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VECTOR & 3-DIMENSIONAL GEOMETRY

Contents

Syllabus

Addition of vectors, scalar multiplication, dot and cross products, scalar triple products and their geometrical interpretations

Direction cosines and direction ratios, equation of a straight line in space, equation of a plane, distance of a point from a plane.

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VECTORS & 3D

KEY CONCEPTS

1. DEFINITIONS :

A Vector may by described as a quantity having both magnitude & direction. A vector is generally represented by a directed line segment, say $\vec{AB}$. A is called the **initial point** & B is called the **terminal point.** The magnitude of vector $\vec{AB}$ is expressed by. $|\vec{AB}|$

ZERO VECTOR a vector of zero magnitude i.e. which has the same initial & terminal point is called a ZERO VECTOR . It is denoted by O.

UNIT VECTOR a vector of unit magnitude in direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\hat{a}$ symbolically $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

EQUAL VECTORS two vectors are said to be equal if they have the same magnitude, direction & represent the same physical quantity.

COLLINEAR VECTORS two vectors are said to be collinear if their directed lines segments are parallel disregards to their direction. Collinear vectors are also called **Parallel vectors**. If they have the same direction they are named as **like vectors** otherwise **unlike vectors.**

Symbolically two non zero vectors $\vec{a}$ & $\vec{b}$ are collinear if and only if, $\vec{a} = K\vec{b}$, where $K \in R$

COPLANAR VECTORS a given number of vectors are called coplanar if their line segment are all parallel to the same plane. Note that **"TWO VECTORS ARE ALWAYS COPLANAR".**

POSITION VECTOR let O be a fixed origin, then the position vector of a point P is the vector $\vec{OP}$. If $\vec{a}$ & $\vec{b}$ are position vectors of two point A and B, then, $\vec{AB} = \vec{b} - \vec{a} = pv$ of B - pv of A.

2. VECTOR ADDITION :

☞ If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\vec{OA}$ & $\vec{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\vec{OC}$, where OC is the diagonal of the parallelogram OACB.

☞ $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (commutative)

☞ $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ (associativity)

☞ $\vec{a}+\vec{0} = \vec{a} = \vec{0}+\vec{a}$

☞ $\vec{a}+(-\vec{a}) = \vec{0} = (-\vec{a})+\vec{a}$

3. MULTIPLICATION OF VECTOR BY SCALARS :

If $\vec{a}$ is a vector & m is a scalar, then m $\vec{a}$ is a vector parallel to $\vec{a}$ whose moduls is $|m|$ times that of $\vec{a}$. This multiplication is called SCALAR MULTIPLICATION. If $\vec{a}$ & $\vec{b}$ are vectors & m, n are scalars, then :

$m(\vec{a}) = (\vec{a})m = m\vec{a}$ $\quad m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$ $\quad (m+n)\vec{a} = m\vec{a} + n\vec{a}$ $\quad m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

4. SECTION FORMULA :

If $\vec{a}$ & $\vec{b}$ are the position vectors of two points A & B then the p.v. of a point which divides AB in the ratio m : n is given by : $\vec{r} = \frac{n\vec{a}+m\vec{b}}{m+n}$. Note p.v. of mid point of $AB = \frac{\vec{a}+\vec{b}}{2}$

5. DIRECTION COSINES :

Let $\vec{a} = a_1 i + a_2 j + a_3 k$ the angles which this vector makes with +ve directions OX, OY & OZ are called DIRECTION ANGLES & their cosines are called DIRECTION COSINES

$\cos\alpha = \frac{a_1}{|\vec{a}|}$, $\cos\beta = \frac{a_2}{|\vec{a}|}$, $\cos\gamma = \frac{a_3}{|\vec{a}|}$,

Note that, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

6. VECTOR EQUATION OF A LINE :

Parametric vector equation of a line passing through two point $A(\vec{a})$ & $B(\vec{b})$ is given by,

