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TRIGONOMETRIC RATIOS
&
IDENTITY AND EQUATION

Contents

Syllabus

Trigonometric functions, their periodicity and graphs, addition and subtraction formulae, formulae involving multiple and submultiple angles, general solution of trigonometric equations.

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TRIGONOMETRIC RATIOS & IDETITIE AND EQUATION

KEY CONCEPTS

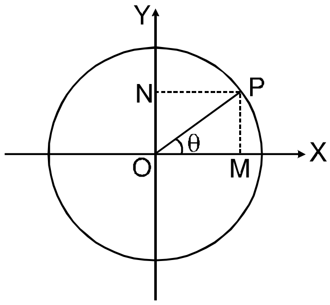
❖ Basic Trigonometric Identities:

(a) $\sin^2\theta + \cos^2\theta = 1$; $-1 \le \sin\theta \le 1$; $-1 \le \cos\theta \le 1 \;\; \forall \;\; \theta \in R$

(b) $\sec^2\theta - \tan^2\theta = 1$; $|\sec\theta| \ge 1 \;\; \forall \;\; \theta \in R - \left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$

(c) $\text{cosec}^2\theta - \cot^2\theta = 1$; $|\text{cosec}\,\theta| \ge 1 \;\; \forall \;\; \theta \in R - \{n\pi,\; n \in I\}$

❖ Circular Definition Of Trigonometric Functions:



$\sin\theta = \frac{PM}{OP}$ $\quad\cos\theta = \frac{OM}{OP}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos\theta \ne 0$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\sin\theta \ne 0$

$\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \ne 0$ $\quad\text{cosec}\,\theta = \frac{1}{\sin\theta}$, $\sin\theta \ne 0$

❖ Trigonometric Functions Of Allied Angles:

If θ is any angle, then $-\theta$, $90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$, $360 \pm \theta$ etc. are called **ALLIED ANGLES**.

(a) $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$

(b) $\sin(90° - \theta) = \cos\theta$; $\cos(90° - \theta) = \sin\theta$

(c) $\sin(90° + \theta) = \cos\theta$; $\cos(90° + \theta) = -\sin\theta$

(d) $\sin(180° - \theta) = \sin\theta$; $\cos(180° - \theta) = -\cos\theta$

(e) $\sin(180° + \theta) = -\sin\theta$; $\cos(180° + \theta) = -\cos\theta$

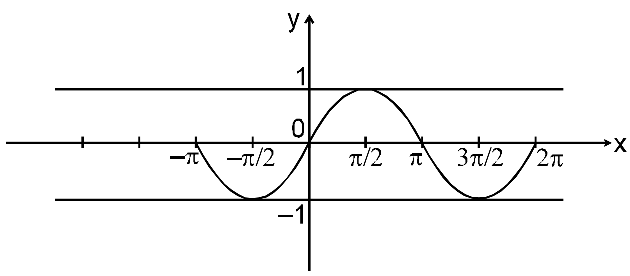
(f) $\sin(270° - \theta) = -\cos\theta$; $\cos(270° - \theta) = -\sin\theta$

(g) $\sin(270° + \theta) = -\cos\theta$; $\cos(270° + \theta) = \sin\theta$

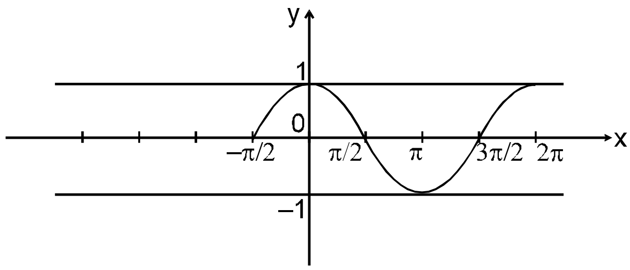
(h) $\tan(90° - \theta) = \cot\theta$; $\cot(90° - \theta) = \tan\theta$

❖ Graphs of Trigonometric functions:

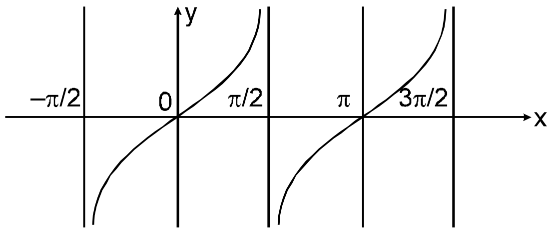
(a) $y = \sin x \quad x \in R;\; y \in [-1, 1]$



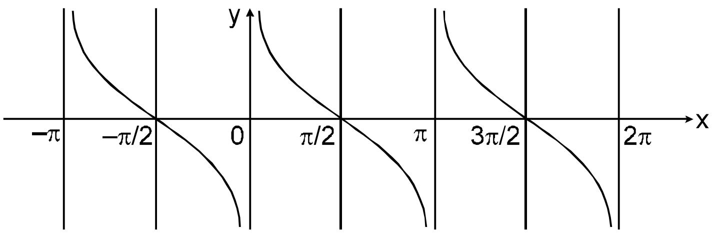
(b) $y = \cos x \quad x \in R; \; y \in [-1, 1]$



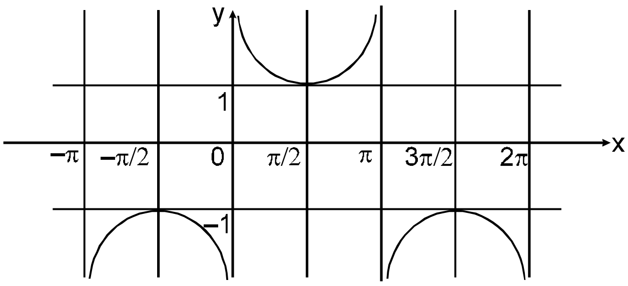
(c) $y = \tan x \quad x \in R - (2n + 1)\,\pi/2,\; n \in I\;;\; y \in R$



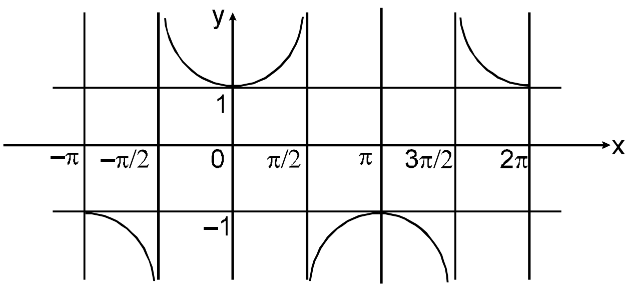
(d) $y = \cot x \quad x \in R - n\pi\,,\; n \in I;\; y \in R$



(e) $y = \operatorname{cosec} x \quad x \in R - n\pi\,,\; n \in I\;;\; y \in (-\infty, -1] \cup [1, \infty)$



(f) $y = \sec x \quad x \in R - (2n + 1)\,\pi/2,\; n \in I\;;\; y \in (-\infty, -1] \cup [1, \infty)$



❖ Trigonometric Functions of Sum or Difference of Two Angles:

(a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B).\sin(A-B)$

(d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B).\cos(A-B)$

(e) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(f) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(g) $\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$.

❖ Factorisation of the Sum or Difference of Two Sines or Cosines:

(a) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

❖ Transformation of Products into Sum or Difference of Sines & Cosines:

(a) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(b) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(c) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(d) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

❖ Multiple and Sub-multiple Angles :

(a) $\sin 2A = 2\sin A\cos A$; $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$; $2\cos^2\dfrac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\dfrac{\theta}{2} = 1 - \cos\theta$.

(c) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$; $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

(d) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$, $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(e) $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$

(g) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

❖ Important Trigonometric Ratios:

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$, where $n \in I$

(b) $\sin 15°$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$ or $\cos\frac{5\pi}{12}$;

$\cos 15°$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$ or $\sin\frac{5\pi}{12}$;

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75°$; $\tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15°$

(c) $\sin\frac{\pi}{10}$ or $\sin 18° = \frac{\sqrt{5}-1}{4}$ & $\cos 36°$ or $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

❖ Conditional Identities:

If $A + B + C = \pi$ then :

(i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

(ii) $\sin A + \sin B + \sin C = 4 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$

(iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

(iv) $\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(vi) $\tan\frac{A}{2} \tan\frac{B}{2} + \tan\frac{B}{2} \tan\frac{C}{2} + \tan\frac{C}{2} \tan\frac{A}{2} = 1$

(vii) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2} . \cot\frac{B}{2} . \cot\frac{C}{2}$

(viii) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

(ix) $A + B + C = \frac{\pi}{2}$ then $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

❖ Range of Trigonometric Expression:

$E = a \sin\theta + b \cos\theta$

$E = \sqrt{a^2+b^2} \sin(\theta + \alpha)$, where $\tan\alpha = \frac{b}{a}$

$= \sqrt{a^2+b^2} \cos(\theta - \beta)$, where $\tan\beta = \frac{a}{b}$

Hence for any real value of θ, $-\sqrt{a^2+b^2} \le E \le \sqrt{a^2+b^2}$

❖ Sine and Cosine Series:

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots + \sin\left(\alpha + \overline{n-1}\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left(\alpha + \frac{n-1}{2}\beta\right)$$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \cos\left(\alpha + \overline{n-1}\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha + \frac{n-1}{2}\beta\right)$$

TRIGONOMETRIC EQUATION

1. DEFINITION :

An equation containing trigonometric function of unknown angles are known as trigonometric equations.

2. PERIODIC FUNCTION :

A function f(x) is said to be periodic if there exists $T > 0$ such that $f(x + T) = f(x)$ for all x in the domain of definitions of f(x). If T is the smallest positive real numbers such that $f(x + T) = f(x)$, then it is called the period of f(x).

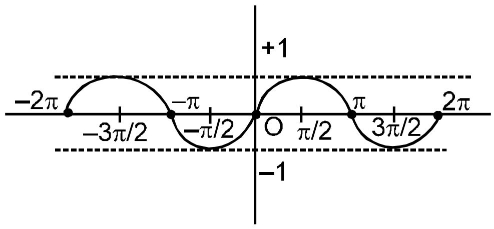
The period of sin x, cos x, sec x, cosec x is 2π and period of tan x and cot x is π.

3. GENERAL SOLUTION OF STANDARD TRIGONOMETRICAL EQUATIONS :

Since Trigonometrical functions are periodic functions, therefore, solutions of Trigonometrical equations can be generalised with the help of periodicity of Trigonometrical functions. The solution consisting of all possible solutions of a Trigonometrical equation is called its general solution.

3.1 General Solution of the equation $\sin\theta = 0$.

By Graphical approach,



The above graph of $\sin\theta$ clearly shows that $\sin\theta = 0$ at

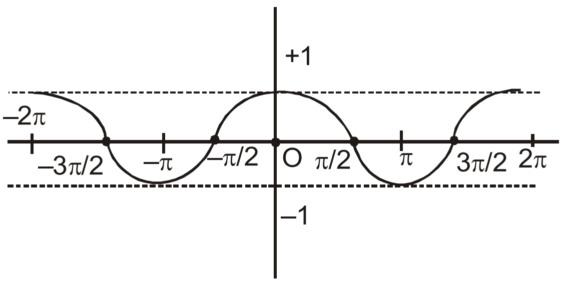
$\theta = 0, \pm\pi, \pm 2\pi, \pm 3\pi$ ....

It follows that when $\sin\theta = 0$ is

$\theta = n\pi \; : \; n \in I$ i.e. $n = 0, \pm 1, \pm 2$ ....

3.2 General solution of $\cos\theta = 0$.

By graphical approach,



The above graph of $\cos\theta$ clearly shows that $\cos\theta = 0$ at

$\theta = \pm \pi/2, \pm 3\pi/2, \pm 5\pi/2$, ...

