4. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of 'c' is : **[AIEEE 2003]**

(1) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (2) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(3) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (4) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

5. Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cos t) and (1, 0), where t is a parameter is : **[AIEEE 2003]**

(1) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (2) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(3) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ (4) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

6. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then : **[AIEEE 2003]**

(1) p = q (2) p = – q (3) pq = 1 (4) pq = – 1

7. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha \left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is : **[AIEEE 2003]**

(1) y (cos α – sin α) – x (sin α – cos α) = a (2) y (cos α + sin α) + x (sin α – cos α) = a

(3) y (cos α + sin α) + x (sin α + cos α) = a (4) y (cos α + sin α) + x (cos α – sin α) = a

8. Let A(2,–3) and B(–2,1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line 2x + 3y = 1, then the locus of the vertex C is the line : **[AIEEE 2004]**

(1) 2x + 3y = 9 (2) 2x – 3y = 7 (3) 3x + 2y = 5 (4) 3x – 2y = 3

9. The equation of the straight line passing through the point (4,3) and making intercepts on the co–ordinate axes whose sum is –1, is : **[AIEEE 2004]**

(1) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$ (2) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(3) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$ (4) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

10. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value : **[AIEEE 2004]**

(1) 1 (2) –1 (3) 2 (4) –2

11. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals : **[AIEEE 2004]**

(1) 1 (2) –1 (3) 3 (4) –3

12. The line parallel to the x–axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a,b) \neq (0,0)$ is : **[AIEEE 2005]**

(1) above the x–axis at a distance of (2/3) from it. (2) above the x–axis at a distance of (3/2) from it.
(3) below the x–axis at a distance of (2/3) from it. (4) below the x–axis at a distance of (3/2) from it.

13. If non–zero numbers a,b,c are in HP, then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is : **[AIEEE 2005]**

(1) $\left(1,-\frac{1}{2}\right)$ (2) (1, – 2) (3) (–1, – 2) (4) (–1, 2)

14. If a vertex of a triangle is (1,1) and the mid–points of two sides through this vertex are (–1,2) and (3,2), then the centroid of the triangle is : **[AIEEE 2005]**

(1) $\left(\frac{1}{3},\frac{7}{3}\right)$ (2) $\left(1,\frac{7}{3}\right)$ (3) $\left(-\frac{1}{3},\frac{7}{3}\right)$ (4) $\left(-1,\frac{7}{3}\right)$

15. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameter of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector, then : **[AIEEE 2005]**

(1) $3a^2 + 2ab + 3b^2 = 0$ (2) $3a^2 + 10ab + 3b^2 = 0$

(3) $3a^2 - 2ab + 3b^2 = 0$ (4) $3a^2 - 10ab + 3b^2 = 0$

16. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is : **[AIEEE 2006]**

(1) $3x - 4y + 7 = 0$ (2) $4x + 3y = 24$ (3) $3x + 4y = 25$ (4) $x + y = 7$

17. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then 'a' belongs to : **[AIEEE 2006]**

(1) $(3, \infty)$ (2) $\left(\frac{1}{2},3\right)$ (3) $\left(-3,-\frac{1}{2}\right)$ (4) $\left(0,\frac{1}{2}\right)$

18. Let A(h, k), B(1, 1) and C(2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of triangle is 1, then the set of values which 'k' can take is given by : **[AIEEE 2007]**

(1) {1, 3} (2) {0, 2} (3) {–1, 3} (4) {–3, – 2}

19. Let P = (–1, 0) Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. The equation of the bisector of the $\angle$PQR is : **[AIEEE 2007]**

(1) $\sqrt{3}$ x + y = 0 (2) x + $\frac{\sqrt{3}}{2}$ y = 0 (3) $\frac{\sqrt{3}}{2}$ x + y = 0 (4) x + $\sqrt{3}$ y = 0

20. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is: **[AIEEE 2007]**

(1) $-\frac{1}{2}$ (2) – 2 (3) ± 1 (4) 2

21. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept – 4. Then a possible value of k is : **[AIEEE 2008]**

(1) – 4 (2) 1 (3) 2 (4) – 2

22. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for : **[AIEEE 2009]**

(1) exactly one value of p. (2) exactly two values of p.
(3) more than two values of p. (4) no value of p.

23. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point : **[AIEEE 2009]**

(1) $\left(\frac{5}{4}, 0\right)$ (2) $\left(\frac{5}{2}, 0\right)$ (3) $\left(\frac{5}{3}, 0\right)$ (4) 0, 0

24.. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is : **[AIEEE 2010]**

(1) $\sqrt{17}$ (2) $\frac{17}{\sqrt{15}}$ (3) $\frac{23}{\sqrt{17}}$ (4) $\frac{23}{\sqrt{15}}$

25.. The lines L_1 : y – x = 0 and L_2 : 2x + y = 0 intersect the line L_3 : y + 2 = 0 at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersect L_3 at R. **[AIEEE - 2011]**

Statement - 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement - 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

26. If the line 2x + y = k passes through the point which divides the line segement joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE - 2012]**

(1) $\frac{29}{5}$ (2) 5 (3) 6 (4) $\frac{11}{5}$

27. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE - 2012]**

(1) $-\frac{1}{4}$ (2) – 4 (3) – 2 (4) $-\frac{1}{2}$

EXERCISE # 4

NCERT BOARD QUESTIONS

Short Answer Type Questions

1. Find the equation of the straight line which passes through the point (1, –2) and cuts off equal intercepts from axes.

2. Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, –1).

3. Find the angle between the lines y = $(2-\sqrt{3})(x+5)$ and y = $(2+\sqrt{3})(x-7)$.

4. Find the equation of the lines which passes through the point (3, 4) and cuts off intercepts from the coordinate axes such that their sum is 14.

5. Find the points on the line x + y = 4 which lie at a unit distance from the line 4x + 3y = 10.

6. Show that the tangent of an angle between the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ is $\frac{2ab}{a^2-b^2}$.

7. Find the equation of lines passing through (1, 2) and making angle 30º with y-axis.

8. Find the equation of the line passing through the point of intersection of 2x + y = 5 and x + 3y + 8 = 0 and parallel to the line 3x + 4y = 7.

9. For what values of a and b the intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x – 3y + 6 = 0 on the axes.

10. If the intercept of a line between the coordinate axes is divided by the point (–5, 4) in the ratio 1 : 2, then find the equation of the line.

11. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120º with the positive direction of x-axis.

12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by 3x + 4y = 4 and the opposite vertex of the hypotenuse is (2, 2).

Long Answer Type

13. If the equation of the base of an equilateral triangle is x + y = 2 and the vertex is (2, –1), then find the length of the side of the triangle.

14. A variable line passes through a fixed point P. the algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P.

15. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line x + y = 4 is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

19. If the sum of the distance of a moving point in a plane from the axes is 1, then find the locus of the point.

20. P_1, P_2 are points on either of the two lines $y-\sqrt{3}\,|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1, P_2 on the bisector of the angle between the given lines.

21. If p is the length of perpendicular from the origin on the line $\frac{x}{a}+\frac{y}{b}=1$ and a^2, p^2, b^2 are in A.P, then show that $a^4 + b^4 = 0$.

ANSWERS

EXERCISE # 1

PART # I

A-1. (C) **A-2.** (B) **A-3*.** (A,C) **A-4.** (B) **A-5.** (A) **A-6.** (A) **A-7*.** (B, D)
A-8. (A) **A-9.** (B)

B-1. (A) **B-2.** (A) **B-3.** (B) **B-4*.** (A, C) **B-5*.** (A,C,D) **B-6.** (D)
B-7. (A) **B-8.** (A) **B-9.** (A)

C-1. (D) **C-2.** (C) **C-3.** (A) **C-4.** (D) **C-5.** (A) **C-6.** (A) **C-7.** (C)
C-8. (A) **C-9.** (B)

PART # II

2. 1 : 3 internally **3.** (4, 5) **4.** (3, –3) **5.** (7, 2) or (1, 0)

6. (i) 15 sq. units (ii) |ab| sq. units **7.** $y = \pm 2x$ **8.** $(3x - 1)^2 + 9y^2 = a^2 + b^2$