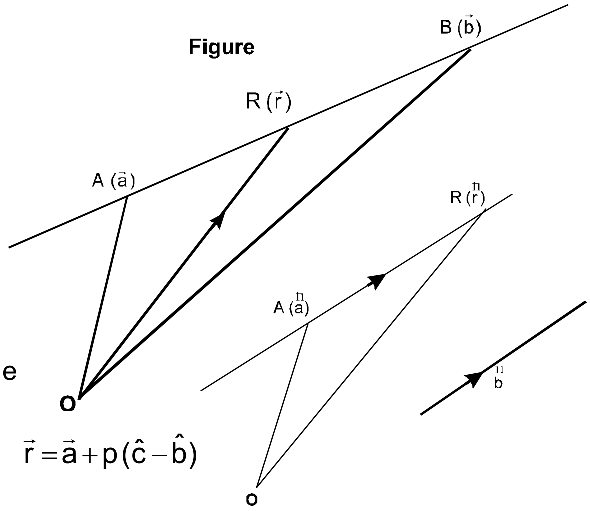
$\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$ where t is a parameter.

If the line pass through the point $A(\vec{a})$ & is parallel to the vector $\vec{b}$ then its equation is.

$\vec{r} = \vec{a} + t\vec{b}$

Note that the equations of the bisectors of the angles between the lines

$\vec{r} = \vec{a} + \lambda\vec{b}$ & $\vec{r} = \vec{a} + \mu\vec{c}$ is, $\vec{r} = \vec{a} + t(\hat{b} + \hat{c})$ & $\vec{r} = \vec{a} + p(\hat{c} - \hat{b})$



7. TEST OF COLLINEARITY :

Three points A, B, C with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively are collinear. If & only if there exist scalars x, y, z not all zero simultaneously such that ; $x\vec{a} + y\vec{b} + z\vec{c} = 0$, where $x + y + z = 0$

8. SCALAR PRODUCT OF TWO VECTORS :

☞ $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta \, (0 \le \theta \le \pi)$,

note that if θ is acute then $\vec{a}.\vec{b} > 0$ & if θ is obtuse then $\vec{a}.\vec{b} < 0$

☞ $\vec{a}.\vec{a} = |\vec{a}|^2 = \vec{a}^2$, $\vec{a}.\vec{b} = \vec{b}.\vec{a}$ (commutative) $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$ (distributive)

☞ $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$; $(\vec{a} \ne 0 \quad \vec{b} \ne 0)$

☞ $i.i = j.j = k.k = 1$; $i.j = j.k = k.i = 0$

☞ projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$.

Note: **(i)** That vector component of $\vec{a}$ along $\vec{b}$ $= \left(\dfrac{\vec{a}.\vec{b}}{\vec{b}^2}\right)\vec{b}$ and perpenducular to $\vec{b} = \vec{a} - \left(\dfrac{\vec{a}.\vec{b}}{\vec{b}^2}\right)\vec{b}$

(ii) The angle ϕ between $\vec{a}$ & $\vec{b}$ is given by $\cos\phi = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ $\quad 0 \le \phi \le \pi$

(iii) If $\vec{a} = a_1 i + a_2 j + a_3 k$ & $\vec{b} = b_1 i + b_2 j + b_3 k$ then $\vec{a}.\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$

(iv) Maximum value of $\vec{a}.\vec{b} = |\vec{a}| \, |\vec{b}|$

(v) Minimum values of $\vec{a}.\vec{b} = -|\vec{a}| \, |\vec{b}|$

(vi) Any vector $\vec{a}$ can be written as, $\vec{a} = (\vec{a}.\hat{i})\,\hat{i} + (\vec{a}.\hat{j})\,\hat{j} + (\vec{a}.\hat{k})\,\hat{k}$

(vii) A vector in the direction of the bisector of the angle between the two vectors $\vec{a}$ & $\vec{b}$

is $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|}$. Hence bisector of the angle between the two vectors $\vec{a}$ & $\vec{b}$ is $\lambda(\hat{a} + \hat{b})$,

where $\lambda \in R^+$. Bisector of the exterior angle between $\vec{a}$ & $\vec{b}$ is $\lambda(\hat{a} - \hat{b})$, $\lambda \in R^+$

9. VECTOR PRODUCT OF TWO VECTORS :

(i) If $\vec{a}$ & $\vec{b}$ are two vectors & θ is the angle between them then $\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ & $\vec{b}$ such that $\vec{a}$, $\vec{b}$ & $\vec{n}$ forms a right handed screw system .