It follows that when $\cos\theta = 0$

$\theta = (2n + 1)\pi/2$, $n \in I$.

i.e. $n = 0, \pm 1, \pm 2$ ....

3.3 General solution of tan θ = 0 .

Proof: If $\tan\theta = 0$

or $$\frac{\sin\theta}{\cos\theta} = 0$$

$\sin\theta = 0$,

it follows that general solution of $\tan\theta = 0$, it same as of $\sin\theta = 0$

so that, general solution of $\tan\theta = 0$ is

$$\theta = n\pi \quad ; \quad n \in I$$

Note : General solution of $\sec\theta = 0$ and $\text{cosec}\theta = 0$ does not exist because $\sec\theta$ and $\text{cosec}\theta$ can never be equal to 0.

3.4 General solution of the equation sin θ = sin α .

Proof : If $\sin\theta = \sin\alpha$

or, $\sin\theta - \sin\alpha = 0$

or, $2\sin\left(\frac{\theta-\alpha}{2}\right)\cos\left(\frac{\theta+\alpha}{2}\right) = 0$

or, $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$ or $\cos\left(\frac{\theta+\alpha}{2}\right) = 0$

or, $\frac{\theta-\alpha}{2} = m\pi$; $m \in I$ and

$\frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2}$; $m \in I$

$\theta = 2m\pi + \alpha$; $m \in I$ or

$\theta = (2m+1)\pi - \alpha$; $m \in I$

θ = (any even multiple of π) + α or

θ = (any odd multiple of π) – α

by combining these two results

$\theta = n\pi + (-1)^n\alpha$; $n \in I$

3.4.1 General solution of the equation sinθ = k, where $-1 \le k \le 1$.

Let α be the numerically least angle such that $k = \sin\alpha$ then, $\sin\theta = \sin\alpha$

$\therefore \quad \theta = n\pi + (-1)^n\alpha$, where $n \in I$

and $\alpha = \sin^{-1}k$

Note: The equation $\text{cosec}\,\theta = \text{cosec}\,\alpha$ is equivalent to $\sin\theta = \sin\alpha$ so these two equations having the same general solution.

3.5 General solution of the equation cos θ = cos α .

Proof : If $\cos\theta = \cos\alpha$

or, $\cos\theta - \cos\alpha = 0$

or, $-2\sin\left(\frac{\theta+\alpha}{2}\right).\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

or, $\sin\left(\frac{\theta+\alpha}{2}\right) = 0$ and $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\frac{\theta+\alpha}{2} = n\pi$; $n \in I$

and $\frac{\theta - \alpha}{2}$ = $n\pi$; $n \in I$

$\theta = 2n\pi - \alpha$; $n \in I$ and

$\theta = 2n\pi + \alpha$; $n \in I$

for the general solution of $\cos\theta = \cos\alpha$, combine these two result which gives.

$\theta = 2n\pi \pm \alpha$, $n \in I$

3.5.1 General solution of the equation $\cos\theta = k$, where $-1 \le k \le 1$.

Let α be the numerically least angle such that $k = \cos\alpha$, then $\cos\theta = \cos\alpha$

$\therefore \quad \theta = 2n\pi \pm \alpha$, where $n \in I$ and $\alpha = \cos^{-1}k$

Note : The equation $\sec\theta = \sec\alpha$ is equivalent to $\cos\theta = \cos\alpha$, so the general solution of these two equations are same.

3.6 General solution of the equation $\tan\theta = \tan\alpha$.

Proof : If $\tan\theta = \tan\alpha$

or, $\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$

or, $\sin\theta.\cos\alpha - \cos\theta \sin\alpha = 0$

or, $\sin(\theta - \alpha) = 0$

or, $\theta - \alpha = n\pi$; $n \in I$

$\theta = n\pi + \alpha$; $n \in I$

3.6.1 General solution of the equation $\tan\theta = k$.

Let α be the numerically least angle such that $k = \tan\alpha$, then

$\tan\theta = \tan\alpha$

$\therefore \quad \theta = n\pi + \alpha$, where $n \in I$ and $\alpha = \tan^{-1}k$

Note : The equation $\cot\theta = \cot\alpha$ is equivalent to $\tan\theta = \tan\alpha$ so these two equations having the same general solution.

4. GENERAL SOLUTION OF SQUARE OF THE TRIGONOMETRICAL EQUATIONS :

4.1 General solution of $\sin^2\theta = \sin^2\alpha$.

Proof : If $\sin^2\theta = \sin^2\alpha$

or, $2\sin^2\theta = 2\sin^2\alpha$

(Both the sides multiple by 2)

or, $1 - \cos2\theta = 1 - \cos2\alpha$

or, $\cos2\theta = \cos2\alpha$

$2\theta = 2n\pi \pm 2\alpha$; $n \in I$

$\theta = n\pi \pm \alpha$; $n \in I$

4.2 General solution of $\cos^2\theta = \cos^2\alpha$.

Proof : If $\cos^2\theta = \cos^2\alpha$

or, $2\cos^2\theta = 2\cos^2\theta$

(multiply both the side by 2)

or, $1 + \cos2\theta = 1 + \cos2\alpha$

or, $\cos2\theta = \cos2\alpha$

or, $2\theta = 2n\pi \pm 2\alpha$

$\theta = n\pi \pm \alpha$; $n \in I$

4.3 General solution of $\tan^2\theta = \tan^2\alpha$.

Proof: If $\tan^2\theta = \tan^2\alpha$

or, $\dfrac{\tan^2\theta}{1} = \dfrac{\tan^2\alpha}{1}$

(using compo. and divid. rule)

$\dfrac{\tan^2\theta+1}{\tan^2\theta-1} = \dfrac{\tan^2\alpha+1}{\tan^2\alpha-1}$ or, $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}$

or, $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$ or, $\cos2\theta = \cos2\alpha$

$\Rightarrow \theta = n\pi \pm \alpha \quad ; n \in I$

5. GENERAL SOLUTION OF TRIGONOMETRICAL EQUATION : $a\cos\theta + b\sin\theta = c$

Consider a trigonometrical equation $a\cos\theta + b\sin\theta = c$,

where $a, b, c \in R$ and $|c| \le \sqrt{a^2+b^2}$

To solve this type of equation, first we reduce them in the form $\cos\theta = \cos\alpha$ or $\sin\theta = \sin\alpha$.

Algorithm to solve equation of the form $a\cos\theta + b\sin\theta = c$.

Step I Obtain the equation $a\cos\theta + b\sin\theta = c$

Step II Put $a = r\cos\alpha$ and $b = r\sin\alpha$,

where $r = \sqrt{a^2+b^2}$ and $\tan\alpha = \dfrac{b}{a}$ i.e. $\alpha = \tan^{-1}\left(\dfrac{b}{a}\right)$

Step III Using the substitution in step - II, the equation reduces $r\cos(\theta - \alpha) = c$

$\Rightarrow \cos(\theta - \alpha) = \dfrac{c}{r}$ $\Rightarrow \cos(\theta - \alpha) = \cos\beta$ (say)

Step IV Solve the equation obtained in step III by using the formula.

General solution of Trigonometrical equation acosq + bsinq = c.

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

** Marked Questions are having more than one correct option.*

Section (A) : Basic Formula and Angle transformation

A-1. The values of the expression $(\sin x + \text{cosec}x)^2 + (\cos x + \sec x)^2 - (\tan x + \cot x)^2$ wherever defined is equal to

(A) 0 (B) 5 (C) 7 (D) 9

A-2. The value of tan 1º tan 2º tan 3º ... tan 89º is

(A) 1 (B) 0 (C) ∞ (D) $\frac{1}{2}$

A-3. The value of $\sin(\pi + \theta)\sin(\pi - \theta)\text{cosec}^2\theta$ is equal to

(A) –1 (B) 0 (C) $\sin\theta$ (D) none of these

A-4. $\dfrac{\tan\left(x - \frac{\pi}{2}\right).\cos\left(\frac{3\pi}{2} + x\right) - \sin^3\left(\frac{7\pi}{2} - x\right)}{\cos\left(x - \frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2} + x\right)}$ when simplified reduces to:

(A) sin x cos x (B) – $\sin^2x$ (C) – sin x cos x (D) $\sin^2x$

A-5. The expression $3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi + \alpha)\right]$ is equal to

(A) 0 (B) 1 (C) 3 (D) $\sin 4\alpha + \sin 6\alpha$

A-6. $\cos(540° - \theta) - \sin(630° - \theta)$ is equal to

(A) 0 (B) $2\cos\theta$ (C) $2\sin\theta$ (D) $\sin\theta - \cos\theta$

A-7. If cos1° cos2° cos3° ..... cos179° = x + 1, then x is equal to

(A) –1 (B) 0 (C) 1 (D) none of these

A-8. Given $\tan\alpha = 2\tan\beta$, then the value of $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

A-9. The value of the expression

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right) \text{ is}$$

(A) $\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $\frac{1}{4}$ (D) 0

A-10. If (sec A + tan A) (sec B + tan B) (sec C + tan C) = (sec A – tan A) (sec B – tan B) (sec C – tan C) then each side is equal to
(A) 1 (B) –1 (C) 0 (D) none

A-11. In a triangle ABC, angle $A = 36^\circ$, AB = AC = 1 & BC = x. If $x = \frac{p+\sqrt{q}}{2}$ then the ordered pair (p , q) is :

Section (B) :Addition of Angles and Multiple Angles Formula

B-1. $\sin\theta\cos^3\theta - \cos\theta\sin^3\theta$ when simplified reduces to

(A) $\frac{1}{4}\sin 4\theta$ (B) $4\sin 4\theta$ (C) $\frac{1}{4}\cos 4\theta$ (D) $4\cos 4\theta$

B-2. If $\theta = \frac{\pi}{19}$, then $\frac{\sin 23\theta - \sin 3\theta}{\sin 16\theta + \sin 4\theta}$ is equal to

(A) -1 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

B-3. Exact value of $\frac{\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ + \cos 157^\circ \cos 97^\circ}$ is-

(A) –1 (B) 1 (C) 0 (D) none of these

B-4. The value of $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ}$ is

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) none

B-5. If A and B are complimentry angles, then :

(A) $\left(1+\tan\frac{A}{2}\right)\left(1+\tan\frac{B}{2}\right) = 2$ (B) $\left(1+\cot\frac{A}{2}\right)\left(1+\cot\frac{B}{2}\right) = 2$

(C) $\left(1+\sec\frac{A}{2}\right)\left(1+\operatorname{cosec}\frac{B}{2}\right) = 2$ (D) $\left(1-\tan\frac{A}{2}\right)\left(1-\tan\frac{B}{2}\right) = 2$

B-6. θ_1 & θ_2 are acute angles such that $\sin\theta_1 = \frac{2}{\sqrt{5}}$ and $\cos\theta_2 = \frac{1}{\sqrt{10}}$, then-

(A) $\tan(\theta_1 + \theta_2) = 1$ (B) $\tan(\theta_1 + \theta_2) = -1$

(C) $\tan(\theta_1 + \theta_2) = 5$ (D) $\cot(\theta_1 + \theta_2) = \frac{1}{6}$

B-7. $4\cos\theta\cos\left(\frac{2\pi}{3}+\theta\right).\cos\left(\frac{2\pi}{3}-\theta\right)$ equals,

(A) $\frac{1}{4}\cos 3\theta$ (B) $\frac{1}{4}\sin 3\theta$ (C) $\cos 3\theta$ (D) $\sin 3\theta$