10. $x - y - 5 = 0,\ x + y + 5 = 0$

11. AB : $2x - 3y + 15 = 0,\ 2\sqrt{13}$ BC : $2x + y - 5 = 0,\ 4\sqrt{5}$, CA : $6x - y - 27 = 0,\ 2\sqrt{37}$

12. $3, -\frac{1}{3}$ **13.** $\left(-\frac{1}{6}, -\frac{23}{9}\right)$ **14.** $\left[\frac{x-2}{-\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = r\right]$ **15.** 2/5

17. 3 sq. unit **18.** $x - 2y + 1 = 0$ **20.** $k = 2$

PART # III

1. (A → s), (B → r), (C → q), (D → p) **2.** (A → q, s), (B → r), (C → p), (D → q, s)

3. (A) **4.** (B) **5.** (D **6.** (C) **7.** (A) **8.** (B) **9.** (C)

10. (D) **11.** (A) **12.** (C)

EXERCISE # 2

PART # I

1.	(A)	**2.**	(D)	**3.**	(B)	**4.**	(B)	**5.**	(D)	**6.**	(A)	**7.**	(A)
8.	(D)	**9.**	(B)	**10.**	(B)	**11.**	(C)	**12.**	(C)	**13.**	(C)	**14.**	(C)
15.	(C)	**16.**	(B)	**17.**	(A)	**18.**	(A)	**19.**	(A)	**20.**	(B)	**21.**	(A)
22.	(C)	**23.**	(B)	**24.**	(A, B, C)	**25.**	(A, C, D)	**26.**	(A, C)				
27.	(A, B, C)	**28.**	(A, D)	**29.**	(C, D)								

PART # II

1. $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{4}$ **4.** 2 **5.** $5/3 \le \beta \le 7/2$ **6.** $3x - y - 11 = 0$

7. $(x - 4y + 3)(x - y) = 0$ or $x^2 - 5xy + 4y^2 + 3x - 3y = 0$

8. (1, – 2), yes (1/3, – 2/3) **9.** $\left(\frac{3}{2},0\right)\left(\frac{9}{4},0\right),\left(\frac{3}{2},\frac{3}{4}\right),\left(\frac{9}{4},\frac{3}{4}\right)$

10. $x + 4y = 4;\ 5x + 2y = 8$ **11.** $x - y = 0$ **12.** $x - y = 0$

14. $2x - y + 3 = 0,\ 2x + y - 7 = 0;\ x - 2y - 6 = 0$

EXERCISE # 3

PART # I

1. 18 **3.** (B) **4.** (C) **5.** (D) **6.** (B) **7.** (A) **8.** (C)

9. (B) **10.** (A) **11.** $y = 2x + 1$ or $y = -2x + 1$ **12.** (C) **13.** (C)

14. (B*) **15.** (D*) **16.** (D*) **17.** $(A \to s), (B \to p,u), (C \to r), (D \to p,q,s)$

PART # II

1.	(3)	**2.**	(2)	**3.**	(2)	**4.**	(1)	**5.**	(2)	**6.**	(4)	**7.**	(4)
8.	(1)	**9.**	(4)	**10.**	(3)	**11.**	(4)	**12.**	(4)	**13.**	(2)	**14.**	(2)
15.	(1)	**16.**	(2)	**17.**	(2)	**18.**	(3)	**19.**	(1)	**20.**	(3)	**21.**	(1)
22.	(1)	**23.**	(1)	**24..**	(3)	**25..**	(2)	**26.**	(3)	**27.**	(3)		

EXERCISE # 4

1. $x + y + 1 = 0$ **2.** $x - 4y + 3 = 0$ **3.** 60º or 120º **4.** $x + y = 7$ or $\frac{x}{6}+\frac{y}{8}=1$

5. (3, 1), (–7, 11) **7.** $y-\sqrt{3}x-2+\sqrt{3}=0$ **8.** $3x + 4y + 3 = 0$ **9.** $a=\frac{-8}{3}, b=4$

10. $8x - 5y + 60 = 0$ **11.** $\sqrt{3}x+y=8$ **12.** $x - 7y - 12 = 0$ **13.** $\sqrt{\frac{2}{3}}$

14. (1, 1) **15.** 15º or 75º **17.** $\left(0,\ 2+\frac{5\sqrt{3}}{2}\right)$