(ii) Lagranges Identity : for any two vectors $\vec{a}$ & $\vec{b}$; $(\vec{a}\times\vec{b})^2 = |\vec{a}|\,|\vec{b}|^2 - (\vec{a}\,.\,\vec{b})^2 = \begin{vmatrix} \vec{a}\,.\,\vec{a} & \vec{a}\,.\,\vec{b} \\ \vec{a}\,.\,\vec{b} & \vec{b}\,.\,\vec{b} \end{vmatrix}$

(iii) Formulation of vector product in terms of scalar product : The vector product $\vec{a}\times\vec{b}$ is the vector $\vec{c}$, such that

(i) $|\vec{c}| = \sqrt{\vec{a}^2\,\vec{b}^2 - (\vec{a}\,.\,\vec{b})^2}$ **(ii)** $\vec{c}\,.\,\vec{a} = 0$; $\vec{c}\,.\,\vec{b} = 0$ and

(iii) $\vec{a}, \vec{b}, \vec{c}$ form a right handed system

(iv) $\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a}\,\&\,\vec{b}$ are parallel (collinear) $(\vec{a}\neq 0$, $\vec{b}\neq 0)$ i.e. $\vec{a} = K\vec{b}$, where K is a scalar

- $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$ (not commutative)
- $(m\vec{a})\times\vec{b} = \vec{a}x\,(m\vec{b}) = m\,(a\times\vec{b})$ where m is a scalar.
- $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$ (distributive)
- $i\times i = j\times j = k\times k = 0$ $i\times j = k,\ j\times k = i,\ k\times i = j$

(v) If $\vec{a} = a_1 i + a_2 j + a_3 k$ & $\vec{b} = b_1 i + b_2 j + b_3 k$ then $a\times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(vi) Geometrically $|\vec{a}\times\vec{b}|$ = area of the parallelogramwhose two adjacent sides are represented by $\vec{a}$ & $\vec{b}$.

(vii) Unit vecor perpendicular to the plane of $\vec{a}$ & $\vec{b}$ is $\hat{n} = \pm\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

- A vector of magnitude 'r' & perpendiculr to the plane of $\vec{a}$ & $\vec{b}$ is $\pm\dfrac{r\,(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$
- If θ is the angle between $\vec{a}$ & $\vec{b}$ then $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}$

(viii) Vector area

- If $\vec{a}$, $\vec{b}$ & $\vec{c}$ are the pv's of 3 points A, B & C then the vector area of triangle $ABC = \dfrac{1}{2}\left[\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right]$. The points A, B & C are collinear if $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 0$
- Area of any quadrilateral whose diagonal vectors are $\vec{d}_1$ & $\vec{d}_2$ is given by $\dfrac{1}{2}|\vec{d}_1\times\vec{d}_2|$

10. SHORTEST DISTANCE BETWEEN TWO LINES :

If two lines in space intersect at a point, then odviously the shortest distance between them is zero. Lines which do not intersect & are also not parallel are called **SKEW LINES.** For Skew lines the direction of the shortest distance vector would be perpendicular to both the lines. The magnitude of the shortest distance vector would be equal to that of the projection of $\vec{AB}$ along the direction of the line of shortest distance , $\vec{LM}$ is parallel to $\vec{p}\times\vec{q}$

i.e. $\vec{LM} = \left|\text{Projection of } \vec{AB} \text{ on } \vec{LM}\right| = \left|\text{Projection of } \vec{AB} \text{ on } \vec{p}\times\vec{q}\right| = \left|\frac{\vec{AB}\,.\,(\vec{p}\times\vec{q})}{\vec{p}\times\vec{q}}\right| = \left|\frac{(\vec{b}-\vec{a})\,.\,(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