B-8. If A + B = 225°, then the value of $\left(\frac{\cot A}{1+\cot A}\right).\left(\frac{\cot B}{1+\cot B}\right)$ is

(A) 2 (B) $\frac{1}{2}$ (C) 3 (D) $\frac{1}{3}$

B-9. If 3 $\sin\alpha$ =5 $\sin\beta$, then $\frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

B-10. In a triangle ABC if tan A < 0 then:
(A) tan B. tan C > 1 (B) tan B. tan C < 1
(C) tan B. tan C = 1 (D) none

B-11. If tan 25° = x, then $\frac{\tan 155^\circ - \tan 115^\circ}{1+\tan 155^\circ \tan 115^\circ}$ is equal to

(A) $\frac{1-x^2}{2x}$ (B) $\frac{1+x^2}{2x}$ (C) $\frac{1+x^2}{1-x^2}$ (D) $\frac{1-x^2}{1+x^2}$

B-12. If $\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$, then the value of $\cot\alpha\tan\beta$ is

(A) –1 (B) 0 (C) 1 (D) none of these

B-13. The value of tan 203° + tan 22° + tan 203° tan 22° is
(A) – 1 (B) 0 (C) 1 (D) 2

B-14. If A lies in the third quadrant and 3 tan A – 4 = 0, then 5 sin 2A + 3sinA + 4 cosA is equal to

(A) 0 (B) $-\frac{24}{5}$ (C) $\frac{24}{5}$ (D) $\frac{48}{5}$

B-15. If $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$, then $\cos 3\theta$ in terms of 'a' is

(A) $\frac{1}{4}\left(a^3+\frac{1}{a^3}\right)$ (B) $\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$ (C) $4\left(a^3+\frac{1}{a^3}\right)$ (D) none

B-16. If $\tan^2\theta = 2\tan^2\phi + 1$, then the value of $\cos 2\theta + \sin^2\phi$ is
(A) 1 (B) 2 (C) – 1 (D) Independent of ϕ

B-17. The value of $\frac{(\cos 11^\circ + \sin 11^\circ)}{(\cos 11^\circ - \sin 11^\circ)}$ is
(A) –tan 304° (B) tan 56° (C) cot 214° (D) cot 34°

Section (C) : Half Angles Formula and Range of Trigonometry Function

C-1. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is

(A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2

C-2. If cos A = 3/4, then the value of 16 $\cos^2$ (A/2) – 32 sin (A/2) sin (5A/2) is

(A) – 4 (B) – 3 (C) 3 (D) 4

C-3. If $\frac{\sin 3\theta}{\sin\theta} = \frac{11}{25}$ then $\tan\frac{\theta}{2}$ can have the value equal to

(A) 2 (B) $\frac{1}{2}$ (C) –2 (D) $-\frac{1}{2}$

C-4. If a cos θ + b sin θ = 3 & a sin θ – b cos θ = 4 then $a^2 + b^2$ has the value =

(A) 25 (B) 14 (C) 7 (D) none

C-5. Given that $5\cos^2\alpha - 2\sin\alpha - 2 = 0\left(\frac{5\pi}{4} < \alpha < \frac{7\pi}{4}\right)$, the value of $\cot\frac{\alpha}{2}$ is-

(A) $1/\sqrt{2}$ (B) $\sqrt{2}$ (C) –1 (D) $\sqrt{3}/2$

C-6. If sin t + cos t = $\frac{1}{5}$ then $\tan\frac{t}{2}$ is equal to:

(A) –1 (B) $-\frac{1}{3}$ (C) 2 (D) $-\frac{1}{6}$

C-7. If $f(\theta) = \sin^4\theta + \cos^2\theta$, then range of $f(\theta)$ is

(A) $\left[\frac{1}{2}, 1\right]$ (B) $\left[\frac{1}{2}, \frac{3}{4}\right]$ (C) $\left[\frac{3}{4}, 1\right]$ (D) None of these

C-8. The maximum value of 2 + $\sqrt{3}\sin\theta - \cos\theta$ is

(A) 4 (B) 0 (C) 2 (D) 1

Section (D) : Miscellaneous

D-1. $\frac{\cos 20^\circ + 8\sin 70^\circ \sin 50^\circ \sin 10^\circ}{\sin^2 80^\circ}$ is equal to:

(A) 1 (B) 2 (C) 3/4 (D) none of these

D-2. If A = tan 6° tan 42° and B = cot 66° cot 78°, then

(A) A = 2B (B) A = $\frac{1}{3}$ B (C) A = B (D) 3A = 2B

D-3. If $\alpha + \beta + \gamma = 2\pi$, then

(A) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(B) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

(C) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 0$

D-4. If x + y = z, then $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x \cos y \cos z$ is equal to
(A) $\cos^2 z$ (B) $\sin^2 z$ (C) $\cos(x + y - z)$ (D) 1

D-5*. In a triangle tan A + tan B + tan C = 6 and tan A tan B = 2, then the values of tan A, tan B and tan C are
(A) 1, 2, 3 (B) 2, 1, 3 (C) 1, 2, 0 (D) none

D-6. If $\tan\alpha + \cot\alpha = a$ then the value of $\tan^4\alpha + \cot^4\alpha =$
(A) a^4+4a^2+2 (B) a^4-4a^2+2 (C) a^4-4a^2-2 (D) none

D-7. If $\pi < 2\theta < \frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ is equal to

(A) $-2\cos\theta$ (B) $-2\sin\theta$ (C) $2\cos\theta$ (D) $2\sin\theta$

D-8. If $\sin x = \cos^2 x$ then $\cos^2 x(1 + \cos^2 x)$ equals to

(A) 0 (B) 1 (C) 2 (D) none of these

Section (E) :Trigonometric Equations

E-1. The general solution of the equation, $2\cos 2x = 3.2\cos^2 x - 4$ is
(A) $x = 2n\pi$ (B) $x = n\pi$ (C) $x = n\pi/4$ (D) $x = n\pi/2$

E-2. Angles A & B are obtuse angles such that, tanA + tanB + tanA tanB = 1. If A – B = 41° then t
(A) A = 133°, B = 92° (B) A = 143°, B = 102°
(C) A = 173°, B = 132° (D) A = 163°, B = 122°

E-3. All solutions of the equation, $2\sin\theta + \tan\theta = 0$ are obtained by taking all integral values of m and n in:

(A) $2n\pi + \frac{2\pi}{3}$ (B) $n\pi$ & $2m\pi \pm \frac{2\pi}{3}$ (C) $n\pi$ & $m\pi \pm \frac{\pi}{3}$ (D) $n\pi$ & $2m\pi \pm \frac{\pi}{3}$

E-4. The general solution of the equation: $\tan^2\alpha + 2\sqrt{3}\tan\alpha = 1$ is given by:

(A) $\alpha = \frac{n\pi}{2}$ (B) $\alpha = (2n+1)\frac{\pi}{2}$ (C) $\alpha = (6n+1)\frac{\pi}{12}$ (D) $\alpha = n\frac{\pi}{12}$

E-5. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:
(A) $x = n\pi + (-1)^{n+1}(\pi/6)$ (B) $x = n\pi + (-1)^n(\pi/6)$
(C) $x = n\pi + (-1)^n(\pi/3)$ (D) $x = n\pi - (-1)^n(\pi/6)$ where $n \in I$

E-6. $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$ if:

(A) $\theta \in \left(0, \frac{\pi}{2}\right)$ (B) $\theta \in \left(\frac{\pi}{2}, \pi\right)$ (C) $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ (D) $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

E-7. The general solution of the equation, $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ is

(A) $\frac{n\pi}{4} + \frac{\pi}{12}$ $n \in I$ (B) $\frac{n\pi}{3} + \frac{\pi}{6}$ $n \in I$ (C) $\frac{n\pi}{3} + \frac{\pi}{12}$ $n \in I$ (D) none

E-8. $\sin 3\theta = 4\sin\theta.\sin 2\theta.\sin 4\theta$ in $0 \le \theta \le \pi$ has:
(A) 2 real solutions (B) 4 real solutions
(C) 6 real solutions (D) 8 real solutions.

E-9. The general solution of the equation, $2\cot\frac{\theta}{2} = (1 + \cot\theta)^2$ is

(A) $n\pi + (-1)^n\frac{\pi}{4}$ (B) $n\pi + (-1)^n\frac{\pi}{3}$ (C) $n\pi + (-1)^n\frac{\pi}{6}$ (D) none

E-10. The general solution of sin x + sin 5x = sin 2x + sin 4x is:

(A) $2n\pi$; $n \in I$ (B) $n\pi$; $n \in I$ (C) $n\pi/3$; $n \in I$ (D) $2n\pi/3$; $n \in I$

E-11. General solution of the equation, $\cot 3\theta - \cot\theta = 0$ is

(A) $\theta = (2n-1)\frac{\pi}{2}$ (B) $\theta = (2n-1)\frac{\pi}{4}$ (C) $\theta = (2n-1)\frac{\pi}{3}$ (D) none

E-12. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1+\tan 3x \tan 2x} = 1$ is

(A) ϕ (B) $\{\pi/4\}$

(C) $\{n\pi + \pi/4 \mid n = 1,2,3....\}$ (D) $\{2n\pi + \pi/4 \mid n = 1,2,3......\}$

E-13. The set of angles between 0 & 2π satisfying the equation $4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$ is

(A) $\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{23\pi}{12}\right\}$ (B) $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}\right\}$

(C) $\left\{\frac{5\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}\right\}$ (D) $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$

E-14. The equation $3\sin^2 x + 10\cos x - 6 = 0$ is satisfied if $(n \in I)$

(A) $x = n\pi + \cos^{-1}(1/3)$ (B) $x = n\pi - \cos^{-1}(1/3)$

(C) $x = 2n\pi \pm \cos^{-1}(1/3)$ (D) $x = \frac{n\pi}{2} - \cos^{-1}(1/3)$

E-15. Number of solutions of the equation tanx + secx = 2 cosx lying in the interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3

E-16. The number of solutions of the equation, $|\cot x| = \cot x + \frac{1}{\sin x}$ $(0 \le x \le 2\pi)$ is :

(A) 0 (B) 1 (C) 2 (D) 3

E-17. The general solution of, sin x - 3 sin 2x + sin 3x = cos x - 3 cos 2x + cos 3x is

(A) $n\pi + \frac{\pi}{8}$ (B) $\frac{n\pi}{2} + \frac{\pi}{8}$

(C) $(-1)^n\left(\frac{n\pi}{2}\right) + \frac{\pi}{8}$ (D) $2n\pi + \cos^{-1}\left(\frac{3}{2}\right)$

E-18. The principal solution set of the equation $2\cos x = \sqrt{2 + 2\sin 2x}$ is

(A) $\left\{\frac{\pi}{8}, \frac{13\pi}{8}\right\}$ (B) $\left\{\frac{\pi}{4}, \frac{13\pi}{8}\right\}$ (C) $\left\{\frac{\pi}{4}, \frac{13\pi}{10}\right\}$ (D) $\left\{\frac{\pi}{8}, \frac{13\pi}{10}\right\}$

E-19. The number of solutions of the equation $|\sin x| = |\cos 3x|$ in $[-2\pi, 2\pi]$ is :
(A) 32 (B) 28 (C) 24 (D) 30

E-20. If $2\tan^2x - 5\sec x - 1 = 0$ has 7 different roots in $\left[0, \frac{n\pi}{2}\right]$, $n \in N$, then greatest value of n is
(A) 8 (B) 10 (C) 13 (D) 15