1. The two lines directed along $\vec{p}$ & $\vec{q}$ will intersect only if shortest distance = 0 i.e. $(\vec{b}-\vec{a})\,.\,(\vec{p}\times\vec{q}) = 0$ i.e. $(\vec{b}-\vec{a})$ lies in the plane containing $\vec{p}$ & $\vec{q}$. $\Rightarrow \left|\left(\vec{b}-\vec{a}\right)\vec{p}\,\vec{q}\right| = 0$

2. If two lines are given by $\vec{r_1} = \vec{a}_1 + K\vec{b}$ & $\vec{r_2} = \vec{a}_2 + K\vec{b}$ i.e. they are parallel then, $d = \left|\frac{\vec{b}\times(\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$

11. SCALAR TRIPLE PRODUCT / BOX PRODUCT / MIXED PRODUCT

☞ The scalar triple product of three vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ is defined as :

$\vec{a}\times\vec{b}\,.\,\vec{c} = |\vec{a}|\,|\vec{b}|\,|\vec{c}|\,\sin\theta\,\cos\phi$ where θ is the angle between $\vec{a}$ & $\vec{b}$ & ϕ is the angle between $\vec{a}\times\vec{b}$ & $\vec{c}$. It is also defined as $[\vec{a}\,\vec{b}\,\vec{c}]$, spelled as box product.

☞ Scalar triple product geometrically respresents the volume of the parallelopiped whose three couterminous edges are represented by $\vec{a}\times\vec{b}$ & $\vec{c}$ i. e. $V = [\vec{a}\,\vec{b}\,\vec{c}]$

☞ In a scalar triple product the position of dot & cross can be interchanged i.e.

$\vec{a}\,.\,(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\,.\,\vec{c}$ OR $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$

☞ $\vec{a}\,.\,(\vec{b}\times\vec{c}) = -a\,.(\vec{c}\times\vec{b})$ i.e. $[\vec{a}\,\vec{b}\,\vec{c}] = -[\vec{a}\,\vec{c}\,\vec{b}]$

If $\vec{a} = a_1 i + a_2 j + a_3 k$; $\vec{b} = b_1 i + b_2 j + b_3 k$ & $\vec{c} = c_1 i + c_2 j + c_3 k$ then $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

In general , if $\vec{a} = a_1\vec{l} + a_2\vec{m} + a_3\vec{n}$; $\vec{b} = b_1\vec{l} + b_2\vec{m} + b_3\vec{n}$; & $\vec{c} = c_1\vec{l} + c_2\vec{m} + c_3\vec{n}$

then $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} [\vec{l}\,\vec{m}\,\vec{n}]$; where $\vec{l}$, $\vec{m}$ & $\vec{n}$ are non coplanar vectors.

☞ If $\vec{a}, \vec{b}, \vec{c}$ are coplanar $\Leftrightarrow$ $[\vec{a}\,\vec{b}\,\vec{c}] = 0$

☞ Scalar product of three vectors, two of which are equal or parallel is 0 i.e. $[\vec{a}\,\vec{b}\,\vec{c}] = 0$

Note : If $\vec{a}, \vec{b}, \vec{c}$ are non- coplanar then $[\vec{a}\,\vec{b}\,\vec{c}] > 0$ for right handed system & $[\vec{a}\,\vec{b}\,\vec{c}] < 0$ for left handed system.