E-21. The solution of $|\cos x| = \cos x - 2\sin x$ is
(A) $x = n\pi, n \in I$
(B) $x = n\pi + \frac{\pi}{4}, n \in I$
(C) $x = n\pi + (-1)^n \frac{\pi}{4}, n \in I$
(D) $x = (2n + 1)\pi + \frac{\pi}{4}, n \in I$

E-22. The number of solutions of $\sin\theta + 2\sin 2\theta + 3\sin 3\theta + 4\sin 4\theta = 10$ in $(0, \pi)$ is
(A) 1 (B) 2 (C) 4 (D) 0

E-23. The solution set of the equation $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is
(A) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (C) $\left\{\frac{3\pi}{4}, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (D) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

E-24. Total number of solutions of equation $\sin x . \tan 4x = \cos x$ belonging to $(0, \pi)$ are :
(A) 4 (B) 7 (C) 8 (D) 5

E-25. If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation $\sin 7x + \sin 4x + \sin x = 0$ is:
(A) 3 (B) 5 (C) 6 (D) None

E-26. If $2\cos^2(\pi + x) + 3\sin(\pi + x)$ vanishes then the values of x lying in the interval from 0 to 2π are
(A) $x = \pi/6$ or $5\pi/6$ (B) $x = \pi/3$ or $5\pi/3$ (C) $x = \pi/4$ or $5\pi/4$ (D) $x = \pi/2$ or $5\pi/2$

E-27. $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$ if
(A) $\theta = n\pi + \frac{\pi}{3}, n \in I$ (B) $\theta = 2n\pi \pm \frac{\pi}{3}, n \in I$ (C) $\theta = 2n\pi \pm \frac{\pi}{6}, n \in I$ (D) $\theta = n\pi + \frac{\pi}{6}, n \in I$

E-28. If $\sin x + \cos x = \sqrt{y + \frac{1}{y}}, x \in [0, \pi]$, then
(A) $x = \pi/4$ (B) $x = \frac{\pi}{2}$ (C) $x = \frac{\pi}{6}$ (D) $x = 3\pi/4$

E-29. If $x^2 - 4x + 5 - \sin y = 0$, $y \in [0, 2\pi)$, then
(A) $x = 1, y = 0$ (B) $x = 1, y = \pi/2$ (C) $x = 2, y = 0$ (D) $x = 2, y = \pi/2$

E-30. The number of integral values of a for which the equation $\cos 2x + a\sin x = 2a - 7$ possesses a solution is
(A) 2 (B) 3 (C) 4 (D) 5

PART - II : MISCELLANEOUS OBJECTIVE QUESTIONS

Comprehension # 1

If θ increases from 0 to $\frac{\pi}{2}$ then value of $\sin\theta$, $\tan\theta$ and $\sec\theta$ increases while $\cos\theta$, $\cot\theta$ and $\text{cosec}\theta$ decreases.

The following pairs $(\sin\theta, \cos\theta)$, $(\tan\theta, \sec\theta)$ and $(\sec\theta, \text{cosec}\theta)$ have the same value at $\theta = \frac{\pi}{4}$

1. If A = 1125° then cosA – sinA will be

(A) positive (B) negative (C) zero (D) $\sqrt{3}$

2. If A = 295° then secA + cosecA will be

(A) positive (B) negative (C) zero (D) not defined

3. If $A = \frac{23\pi}{8}$ then cotA – tanA will be-

(A) positive (B) negative (C) zero (D) two

Comprehension # 2

Consider the equation $\frac{\sin^4 x}{a} + \frac{\cos^4 x}{b} = \frac{1}{a+b}, 0 \le x \le \frac{\pi}{2}$ and answer the following questions

4. From the given equation

(A) $\frac{\sin^4 x}{a} = \frac{\cos^4 x}{b}$ (B) $\frac{\sin x}{a} = \frac{\cos x}{b}$ (C) $\frac{\sin^4 x}{a^2} = \frac{\cos^4 x}{b^2}$ (D) none of these

5. In terms of a and b the value of $\sin^2 x$ must be

(A) $\sqrt{ab}$ (B) $\frac{a}{a+b}$ (C) $\frac{a^2 - b^2}{a^2 + b^2}$ (D) none of these

6. The value of $\frac{\sin^8 x}{a^3} + \frac{\cos^8 x}{b^3}$ must be

(A) $\frac{1}{(a+b)^2}$ (B) $\frac{1}{(a+b)^3}$ (C) $\frac{1}{(a+b)^4}$ (D) none of these

Comprehension # 3

Let a, b, c, d $\in$ R. Then the cubic equation of the type $ax^3 + bx^2 + cx + d = 0$ has either one root real or all three roots are real. But in case of trigonometric equations of the type $a \sin^3 x + b \sin^2 x + c \sin x + d = 0$ can possess several solutions depending upon the domain of x.

To solve an equation of the type $a \cos\theta + b \sin\theta = c$. The equation can be written as $\cos(\theta - \alpha) = c/\sqrt{(a^2 + b^2)}$.

The solution is $\theta = 2n\pi + \alpha \pm \beta$, where $\tan\alpha = b/a$, $\cos\beta = c/\sqrt{(a^2+b^2)}$.

7. On the domain $[-\pi, \pi]$ the equation $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$ possess

(A) only one real root (B) three real roots
(C) four real roots (D) six real roots

8. In the interval $[-\pi/4, \pi/2]$, the equation, $\cos 4x + \frac{10\tan x}{1+\tan^2 x} = 3$ has

(A) no solution (B) one solution (C) two solutions (D) three solutions

9. $|\tan x| = \tan x + \frac{1}{\cos x}$ $(0 \le x \le 2\pi)$ has

(A) no solution (B) one solution (C) two solutions (D) three solutions

Comprehension # 4

To solve a trigonometric inequation of the type sin $x \geq a$ where $|a| \leq 1$, we take a hill of length 2π in the sine curve and write the solution within that hill. For the general solution, we add $2n\pi$. For instance, to solve $\sin x \geq -\frac{1}{2}$, we take the hill $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ over which solution is $-\frac{\pi}{6} < x < \frac{7\pi}{6}$. The general solution is $2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{7\pi}{6}$, n is any integer. Again to solve an inequation of the type $\sin x \leq a$, where $|a| \leq 1$, we take a hollow of length 2π in the sine curve. (since on a hill, $\sin x \leq a$ is satisfied over two intervals). Similarly $\cos x \geq a$ or $\cos x \leq a$, $|a| \leq 1$ are solved.

10. Solution to the inequation $\sin^6 x + \cos^6 x < \frac{7}{16}$ must be

(A) $n\pi + \frac{\pi}{3} < x < n\pi + \frac{\pi}{2}$ (B) $2n\pi + \frac{\pi}{3} < x < 2n\pi + \frac{\pi}{2}$

(C) $\frac{n\pi}{2} + \frac{\pi}{6} < x < \frac{n\pi}{2} + \frac{\pi}{3}$ (D) none of these

11. Solution to inequality $\cos 2x + 5\cos x + 3 \geq 0$ over $[-\pi, \pi]$ is

(A) $[-\pi, \pi]$ (B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$ (C) $[0, \pi]$ (D) $\left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right]$

12. Over $[-\pi, \pi]$, the solution of $2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x \geq 0$ is

(A) $[-\pi, \pi]$ (B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$

(C) $[0, \pi]$ (D) $\left[-\pi, \frac{-7\pi}{12}\right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

Match the Column :

13. **Match the following**

Column-I	Column-II
(A) Given $\tan\alpha = 2\tan\beta$, then the value of $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ is equal to	(P) 1
(B) If $a \leq \sin\left(x + \frac{\pi}{6}\right) + 3\cos\left(x - \frac{\pi}{3}\right) + 2 \leq b$ then the value of a + b is equal to	(Q) 3
(C) $\frac{\sqrt{3}}{\cos 290^\circ} + \frac{1}{\sin 250^\circ}$ is equal to	(R) 4
(D) $\cos 20^\circ + 2\sin^2 55^\circ - \sqrt{2}\sin 65^\circ$ is equal to	(S) the value of $(a^2 + 4)(b^2 - 1)^2$ where $\tan\theta - \cot\theta = a$ and $\cos\theta - \sin\theta = b$

Match the Column :

14. **Match the following**

Let $E=\sqrt{1+\sin A}+\sqrt{1-\sin A}$, match the following values of E :

Column-I	Column-II
(A) $0 \le A \le 180°$	(p) $2\sin\frac{A}{2}$
(B) $180° \le A \le 360°$	(q) $2\cos\frac{A}{2}$
(C) $360° \le A \le 450°$	(r) $-2\sin\frac{A}{2}$
(D) $450° \le A \le 720°$	(s) $-2\cos\frac{A}{2}$

15. **Match the following**

If $\cos A = \mu$ $(0 < \mu < 1)$ where A lies between 1350° and 1440° then match the following

Column-I	Column-II
(A) $\sin A$	(p) $-\sqrt{\frac{1-\mu}{1+\mu}}$
(B) $\sin\frac{A}{2}$	(q) $-\sqrt{1-\mu}$
(C) $\cos\frac{A}{2}$	(r) $-\sqrt{\frac{1-\mu}{2}}$
(D) $\tan\frac{A}{2}$	(s) $+\sqrt{\frac{1+\mu}{2}}$

16. If α and β are the roots of the equation, $a\cos\theta + b\sin\theta = c$ then match the entries of **column-I** with the entries of **column-II**.

Column-I	Column-II
(A) $\sin\alpha + \sin\beta$	(p) $\frac{2b}{a+c}$
(B) $\sin\alpha \,.\, \sin\beta$	(q) $\frac{c-a}{c+a}$
(C) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}$	(r) $\frac{2bc}{a^2+b^2}$
(D) $\tan\frac{\alpha}{2} \,.\, \tan\frac{\beta}{2} =$	(s) $\frac{c^2-a^2}{a^2+b^2}$

17. Solve the equation for 'x' given in **Column-I** and match with the entries of **Column-II**

	Column-I		**Column-II**
(A)	$\cos 3x \,.\, \cos^3 x + \sin 3x \,.\, \sin^3 x = 0$	(p)	$n\pi \pm \frac{\pi}{3}$
(B)	$\sin 3\alpha = 4 \sin \alpha \sin(x + \alpha) \sin(x - \alpha)$ where α is a constant $\neq n\pi$.	(q)	$n\pi + \frac{\pi}{4}$, $n \in I$
(C)	$\lvert 2 \tan x - 1 \rvert + \lvert 2 \cot x - 1 \rvert = 2$.	(r)	$\frac{n\pi}{4} + \frac{\pi}{8}$, $n \in I$
(D)	$\sin^{10}x + \cos^{10}x = \frac{29}{16} \cos^4 2x$.	(s)	$\frac{n\pi}{2} \pm \frac{\pi}{4}$

Assersion-Reason Type

(A) Statement -1 is true, Statement - 2 is true ; Statement - 2 is correct explanation for Statement - 1

(B) Statement -1 is true, Statement - 2 is true ; Statement - 2 is **NOT** correct explanation for Statement - 1

(C) Statement -1 is true, Statement - 2 is false.

(D) Statement -1 is false, Statement - 2 is true.

18. Statement-1 If A, B, C are the angles of a triangle such that angle A is obtuse, then tanB tan C > 1

Statement-2: In any triangle $\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$

19. Statement-1 The number sin18° and –sin54° are the roots of same quadratic equation with integer co-efficients.

Statement-2: If x = 18°, then 5x = 90°, if y = –54°, then 5y = –270°

20. Statement-1 The number of roots of the equation $\sin \pi x = x^2 - x + \frac{5}{4}$ is 2.

Statement-2: In $[0, 2\pi]$, $\sin x = \frac{1}{2}$ has exactly two solutions.

21. Statement-1: In $(0, \pi)$, the number of solutions of the equation $\tan\theta + \tan 2\theta + \tan 3\theta = \tan \theta.\tan 2\theta.\tan 3\theta$ is 2.

Statement-2: Each solution of $\tan 6\theta = 0$ is a solution of $\tan\theta + \tan 2\theta + \tan 3\theta = \tan \theta.\tan 2\theta.\tan 3\theta$.

22. Statement-1 If $0 \le x \le 3\pi$, $0 \le y \le 3\pi$ and cos x . sin y = 1, then the possible number of values of the ordered pair (x, y) is 3.

Statement-2: $-1 \le \sin x \le 1$, $-1 \le \cos x \le 10$.

23. Statement-1 The number of integral values of n so that sinx(sinx + cosx) = n has at least one solution are 2.

Statement-2: $-\sqrt{a^2 + b^2} \le a \sin x + b \cos x \le \sqrt{a^2 + b^2}$

EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

1. The value of $\cos\frac{\pi}{10}\ \cos\frac{2\pi}{10}\ \cos\frac{4\pi}{10}\ \cos\frac{8\pi}{10}\ \cos\frac{16\pi}{10}$ is :

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$ (C) $\frac{\cos(\pi/10)}{16}$ (D) $-\frac{\sqrt{10+2\sqrt{5}}}{64}$

2. If $A = 340^\circ$ then $2\sin\frac{A}{2}$ is identical to

(A) $\sqrt{1+\sin A} + \sqrt{1-\sin A}$ (B) $-\sqrt{1+\sin A} - \sqrt{1-\sin A}$

(C) $\sqrt{1+\sin A} - \sqrt{1-\sin A}$ (D) $-\sqrt{1+\sin A} + \sqrt{1-\sin A}$

3. A man running on a circular track at the rate of 11 Km/hr traverses an arc which subtends an angle of 56° at the centre in 40 seconds. The diameter of the circle is– (Assume π is approximately equal to $\frac{22}{7}$)

(A) 125 meters (B) 250 meters (C) 375 meters (D) 500 meters

4. $\tan A = \frac{-1}{2}$, $\tan B = \frac{-1}{3}$, if A and B are positive then A + B is–

(A) $\frac{-\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{4}$ (D) none

5. $\alpha+\beta+\gamma=\pi$, $S_1 = \sin(\alpha+\beta)\sin\gamma + \sin(\beta+\gamma)\sin\alpha + \sin(\gamma+\alpha)\sin\beta$ and $C_1 = \cos(\alpha+\beta)\cos\gamma + \cos(\beta+\gamma)\cos\alpha + \cos(\gamma+\alpha)\cos\beta$ · then $S_1 - C_1$ equals :

(A) -1 (B) 0 (C) 3 (D) 1

6. $\cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{7\pi}{15}\cdot$ is equal to

(A) $-\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $-\frac{1}{16}$ (D) $\frac{1}{8}$

7. The value of $\tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ$ is equal to –

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $2/\sqrt{3}$

8. $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$ is equal to-

(A) 0 (B) 1 (C) 2 (D) 3

9. The expression $\frac{\sin(\alpha+\theta)-\sin(\alpha-\theta)}{\cos(\beta-\theta)-\cos(\beta+\theta)}$ is

(A) independent of α (B) independent of β

(C) independent of θ (D) independent of α and β

10. $8\{\sin^2 72° - \sin^2 60°\} + 1$ is-

(A) $2\sqrt{5}$ (B) $\frac{\sqrt{5}}{2}$ (C) 5 (D) $\sqrt{5}$

11. $8\sin 12° \sin 48° \sin 54°$ is equal to

(A) $\frac{1}{8}$ (B) 8 (C) 1 (D) $\frac{1}{16}$

12. If A + B + C = π & $\sin\left(A + \frac{C}{2}\right) = k \sin\frac{C}{2}$, then $\tan\frac{A}{2}\tan\frac{B}{2}$ is-

(A) $\frac{k-1}{k+1}$ (B) $\frac{k+1}{k-1}$ (C) $\frac{k}{k+1}$ (D) $\frac{k+1}{k}$

13. Value of expression tan81° + cot81° + tan117° + cot117° is

(A) –4 (B) 4 (C) 3 (D) $4\sqrt{5}$

14. $\frac{\cos 20° + 8\sin 70° \sin 50° \sin 10°}{\sin^2 80°}$ is equal to :

(A) 1 (B) 2 (C) 3/4 (D) None

15. $\cot\theta - \tan\theta - 2\tan 2\theta - 4\tan 4\theta$ is equal to-

(A) 0 when $\theta = \frac{\pi}{16}$ (B) 8 when $\theta = \frac{\pi}{32}$

(C) 8 when $\theta = \frac{\pi}{16}$ (D) 0 when $\theta = \frac{\pi}{32}$

16. cos24° + cos48° – cos12° – cos84° is equal to-

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) 10

17. If $\cot(\alpha + \beta) = 0$, then $\sin(\alpha + 2\beta)$ is equal to-

(A) $\cos\alpha$ (B) $\cos\beta$ (C) $\sin\alpha$ (D) $\sin\beta$

18. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are

(A) $\frac{\pi}{3}$ & $\frac{\pi}{6}$ (B) $\frac{\pi}{8}$ & $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ & $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ & $\frac{3\pi}{10}$

19. If $\pi < \alpha < \frac{3\pi}{2}$, then, $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} + \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$ is-

(A) $\frac{2}{\sin\alpha}$ (B) $-\frac{2}{\sin\alpha}$ (C) $\frac{1}{\sin\alpha}$ (D) $-\frac{1}{\sin\alpha}$

20. The expression $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to

(A) cos 2x (B) 2 cos x (C) $\cos^2 x$ (D) 1 + cos x

21. If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\alpha - \beta = 2\theta$, then $\frac{\cos 3\theta}{\cos\theta} =$

(A) $a^2 + b^2 - 2$ (B) $a^2 + b^2 - 3$ (C) $3 - a^2 - b^2$ (D) $(a^2 + b^2)/4$

22. If $\alpha \in \left[\frac{\pi}{2}, \pi\right]$ then the value of $\sqrt{1+\sin\alpha} - \sqrt{1-\sin\alpha}$ is equal to:

(A) $2\cos\frac{\alpha}{2}$ (B) $2\sin\frac{\alpha}{2}$ (C) 2 (D) none of these

23. If A + B + C = π & $\sin\left(A + \frac{C}{2}\right) = k\sin\frac{C}{2}$, then $\tan\frac{A}{2}\tan\frac{B}{2}$ =

(A) $\frac{k-1}{k+1}$ (B) $\frac{k+1}{k-1}$ (C) $\frac{k}{k+1}$ (D) $\frac{k+1}{k}$

24. If $x \in \left(\pi, \frac{3\pi}{2}\right)$ then $4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$ is always equal to

(A) 1 (B) 2 (C) – 2 (D) none of these

25. If three angles A, B, C are such that
cos A + cos B + cos C = 0 and if cos A cos B cos C =
λ (cos 3A + cos 3B + cos 3C), then value of λ is :

(A) $\frac{1}{12}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

26. In any triangle ABC, which is not right angled $\sum$ cosA .cosecB.cosecC is equal to
(A) 1 (B) 2 (C) 3 (D) none of these

27. If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$, then xy + yz + zx is equal to

(A) – 1 (B) 0 (C) 1 (D) 2

28. If $\cos(A - B) = \frac{3}{5}$ and tan A tan B = 2,

(A) $\cos A\cos B = -\frac{1}{5}$ (B) $\sin A\sin B = -\frac{2}{5}$ (C) $\cos(A + B) = -\frac{1}{5}$ (D) $\sin A\cos B = \frac{4}{5}$

29. If $A + B + C = \frac{3\pi}{2}$, then cos 2A + cos2B + cos2C is equal to

(A) 1 – 4cos A cosB cosC (B) 4 sin A sin B sin C
(C) 1 + 2 cos A cos B cos C (D) 1– 4 sin A sin B sin C

30. If A + B + C = π & cosA = cosB. cosC then tanB. tanC has the value equal to:
(A) 1 (B) 1/2 (C) 2 (D) 3

MULTIPLE OPTIONS CORRECT

31*. If the sides of a right angled triangle are $\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta)\}$ and $\{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha + \beta)\}$, then the length of the hypotenuse is:

(A) $2[1+\cos(\alpha - \beta)]$ (B) $2[1 - \cos(\alpha + \beta)]$ (C) $4\cos^2\frac{\alpha - \beta}{2}$ (D) $4\sin^2\frac{\alpha + \beta}{2}$

32*. If $\tan x = \frac{2b}{a - c}$, $(a \neq c)$

$y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$
$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$, then

(A) y = z (B) y + z = a + c (C) y – z = a – c (D) $y - z = (a - c)^2 + 4b^2$

33*. For $0 < \theta < \pi/2$, $\tan\theta + \tan 2\theta + \tan 3\theta = 0$ if

(A) $\tan\theta = 0$ (B) $\tan 2\theta = 0$ (C) $\tan 3\theta = 0$ (D) $\tan\theta \tan 2\theta = 2$

34*. $(a + 2)\sin\alpha + (2a - 1)\cos\alpha = (2a + 1)$ if $\tan\alpha =$

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{2a}{a^2+1}$ (D) $\frac{2a}{a^2-1}$

35*. The value of $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$ is

(A) $2\tan^n \frac{A-B}{2}$ (B) $2\cot^n \frac{A-B}{2}$: n is even

(C) 0 : n is odd (D) none

36*. The extreme values of $f(\theta) = 2\sin^2\theta + 3\sin\theta.\cos\theta + 4\cos^2\theta$, $\forall\, \theta \in R$, are

(A) 0 (B) 6 (C) $2\sqrt{5}$ (D) 8

37*. Which of the following when simplified reduces to unity ?