☞ $[i\ j\ k] = 1$ $[K\ \vec{a}\ \vec{b}\ \vec{c}] = K[\vec{a}\ \vec{b}\ \vec{c}]$ $[(\vec{a}+\vec{b})\ \vec{c}\ \vec{d}] = [\vec{a}\ \vec{c}\ \vec{d}] + [\vec{b}\ \vec{c}\ \vec{d}]$

The Volume of the tetrahedron OABC with O as origin & the pv's of A, B and C being $\vec{a}, \vec{b}$ & $\vec{c}$ are given by $V = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

The position vector of the centroid of of a tetrahedron if the pv's of its angular vertices are $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are given by $\frac{1}{4}[\vec{a}+\vec{b}+\vec{c}+\vec{d}]$

Note that this is also the point of concurrency of the lines joining the vertices to the centroids of the opposite faces and is also called ther centre of the tetrahedron. In case the tetrahedron is regular it is equidistant from the vertices and the four faces of the tetrahedron.

Remember that : $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] = 0$ & $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$ emember that

12. VECTOR TRIPLE PRODUCT :

Let $\vec{a}, \vec{b}$ & $\vec{c}$ be any three vectors, then that expression $\vec{a} \times (\vec{b} \times \vec{c})$ is a vector & is called a vector triple product.

GEOMETRICAL INTERPRETATION OF $\vec{a} \times (\vec{b} \times \vec{c})$

Consider the expression $\vec{a} \times (\vec{b} \times \vec{c})$ which itself is a vector, since it is a cross product of two vectors $\vec{a}$ & $(\vec{b} \times \vec{c})$. Now $\vec{a} \times (\vec{b} \times \vec{c})$ is vector perpendicular to the plane containing $\vec{a}$ & $(\vec{b} \times \vec{c})$ but $(\vec{b} \times \vec{c})$ is a vector perpendicular to the plane $\vec{b}$ & $\vec{c}$, therefore $a \times (\vec{b} \times \vec{c})$ is vector lies in the plane of $\vec{b}$ & $\vec{c}$ and perpendicular to $\vec{a}$. Hence we can express $a \times (\vec{b} \times \vec{c})$ in terms of $\vec{b}$ & $\vec{c}$ i.e. $a \times (\vec{b} \times \vec{c}) = x\vec{b} + y\vec{c}$ where x & y are scalars .

☞ $a \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$ ☞ $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$

☞ $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$

13. LINEAR COMBINATIONS / LINEARLY INDEPENDENCE AND DEPENDENCE OF VECTORS :

Given a finite set of vectors $\vec{a}, \vec{b}, \vec{c}$ **,........** then the vector $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c} + ...$ is called a linear combination of $\vec{a}, \vec{b}, \vec{c}$ **,........** for any x, y, z......... $\in R$. We have the following results.

(a) FUNDAMENTAL THEOREM IN PLANE : Let $\vec{a}\ \vec{b}$ be non zero, non collinear vectors. Then any vector $\vec{r}$ coplanar with $\vec{a}, \vec{b}$ can be expressed uniquely as a linear combination of $\vec{a}, \vec{b}$ i.e. There exist some unique $x, y \in R$ such that $x\vec{a} + y\vec{b} = \vec{r}$

(b) FUNDAMENTAL THEOREM IN SPACE : Let $\vec{a}, \vec{b}, \vec{c}$ be non-zero, non-coplanar vectors in space. Then any vector $\vec{r}$, can be uniquily expressed as a linear combination of $\vec{a}, \vec{b}, \vec{c}$ i.e. There exist some unique $x, y \in R$ such that $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$.

(c) If $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ are n non zero vectors, & $k_1, k_2, ...k_n$ are n scalars & if the linear combination $k_1\vec{x}_1, k_2\vec{x}_2 + ...k_n\vec{x}_n = 0 \Rightarrow k_1 = 0, k_2 = 0.....k_n = 0$ then we say that vectors $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ are **LINEARLY INDEPENDENT VECTORS**

(d) If $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ are not **LINEARLY INDEPENDENT** then they are said to be LINEAR DEPENDENT vectors. i.e. if $k_1\vec{x}_1, k_2\vec{x}_2 + ...k_n\vec{x}_n = 0$ & if there exists at least one $k_r \neq 0$ then $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ are said to be **LINEARLY DEPENDENT.**

Note : ☞ If $\vec{a} = 3i + 2j + 5k$ then $\vec{a}$ is expressed as a **LINEAR COMBINATION** of vectors i, j, k . Also, $\vec{a}$, i, j, k form a linearly dependent set of vectors. In general, every set of four vectors is a linearly dependent system.

☞ i, j, k are LINEARLY INDEPENDENT set of vectors. For $K_1 i + K_2 j + K_3 k = 0 \Rightarrow K_1 = 0 = K_2 = K_3$

☞ Two vectors $\vec{a}$ & $\vec{b}$ are linearly dependent $\Rightarrow \vec{a}$ is a parallel to $\vec{b}$ i,.e. $\vec{a} \times \vec{b} = 0 \Rightarrow$ linear dependence of $\vec{a}$ & $\vec{b}$. Conversely if $\vec{a} \times \vec{b} \neq 0$ then $\vec{a}$ & $\vec{b}$ are linearly independent.

☞ If three vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent , then they are coplanar i.e. $[\vec{a}, \vec{b}, \vec{c}]$ =0, conversely, if $[\vec{a}, \vec{b}, \vec{c}] \neq 0$, then the vectors are linearly independent.

14. COPLANARITY OF VECTORS :

Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are coplanar if and only if there exist scalars x, y, z, w not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$
where, x + y + z + w = 0

15. RECIPROCAL SYSTEM OF VECTORS :

If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{a}', \vec{b}', \vec{c}'$ are two sets of non coplanar vectors such that $\vec{a} . \vec{a}' = \vec{b} . \vec{b}' = \vec{c} . \vec{c}' = 1$ then the two systems are called Reciprocal System of vectors.

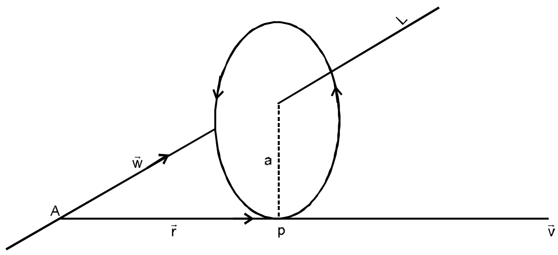
Note : $a' = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $b' = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $c' = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

16. EQUATION OF A PLANE :

(a) The equation $(\vec{r} - \vec{r}_0) . \vec{n} = 0$ reprenents a plane containing the points with p.v. $\vec{r}_0$ where $\vec{n}$ is a vector normal to the plane . $\vec{r} . \vec{n} = d$ is the general equation of a plane.

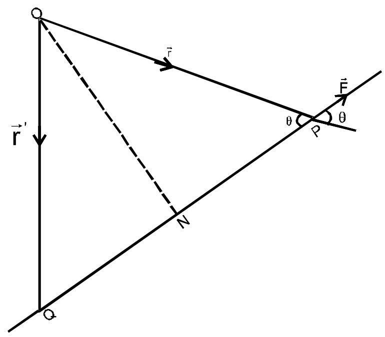
(b) Angle between the 2 planes is the angle between 2 normals drawn to the planes and the angle between a line and the plane is the compliment of the angle between the line and the normal to the plane.

17. APPLICATION OF VECTORS :



(a) Work done against a constant force $\vec{F}$ over a displacement $\vec{s}$ is defined as $W = \vec{F} . \vec{s}$

(b) The tangential velocity $\vec{V}$ of a body moving in a circle is given by $\vec{V} = \vec{w} \times \vec{r}$ where $\vec{r}$ is the pv of the point P .



(c) The moment of $\vec{F}$ about 'O' is defined as $\vec{M} = \vec{r} \times \vec{F}$ where $\vec{r}$ is the pv of P wrt 'O'.
The direction of $\vec{M}$ is along to the normal to the plane OPN such that $\vec{r}, \vec{F}, \& \vec{M}$ form a right handed system .