(A) $\dfrac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$ (B) $\dfrac{\sin(\pi-\alpha)}{\sin\alpha - \cos\alpha\tan\frac{\alpha}{2}} + \cos(\pi-\alpha)$

(C) $\dfrac{1}{4\sin^2\alpha\cos^2\alpha} + \dfrac{(1-\tan^2\alpha)^2}{4\tan^2\alpha}$ (D) $\dfrac{1+\sin 2\alpha}{(\sin\alpha+\cos\alpha)^2}$

38*. If $\sin\alpha = \frac{3}{5}$, then the value of expression $\frac{\tan\alpha}{\sin\alpha - \cos\alpha}$ may be -

(A) $\frac{15}{4}$ (B) $-\frac{15}{4}$ (C) $\frac{15}{28}$ (D) $-\frac{15}{28}$

39*. If $\cos(A - B) = 3/5$, and $\tan A \tan B = 2$, then

(A) $\cos A\cos B = \frac{1}{5}$ (B) $\sin A\sin B = \frac{-2}{5}$

(C) $\cos(A+B) = \frac{-1}{5}$ (D) none of these

40*. The value of $\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cos\frac{4\pi}{10}\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$ is :

(A) $\frac{\sqrt{10+2\sqrt{5}}}{64}$ (B) $-\frac{\cos(\pi/10)}{16}$ (C) $\frac{\cos(\pi/10)}{16}$ (D) $-\frac{\sqrt{10+2\sqrt{5}}}{64}$

41. The solutions set of $(2\cos x - 1)(3 + 2\cos x) = 0$ in the interval $0 \le x \le 2\pi$ is

(A) $\left\{\frac{\pi}{3}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

(C) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \cos^{-1}\left(\frac{-3}{2}\right)\right\}$ (D) none of these

42. The number of ordered pair (x, y) where x and y satisfy $x + y = 2\pi/3$ and $\cos x + \cos y = 3/2$ is

(A) 0 (B) 1 (C) 2 (D) infinity

43. The number of solution of $\cos^2\theta + \sin\theta + 1 = 0$, for ($\theta \in [0, 2\pi]$) is

(A) 0 (B) 1 (C) 2 (D) infinity

44. The number of values of x for which $\sin 2x + \sin 4x = 2$ is

(A) 0 (B) 1 (C) infinite (D) none of these

45. The number of solutions of $2\cos(x/2) = 3^x + 3^{-x}$, $x \in [0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) infinite

46. If $\cos 2\theta + 3\cos\theta = 0$, then

(A) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}-3}{4}\right)$

(B) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{-\sqrt{17}-3}{4}\right)$

(C) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\pm\sqrt{17}-3}{4}\right)$

(D) none of these

47. If $\sin\theta + 7\cos\theta = 5$, then $\tan(\theta/2)$ is a root of the equation

(A) $x^2 - 6x + 1 = 0$ (B) $6x^2 - x - 1 = 0$ (C) $6x^2 + x + 1 = 0$ (D) $x^2 - x + 6 = 0$

48. The most general solution of $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$ is :

(A) $n\pi + \frac{7\pi}{4}$, $n \in I$

(B) $n\pi + (-1)^n \frac{7\pi}{4}$, $n \in I$

(C) $2n\pi + \frac{7\pi}{4}$, $n \in I$

(D) none of these

49.* $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if

(A) $\cos 12x = \cos 14x$

(B) $\sin 13x = 0$

(C) $\sin x = 0$

(D) $\cos x = 0$

50.* If $\sin(x - y) = \cos(x + y) = 1/2$ then the values of x & y lying between 0 and π are given by:

(A) $x = \pi/4$, $y = 3\pi/4$

(B) $x = \pi/4$, $y = \pi/12$

(C) $x = 5\pi/4$, $y = 5\pi/12$

(D) $x = 11\pi/12$, $y = 3\pi/4$

51.* The equation $2\sin\frac{x}{2} . \cos^2 x + \sin^2 x = 2\sin\frac{x}{2} . \sin^2 x + \cos^2 x$ has a root for which

(A) $\sin 2x = 1$ (B) $\sin 2x = -1$ (C) $\cos x = \frac{1}{2}$ (D) $\cos 2x = -\frac{1}{2}$

52.* $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:

(A) $x = n\pi + (-1)^{n+1}(\pi/6)$, $n \in I$

(B) $x = n\pi + (-1)^n(\pi/6)$, $n \in I$

(C) $x = n\pi + (-1)^n(\pi/3)$, $n \in I$

(D) $x = n\pi - (-1)^n(\pi/6)$, $n \in I$

53.* The general solution of the equation $\cos x . \cos 6x = -1$, is :

(A) $x = (2n + 1)\pi$, $n \in I$

(B) $x = 2n\pi$, $n \in I$

(C) $x = (2n - 1)\pi$, $n \in I$

(D) none of these

54.* The general value of θ, satisfying the equation

$$2\cos 2\theta + \sqrt{2\sin\theta} = 2, \text{ is}$$

(A) $n\pi$ (B) $n\pi + (-1)^n \frac{\pi}{3}$ (C) $n\pi + (-1)^n \frac{\pi}{6}$ (D) none of these

PART - II : SUBJECTIVE QUESTIONS

1. **Prove that :**

(a) $\sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A = 1$

(b) $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A = \tan^2 A \sin^2 A$

(c) $\dfrac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$

(d) $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

(e) $\dfrac{\cot^2\theta\,(\sec\theta - 1)}{1 + \sin\theta} = \sec^2\theta \cdot \dfrac{1 - \sin\theta}{1 + \sec\theta}$

(f) $(1 + \cot A + \tan A)(\sin A - \cos A) = \dfrac{\sec A}{\operatorname{cosec}^2 A} - \dfrac{\operatorname{cosec} A}{\sec^2 A}$

(g) $\dfrac{2\sin\theta\tan\theta\,(1 - \tan\theta) + 2\sin\theta\sec^2\theta}{(1 + \tan\theta)^2} = \dfrac{2\sin\theta}{(1 + \tan\theta)}$

2. If the arcs of the same length in two circles subtend angles 75° and 120° at the centre, find the ratio of their radii.

3. Find the radian measures corresponding to the following degree measures

(i) 15° (ii) 240° (iii) 530°

4. Find the degree measures corresponding to the following radian measures

(i) $\dfrac{3\pi}{4}$ (ii) -4π (iii) $\dfrac{5\pi}{3}$ (iv) $\dfrac{7\pi}{6}$

5. **Prove that :**

(a) $\sin^2\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{3} - \tan^2\dfrac{\pi}{4} = -\dfrac{1}{2}$

(b) $2\sin^2\dfrac{\pi}{6} + \operatorname{cosec}\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3} = 0$

(c) $3\cos^2\dfrac{\pi}{4} + \sec\dfrac{2\pi}{3} + 5\tan^2\dfrac{\pi}{3} = \dfrac{29}{2}$

(d) $\cot^2\dfrac{\pi}{6} + \operatorname{cosec}\dfrac{5\pi}{6} + 3\tan^2\dfrac{\pi}{6} = 6$

(e) $2\sin^2\dfrac{3\pi}{4} + 2\cos^2\dfrac{\pi}{4} + 2\sec^2\dfrac{\pi}{3} = 10$

6. Find the value of :

(i) cos 210° (ii) sin 225° (iii) tan 330° (iv) cot (– 315°)

7. $\dfrac{\cos(\pi + \theta)\cos(-\theta)}{\sin(\pi - \theta)\cos\left(\dfrac{\pi}{2} + \theta\right)} = \cot^2\theta$.

8. $\cos\theta + \sin(270° + \theta) - \sin(270° - \theta) + \cos(180° + \theta) = 0$.

9. $\cos\left(\dfrac{3\pi}{2} + \theta\right)\cos(2\pi + \theta)\left[\cot\left(\dfrac{3\pi}{2} - \theta\right) + \cot(2\pi + \theta)\right] = 1$.

10. $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right) = \left(\frac{1}{\sqrt{2}}\right)\sin A$

11. $\cos 2\theta \cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2} = \sin 5\theta \sin\frac{5\theta}{2}$.

12. $\cos^2\alpha + \cos^2(\alpha+\beta) - 2\cos\alpha\cos\beta\cos(\alpha+\beta) = \sin^2\beta$.

13. For all values of α, β, γ prove that,

$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\frac{\alpha+\beta}{2} . \cos\frac{\beta+\gamma}{2} . \cos\frac{\gamma+\alpha}{2}$.

14. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, find the value of $\sin\frac{x}{2}$ and $\cos\frac{x}{2}$.

15. If $\cos(\alpha+\beta) = \frac{4}{5}$; $\sin(\alpha-\beta) = \frac{5}{13}$ & α, β lie between 0 & $\frac{\pi}{4}$, then find the value of $\tan 2\alpha$.

16. If $\sin(\theta+\alpha) = a$ & $\sin(\theta+\beta) = b$ $(0 < \alpha, \beta, \theta < \pi/2)$ then find the value of $\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta)$

17. Prove that, $\sin 3x . \sin^3 x + \cos 3x . \cos^3 x = \cos^3 2x$.

18. prove that $\left\{\frac{1-\cot^2\left(\frac{\alpha-\pi}{4}\right)}{1+\cot^2\left(\frac{\alpha-\pi}{4}\right)} + \cos\frac{\alpha}{2}\cot 4\alpha\right\}\sec\frac{9\alpha}{2} = \operatorname{cosec} 4\alpha$.

19. Prove that $\frac{1}{\tan 3\alpha - \tan\alpha} - \frac{1}{\cot 3\alpha - \cot\alpha} = \cot 2\alpha$.

20. Find the greatest and least value of y

(i) $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$

(ii) $y = 1 + 2\sin x + 3\cos^2 x$

(iii) $y = 3\cos\left(\theta+\frac{\pi}{3}\right) + 5\cos\theta + 3$

21. If ϕ is the exterior angle of a regular polygon of n sides and θ is any constant, then prove that $\sin\theta + \sin(\theta+\phi) + \sin(\theta+2\phi) + \ldots\ldots\ldots\ldots$ up to n terms = 0

22. If $x + y + z = \frac{\pi}{2}$ show that, $\sin 2x + \sin 2y + \sin 2z = 4\cos x\cos y\cos z$.

23. If $x + y = \pi + z$, then prove that $\sin^2 x + \sin^2 y - \sin^2 z = 2\sin x\sin y\cos z$.

24. Prove that :

(i) $\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$

(ii) $\frac{1}{\sec\alpha - \tan\alpha} - \frac{1}{\cos\alpha} = \frac{1}{\cos\alpha} - \frac{1}{\sec\alpha + \tan\alpha}$

(iii) $\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$

25. Show that:

(i) $\cot 7\frac{1^\circ}{2}$ or $\tan 82\frac{1^\circ}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$ or $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

(ii) $\tan 142\frac{1^\circ}{2} = 2 + \sqrt{2}-\sqrt{3}-\sqrt{6}$.

26. Prove that, $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$.

27. Calculate the following without using trigonometric tables:

(i) $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

(ii) $\operatorname{cosec} 10° - \sqrt{3}\sec 10°$

(iii) $2\sqrt{2}\sin 10°\left[\frac{\sec 5°}{2}+\frac{\cos 40°}{\sin 5°}-2\sin 35°\right]$

(iv) $\cot 70° + 4\cos 70°$

(v) $\tan 10° - \tan 50° + \tan 70°$

28. Let $A_1, A_2, \ldots, A_n$ be the vertices of an n–sided regular polygon such that; $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$.
Find the value of n.

29. Prove that % $\cos 2\alpha = 2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta)$

30. If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = \frac{-3}{2}$, prove that
$\cos\alpha + \cos\beta + \cos\gamma = 0$, $\sin\alpha + \sin\beta + \sin\gamma = 0$.

31. If $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$, $\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$.
Show that $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

32. If $m\tan(\theta - 30°) = n\tan(\theta + 120°)$, show that $\cos 2\theta = \frac{m+n}{2(m-n)}$.

33. If $\tan\beta = \frac{\tan\alpha+\tan\gamma}{1+\tan\alpha.\tan\gamma}$, prove that $\sin 2\beta = \frac{\sin 2\alpha+\sin 2\gamma}{1+\sin 2\alpha.\sin 2\gamma}$.

34. If $\sin x + \sin y = a$ & $\cos x + \cos y = b$, show that,

$\sin(x+y) = \frac{2ab}{a^2+b^2}$ and $\tan\frac{x-y}{2} = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$.