(d) Moment of couple = $(\vec{r}_1 - \vec{r}_2) \times \vec{F}$ where r_1 & r_2 are pv's of the point of the application of the forces $\vec{F}$ & $-\vec{F}$

3-D CO-ORDINATE GEOMETRY

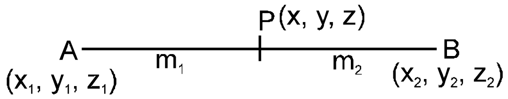
USEFUL RESULTS

A General :

(1) Distance (d) between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)

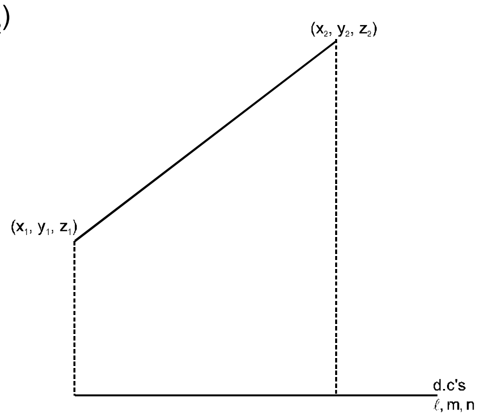
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(2) SECTION FORMULA



$$x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2} \quad ; \quad y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2} \quad ; \quad z = \frac{m_2 z_1 + m_1 z_2}{m_1 + m_2}$$

(For external division take -ve sign of either m_1 or m_2)



(3) Direction cosine of a line has the same meaning as d.c's of a vector.

(a) Any three numbers a, b, c propotional to the direction cosines are called the direction ratios i.e.

$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$ same sign either +ve or -ve should be taken through out.

note that d.r's of a line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) are propotional to $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$

(b) If θ is the angle between the two lines whose d.c's are l_1, m_1, n_1 and l_2, m_2, n_2

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ hence if lines are perpendicular then $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

If lines are parallel then $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ note that if three lines are coplanar then $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$

(4) Projection of the join of two points on a line with d.c's l, m, n are

$l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

B PLANE

(i) General equation of degree one in x, y, z i.e. ax + by + cz + d = 0 represents a plane,

(ii) Equation of a plane passing through (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ where a, b, c are the direction ratio of the normal to the plane

(iii) Equation of a plane if its intercepts on the co-ordinate axes are x_1, y_1, z_1 is $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$

(iv) Equation of a plane if the length of the perpendicular from the origin on the plane is p and d.c's of the perpedicular as l, m, n is $lx + my + nz = p$

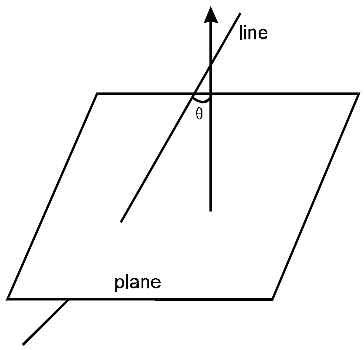
(v) Parallel and perpendicular planes - Two planes $a_1x + b_1y + c_1z + d_1 = 0$ and are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and

coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$

(vi) Angle between a plane and a line is the compliment of the angle between the normal to the plane and the line. If $\left.\begin{array}{l}\text{Line } : \vec{r} = \vec{a} + \lambda\vec{b} \\ \text{Plane} : \vec{r}\,.\,\vec{n} = d\end{array}\right]$ then $\cos(90 - \theta) = \sin\theta = \frac{\vec{b}\,.\,\vec{n}}{|\vec{b}|\,.\,|\vec{n}|}$

Where θ is the angle between the line and normal to the plane.



(vii) Length of the perpendicular from a point (x_1, y_1, z_1) to a plane ax + by + cz +d = 0 is

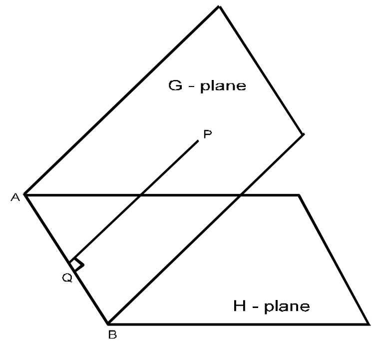
$$p = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

(viii) Distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $\left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right|$

(ix) Planes bisecting the angle between two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left|\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}\right| = \pm\left|\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

of these two bisecting planes, one bisects the acute and the other obtuse angle between the given planes.