35. If $\sin(\theta+\alpha) = a$ & $\sin(\theta+\beta) = b$ $(0 < \alpha, \beta, \theta < \pi/2)$ then find the value of
$\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta)$

36. If $P_n = \cos^n\theta + \sin^n\theta$ and $Q_n = \cos^n\theta - \sin^n\theta$, then show that
$P_n - P_{n-2} = -\sin^2\theta\cos^2\theta\, P_{n-4}$
$Q_n - Q_{n-2} = -\sin^2\theta\cos^2\theta\, Q_{n-4}$
and hence show that
$P_4 = 1 - 2\sin^2\theta\cos^2\theta$
$Q_4 = \cos^2\theta - \sin^2\theta$

Trignometric Equation

37. What are the most general values of θ which satisfy the equations:

(a) $\sin\theta = \frac{1}{\sqrt{2}}$ (b) $\tan(x - 1) = \sqrt{3}$ (c) $\tan\theta = -1$ (d) $\operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$

(e) $2\cot^2\theta = \operatorname{cosec}^2\theta$

38. Solve $\sin 9\theta = \sin\theta$

39. Solve $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$

40. Solve $\sin 2\theta = \cos 3\theta$

41. Solve $\cot\theta = \tan 8\theta$

42. Solve $\cot\theta - \tan\theta = 2$

43. Solve $\operatorname{cosec}\theta = \cot\theta + \sqrt{3}$

44. Solve $\tan 2\theta \tan\theta = 1$

45. Solve $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

46. Solve $\sin\theta + \sin 3\theta + \sin 5\theta = 0$.

47. Solve $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$

48. find all values of θ between 0° and 180° satisfying the equation $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.

49. Solve $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$.

50. Solve $\sin^2 n\theta - \sin^2 (n-1)\theta = \sin^2\theta$, where n is constant and $n \neq 0, 1$

51. find the most general solution of the following :

(i) $\sin 6x = \sin 4x - \sin 2x$

(ii) $\sec 4x - \sec 2x = 2$

52. If sinA = sinB and cosA = cosB, find all the value of A in terms of B.

53. Solve $\tan 2x + \tan 3x = 0$

54. Solve $(1 - \tan\theta)(1 + \sin 2\theta) = 1 + \tan\theta$.

55. Solve $\sin x + \sqrt{3}\cos x = \sqrt{2}$.

56. Solve $\sin\theta = \frac{1}{2}$, $\tan\theta = \frac{1}{\sqrt{3}}$

57. Solve the equality: $2\sin 11x + \cos 3x + \sqrt{3}\sin 3x = 0$

58. Find all value of θ, between 0 & π, which satisfy the equation; $\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = 1/4$.

59. Find the general solution of the equation , $2 + \tan x \cdot \cot\frac{x}{2} + \cot x \cdot \tan\frac{x}{2} = 0$

60. Solve for x , the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

61. Determine the smallest positive value of x which satisfy the equation, $\sqrt{1 + \sin 2x} - \sqrt{2}\cos 3x = 0$.

62. $2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x \cdot \cos^2 2x}$

63. Find the number of principal solution of the equation, $\sin x - \sin 3x + \sin 5x = \cos x - \cos 3x + \cos 5x$.

64. Find the general solution of the trigonometric equation $3^{\left(\frac{1}{2} + \log_3(\cos x + \sin x)\right)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$.

65. Find all values of θ between 0° & 180° satisfying the equation;
$\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

1. Let $f(\theta) = \sin\theta\,(\sin\theta + \sin 3\theta)$. Then **[IIT-JEE-2000, Scr., 1/35]**

(A) $f(\theta) \geq 0$ only when $\theta \geq 0$
(B) $f(\theta) \leq 0$ for all real θ
(C) $f(\theta) \geq 0$ for all real θ
(D) $f(\theta) \leq 0$ only when $\theta \leq 0$.

2. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)\ldots\ldots\ldots(\cos\alpha_n)$ under the restrictions,

$$0 \leq \alpha_1, \alpha_2, \ldots\ldots, \alpha_n \leq \frac{\pi}{2} \text{ and } (\cot\alpha_1).(\cot\alpha_2)\ldots\ldots\ldots(\cot\alpha_n) = 1 \text{ is:}$$

[IIT-JEE - 2001, Scr - (1- M), 35]

(A) $1/2^{n/2}$ (B) $1/2^n$ (C) $1/2n$ (D) 1

3. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals **[IIT-JEE - 2001,Scr - (1- M), 35]**

(A) $2(\tan\beta + \tan\gamma)$ (B) $\tan\beta + \tan\gamma$ (C) $\tan\beta + 2\tan\gamma$ (D) $2\tan\beta + \tan\gamma$

4. The number of integral values of ' k ' for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is:
[IIT-JEE-2002, Scr., (3,–1)/90]

(A) 4 (B) 8 (C) 10 (D) 12

5. If $\sin\alpha = 1/2$ and $\cos\theta = 1/3$, then the values of $\alpha + \theta$ (if θ, α are both acute) will lie in the interval
[IIT-JEE-2004, Scr., (3,–1)/84]

(A) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$ (C) $\left[\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left[\frac{5\pi}{6}, \pi\right]$

6. Find the range of values of ' t ' for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
[IIT-JEE - 2005, Main, (2-M), 60]

7. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is **[IIT-JEE - 2005, Scr- (3, – 1), 84]**

(A) $4 + 2\sqrt{3}$ (B) $6 + 4\sqrt{3}$ (C) $12 + \frac{7\sqrt{3}}{4}$ (D) $3 + \frac{7\sqrt{3}}{4}$

8. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, where $\alpha, \beta \in [-\pi, \pi]$. Pairs α, β which satisfy both the equations is/are **[IIT-JEE-2005, Scr., (3,–1)/84]**

(A) 0 (B) 1 (C) 2 (D) 4

9. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then
[IIT-JEE - 2006 , Main - (3, –1), 184]

(A) $t_1 > t_2 > t_3 > t_4$ (B) $t_2 < t_1 < t_3 < t_4$ (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

10. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0$, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is **[IIT-JEE-2007, Paper-1, (3,–1)/81]**

(A) zero (B) one (C) two (D) four

11. For $0 < \theta < \pi/2$, the solution(s) of $\sum_{m=1}^{6} \text{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\text{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is/are :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

[IIT-JEE 2009, (4, –1)/82]

12.* If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then **[IIT-JEE - 2009 ,Paper-1, (4, –1), 80]**

(A) $\tan^2 x = \frac{2}{3}$

(B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$

(D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

13. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

14. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ±1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **[IIT-JEE-2010, Paper-1, (3, 0)/84]**

15. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

[IIT-JEE-2011, Paper-1/80]

(A) $P \subset Q$ and $Q - P \neq \varnothing$

(B) $Q \not\subset P$

(C) $P \not\subset Q$

(D) $P = Q$

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to **[AIEEE - 2002]**

(1) $\frac{24}{25}$ (2) $-\frac{24}{25}$ (3) $\frac{13}{18}$ (4) $-\frac{13}{18}$

2. The equation $a\sin x + b\cos x = c$, where $|c| > \sqrt{a^2 + b^2}$ has **[AIEEE - 2002]**

(1) a unique solution
(2) infinite number of solutions
(3) no solution
(4) None of the above

3. If $y = \sin^2\theta + \text{cosec}^2\theta$, $\theta \neq 0$, then **[AIEEE - 2002]**

(1) $y = 0$ (2) $y \leq 2$ (3) $y \geq -2$ (4) $y \geq 2$

4. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$ then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$ is equal to **[AIEEE - 2002]**

(1) 1 (2) –1 (3) 0 (4) None of these

5. If $\tan\theta = -\frac{4}{3}$ then $\sin\theta$ is **[AIEEE - 2002]**

(1) $-\frac{4}{5}$ but not $\frac{4}{5}$ (2) $-\frac{4}{5}$ or $\frac{4}{5}$ (3) $\frac{4}{5}$ but not $-\frac{4}{5}$ (4) None of these

6. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is **[AIEEE - 2002]**

(1) 1 (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) 2

7. $\sin^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if **[AIEEE - 2002]**

(1) $x - y \neq 0$ (2) $x = y, x \neq 0$ (3) $x = y$ (4) $x \neq 0, y \neq 0$

8. In a ΔABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ & $\angle ABE = \frac{\pi}{3}$, then the area of ΔABC is

[AIEEE - 2003]

(1) $\frac{8}{3}$ squre unit (2) $\frac{16}{3}$ squre unit (3) $\frac{32}{3\sqrt{3}}$ squre unit (4) $\frac{64}{3}$ squre unit

9. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of

$\cos\left(\frac{\alpha-\beta}{2}\right)$ is **[AIEEE - 2004]**

(1) $\frac{-3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$ (3) $\frac{6}{65}$ (4) $\frac{-6}{65}$

10. In a triangle PQR , $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$; $a \neq 0$ then

[AIEEE - 2005]

(1) b = a + c (2) b = c (3) c = a + b (4) a = b + c

11. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then tan x is **[AIEEE-2006]**

(1) $\frac{4-\sqrt{7}}{3}$ (2) $-\left(\frac{4+\sqrt{7}}{3}\right)$ (3) $\frac{1+\sqrt{7}}{4}$ (4) $\frac{1-\sqrt{7}}{4}$

12. A triangular part is enclosed on two sides by a fence and the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is **[AIEEE-2006]**

(1) $\sqrt{\frac{x^3}{8}}$ (2) $\frac{x^2}{2}$ (3) πx^2 (4) $\frac{3x^2}{2}$

13. Let A and B denote the statements **[AIEEE-2009]**

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then :

(A) A is false and B is true (B) both A and B are true
(C) both A and B are false (D) A is true and B is false

14. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ =

(1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

15. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is **[AIEEE 2002]**

(A) $-\frac{4}{5}$ but not $\frac{4}{5}$ (B) $-\frac{4}{5}$ or $\frac{4}{5}$ (C) $\frac{4}{5}$ but not $-\frac{4}{5}$ (D) None of these

16. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to **[AIEEE 2002]**

(A) $\frac{24}{25}$ (B) $-\frac{24}{25}$ (C) $\frac{13}{18}$ (D) $-\frac{13}{18}$

17. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$, then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$ is equal to **[AIEEE 2002]**

(A) 1 (B) –1 (C) zero (D) none of these

18. If $y = \sin^2\theta + \text{cosec}^2\theta$, $\theta \neq 0$, then **[AIEEE 2002]**

(A) $y = 0$ (B) $y \le 2$ (C) $y \ge -2$ (D) $y \ge 2$

19. The equation $a\sin x + b\cos x = c$ where $|c| > \sqrt{a^2 + b^2}$ has **[AIEEE 2002]**

(A) a unique solution (B) infinite number of solutions
(C) no solution (D) none of the above

20. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of cos $\left(\frac{\alpha - \beta}{2}\right)$ is **[AIEEE 2004]**

(A) $-\frac{3}{\sqrt{130}}$ (B) $\frac{3}{\sqrt{130}}$ (C) $\frac{6}{65}$ (D) $-\frac{6}{65}$

21. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[AIEEE 2006]**

(A) 6 (B) 1 (C) 2 (D) 4

22. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[AIEEE 2006]**

(A) $\frac{(4 - \sqrt{7})}{3}$ (B) $-\frac{(4 + \sqrt{7})}{3}$ (C) $\frac{(1 + \sqrt{7})}{4}$ (D) $\frac{(1 - \sqrt{7})}{4}$

23. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as **[JEE MAINS 2013]**

(1) $\sin A\cos A + 1$ (2) $\sec A\,\text{cosec}\,A + 1$ (3) $\tan A + \cot A$ (4) $\sec A + \text{cosec}\,A$

EXERCISE # 4

NCERT BOARD QUESTIONS

1. Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

2. If $\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$, then prove that $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ is also equal to y.

3. If m sin θ = n sin (θ + 2α) , then prove that tan (θ + α) cot α = $\frac{m + n}{m - n}$

4. If cos (α + β) = $\frac{4}{5}$ and sin (α - β) = $\frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, find the value of tan 2α

5. If tan x = $\frac{b}{a}$, then find the value of $\sqrt{\frac{a + b}{a - b}} + \sqrt{\frac{a - b}{a + b}}$

6. Prove that cos θ cos $\frac{\theta}{2}$ - cos 3θ cos $\frac{9\theta}{2}$ =sin 7θ sin 8θ.

7. If a cos θ + b sin θ = m and a sin θ - b cos θ = n, then show that $a^2 + b^2 = m^2 + n^2$.

8. Find the value of tan 22º30'.

9. Prove that sin 4A = 4 sin A $\cos^3 A$ - 4 cosA $\sin^3 A$.

10. If tan θ + sin θ = m and tan θ - sin θ = n, then prove that $m^2 - n^2$ = 4 sin θ tanθ.

11. If tan (A + B) = p, tan (A –B) = q, then show that tan 2 A = $\frac{p + q}{1 - pq}$

12. If cosα + cosβ = 0 = sin α + sin β, then prove that cos 2α + cos 2β = – 2 cos (α + β).

13. If $\frac{\sin(x + y)}{\sin(x - y)} = \frac{a + b}{a - b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$

14. If tan θ = $\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that sin α + cos α = $\sqrt{2}$ cosθ.

15. If sin θ + cos θ = 1, then find the general value of θ.

16. Find the most general value of θ satisfying the equation $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$

17. If $\cot\theta + \tan\theta = 2\,\text{cosec}\,\theta$, then find the general value of θ.

18. If $2\sin^2\theta = 3\cos\theta$, where $0 \le \theta \le 2\pi$, then find the value of θ.

19. If $\sec x \cos 5x + 1 = 0$, where $0 < x \le \frac{\pi}{2}$, then find the value of x.

20. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$

21. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then prove that $\tan\theta = \frac{1-m}{1+m}\cot\phi$.

22. Find the value of the expression

$$3\,[\sin^4(\frac{3\pi}{2} - \alpha) + \sin^4(3\pi + \alpha)] - 2\{\sin^6(\frac{\pi}{2} + \alpha) + \sin^6(5\pi - \alpha)]$$

23. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β at its roots, then prove that $\tan\alpha + \tan\beta = \frac{2b}{a+c}$

24. If $x = \sec\phi - \tan\phi$ and $y = \text{cosec}\,\phi + \cot\phi$ then show that $xy + x - y + 1 = 0$

25. If θ lies in the first quadrant and $\cos\theta = \frac{8}{17}$, then find the value of $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$.

26. Find the value of the expression $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

27. Find the general solution of the equation $5\cos^2\theta + 7\sin^2\theta - 6 = 0$

28. Find the general solution of the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

29. Find the general solution of the equation $\left(\sqrt{3} - 1\right)\cos\theta + \left(\sqrt{3} + 1\right)\sin\theta = 2$

ANSWERS

EXERCISE # 1

PART # I

A-1.	(C)	**A-2.**	(A)	**A-3.**	(A)	**A-4.**	(D)	**A-5.**	(B)	**A-6.**	(A)	**A-7.**	(D)
A-8.	(D)	**A-9.**	(B)	**A-10.**	(A) (B)	**A-11.**	(C)						
B-1.	(A)	**B-2.**	(A)	**B-3.**	(B)	**B-4.**	(B)	**B-5.**	(A)	**B-6.**	(B)	**B-7.**	(C)
B-8.	(B)	**B-9.**	(D)	**B-10.**	(B)	**B-11.**	(A)	**B-12.**	(A)	**B-13.**	(C)	**B-14.**	(A)
B-15.	(B)	**B-16.**	(D)	**B-17.**	(A, B, C, D)								
C-1.	(C)	**C-2.**	(C)	**C-3.**	(C)	**C-4.**	(A)	**C-5.**	(C)	**C-6.**	(B, C)	**C-7.**	(C)
C-8.	(A)												
D-1.	(B)	**D-2.**	(C)	**D-3.**	(A)	**D-4.**	(C)	**D-5*.**	(A, B)	**D-6.**	(B)	**D-7.**	(D)
D-8.	(B)												
E-1.	(B)	**E-2.**	(A)	**E-3.**	(B)	**E-4.**	(C)	**E-5.**	(A)	**E-6.**	(B)	**E-7.**	(C)
E-8.	(D)	**E-9.**	(C)	**E-10.**	(C)	**E-11.**	(A)	**E-12.**	(A)	**E-13.**	(B)	**E-14.**	(C)
E-15.	(C)	**E-16.**	(C)	**E-17.**	(B)	**E-18.**	(A)	**E-19.**	(C)	**E-20.**	(D)	**E-21.**	(D)
E-22.	(D)	**E-23.**	(D)	**E-24.**	(D)	**E-25.**	(B)	**E-26.**	(A)	**E-27.**	(B)	**E-28.**	(A)
E-29.	(D)	**E-30.**	(D)										

PART # II

1.	(C)	**2.**	(A)	**3.**	(B)	**4.**	(C)	**5.**	(B)	**6.**	(B)	**7.**	(D)
8.	(C)	**9.**	(B)	**10.**	(C)	**11.**	(D)	**12.**	(D)	**13.**	A-q B-rs C-rs D-p		

14. A-q, B-p C-s D-r **15.** (A-q) (B-r) (C-s) (D-p) **16.** (A-r) (B-s) (C -p) (D-q)

17. (A-s) (B-p) (C-q) (D-r) **18.** (D) **19.** (A) **20.** (B) **21.** (C)

22. (D) **23.** (A)

EXERCISE # 2

PART # I

1.	(D)	**2.**	(D)	**3.**	(B)	**4.**	(D)	**5.**	(C)	**6.**	(B)	**7.**	(C)
8.	(A)	**9.**	(C)	**10.**	(D)	**11.**	(C)	**12.**	(A)	**13.**	(B)	**14.**	(B)
15.	(A)	**16.**	(B)	**17.**	(C)	**18.**	(B)	**19.**	(B)	**20.**	(B)	**21.**	(B)
22.	(A)	**23.**	(A)	**24.**	(B)	**25.**	(A)	**26.**	(B)	**27.**	(B)	**28.**	(C)
29.	(D)	**30.**	(C)	**31*.**	(A,C)	**32*.**	(B, C)	**33*.**	(C, D)	**34*.**	(B, D)		
35*.	(B,C)	**36*.**	(A , B)	**37*.**	(A, B, D)			**38*.**	(B,D)			**39*.**	(A,C)
40*.	(B)	**41.**	(B)	**42.**	(A)	**43.**	(B)	**44.**	(A)	**45.**	(B)	**46.**	(A)
47.	(B)	**48.**	(C)	**49.***	(A, B, C)			**50.***	(B, D)	**51.***	(A, B, C, D)	**52.***	(A, D)
53.*	(A, C)	**54.***	(A, C)										

PART # II

20. (i) $y_{max} = 11;\ y_{min} = 1$ (ii) $y_{max} = \frac{13}{3};\ y_{min} = -1$ (iii) $y_{max} = 10;\ y_{min} = -4$

27. (i) 4 (ii) 4 (iii) 4 (iv) $\sqrt{3}$ (v) $\sqrt{3}$

28. n = 7 **35.** $1 - 2a^2 - 2b^2$

37. (a) $n\pi + (-1)^n \frac{\pi}{4}, n \in I$ (b) $n\pi + \frac{\pi}{3} + 1,\ n \in I$ (c) $n\pi - \frac{\pi}{4},\ n \in I$ (d) $n\pi + (-1)^n \frac{\pi}{3},\ n \in I$ (e) $n\pi \pm \frac{\pi}{4},\ n \in I$

38. $\frac{m\pi}{4}, m \in I$ **or** $\frac{(2m+1)\pi}{10}, m \in I$ **39.** $2n\pi \pm \frac{\pi}{3}, n \in I$ **40.** $\left(2n + \frac{1}{2}\right)\frac{\pi}{5}, n \in I$ or $2n\pi - \frac{\pi}{2}, n \in I$

41. $\left(n + \frac{1}{2}\right)\frac{\pi}{9}, n \in I$ **42.** $\left(n + \frac{1}{4}\right)\frac{\pi}{2}, n \in I$ **43.** $2n\pi + \frac{2\pi}{3},\ n \in I$

44. $(2n+1)\frac{\pi}{6}, n \in I$ **45.** $\left(n + \frac{1}{3}\right)\frac{\pi}{3}, n \in I$ **46.** $\frac{n\pi}{3}, n \in I$ or $\left(n \pm \frac{1}{3}\right)\pi,\ n \in I$

47. $2n\pi,\ n \in I$ or $\frac{2n\pi}{3} + \frac{\pi}{6}, n \in I$ **48.** 30º, 45º, 90º, 135º, 150º

49. $x = (2n+1)\frac{\pi}{4}, n \in I$ or $x = (2n+1)\frac{\pi}{2}\ n \in I$ or $x = n\pi \pm \frac{\pi}{6}, n \in I$

50. $m\pi, m \in I$ or $\frac{m\pi}{n-1}, m \in I$ or $\left(m+\frac{1}{2}\right)\frac{\pi}{n}, m \in I$

51. (i) $(2n+1)\frac{\pi}{6}, (2m+1)\frac{\pi}{4}, k\pi$ (ii) $(2n+1)\frac{\pi}{10}, (2m+1)\frac{\pi}{2}$

52. $A = 2n\pi + B$ **53.** $x = \frac{n\pi}{3}, n\pi \pm \tan^{-1}\frac{1}{\sqrt{2}}$ **54.** $\theta = n\pi, n\pi - \frac{\pi}{4}$

55. $x = 2n\pi + \frac{5\pi}{12}, 2n\pi - \frac{\pi}{12}$ **56.** $\theta = 2n\pi + \frac{\pi}{6}$ **57.** $x = \frac{n\pi}{7} - \frac{\pi}{84}$ or $x = \frac{n\pi}{4} + \frac{7\pi}{48}, n \in I$

58. $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$ **60.** $\alpha - 2\pi ; \alpha - \pi, \alpha, \alpha + \pi$ where $\tan\alpha = \frac{2}{3}$ **61.** $x = \pi/16$

62. $x = 2n\pi + \frac{\pi}{12}$ or $2n\pi + \frac{17\pi}{12}; n \in I$ **63.** 10 solutions **64.** $x = 2n\pi + \frac{\pi}{12}$

65. 30°, 45°, 90°, 135°, 150°

EXERCISE # 3

PART # I

1.	(C)	**2.**	(A)	**3.**	(C)	**4.**	(B)	**5.**	(B)	**6.**	$\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$
7.	(B)	**8.**	(D)	**9.**	(B)	**10.**	(C)	**12.***	(A, B)	**13.**	2
14.	3	**15.**	**(D)**								

PART # II

1.	(2)	**2.**	(3)	**3.**	(4)	**4.**	(1)	**5.**	(2)	**6.**	(3)	**7.**	(3)
8.	(3)	**9.**	(1)	**10.**	(3)	**11.**	(2)	**12.**	(2)	**13.**	(B)	**14.**	(1)
15.	(B)	**16.**	(B)	**17.**	(A)	**18.**	(D)	**19.**	(C)	**20.**	(A)	**21.**	(D)
22.	(B)	**23.**	(2)										

EXERCISE # 4

4. $\frac{56}{33}$ **5.** $\frac{2\cos x}{\sqrt{\cos 2x}}$ **8.** $\frac{1}{\sqrt{2}+1}$ **15.** $\theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$

16. $\theta = 2n\pi + \frac{7\pi}{4}$ **17.** $\theta = 2n\pi \pm \frac{\pi}{3}$ **18.** $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **19.** $x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}$ **22.** 1

25. $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right)$ **26.** $\frac{3}{2}$ **27.** $n\pi \pm \frac{\pi}{4}$ **28.** $\frac{n\pi}{2} + \frac{\pi}{8}$

29. $\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$