(x) Equation of a plane through the intersection of two planes P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$

C STRAIGHT LINE IN SPACE

(i) Equation of a line through A (x_1, y_1, z_1) and having direction cosines l, m, n are $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ and the lines through (x_1, y_1, z_1) and (x_2, y_2, z_2) $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

(ii) Intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ together represent the usymmetrical form of the straight line.

(iii) General equation of the plane containing the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is

$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$ where $Al + Bm + Cn = 0$

LINE OF GREATEST SLOPE

AB is the line of intersection of G-plane and H is the horizontal plane, Line of greatest slope on a given plane, drawn through a given point on the plane, is the line through the point 'P' perpendicular to the line of intersection of the given plane with any horizontal plan

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

*** Marked Questions are having more than one correct option.***
Section (A) : Basic, section formula & Direction ratios

A-1. If the vector $\vec{b}$ is collinear with the vector $\vec{a} = (2\sqrt{2}, -1, 4)$ and $|\vec{b}|$ = 10, then:

(A) $\vec{a} \pm \vec{b} = 0$ (B) $\vec{a} \pm 2\vec{b} = 0$ (C) $2\vec{a} \pm \vec{b} = 0$ (D) none

A-2. OABCDE is a regular hexagon of side 2 units in the XY–plane. O being the origin and OA taken along the X–axis. A point P is taken on a line parallel to Z–axis through the centre of the hexagon at a distance of 3 units from O. Then vector $\overrightarrow{AP}$ is:

(A) $-\hat{i} + 3\hat{j} + \sqrt{5}\hat{k}$ (B) $\hat{i} - \sqrt{3}\hat{j} + 5\hat{k}$ (C) $-\hat{i} + \sqrt{3}\hat{j} + \sqrt{5}\hat{k}$ (D) $\hat{i} + \sqrt{3}\hat{j} + \sqrt{5}\hat{k}$

A-3. If the sum of the squares of the distances of a point from the three coordinate axes be 36, then its distance from the origin is

(A) 6 (B) $3\sqrt{2}$ (C) $2\sqrt{3}$ (D) $6\sqrt{2}$

A-4. A line makes angles α, β, γ with the coordinate axes. If $\alpha + \beta = 90°$, then γ =
(A) 0 (B) 90° (C) 180° (D) None of these

A-5. 'P' is a point inside the triangle ABC, such that $BC\left(\overrightarrow{PA}\right) + CA\left(\overrightarrow{PB}\right) + AB\left(\overrightarrow{PC}\right) = 0$, then for the tiangle ABC the point P is its :
(A) incentre (B) circumcentre (C) centroid (D) orthocentre

Section (B) : Dot and cross product

B-1. If $|\vec{a}| = 5$, $|\vec{a} - \vec{b}| = 8$ and $|\vec{a} + \vec{b}| = 10$, then $|\vec{b}|$ is equal to :

(A) 1 (B) $\sqrt{57}$ (C) 3 (D) none of these

B-2. If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then the angle between $\vec{a}$ & $\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{2\pi}{3}$ (C) $\frac{5\pi}{3}$ (D) $\frac{\pi}{3}$

B-3. Angle between diagonals of a parallelogram whose side are represented by $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$ (C) $\cos^{-1}\left(\frac{4}{9}\right)$ (D) $\cos^{-1}\left(\frac{5}{9}\right)$

B-4. If Vector $\vec{a}$ of magnitude 50 is collinear with vectors $\vec{b} = 6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}$ and makes an acute angle with positive z-axis then :

(A) $\vec{a} = 4\vec{b}$ (B) $\vec{a} = -4\vec{b}$ (C) $\vec{b} = 4\vec{a}$ (D) none

B-5. Let $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a}+\hat{b}+\hat{c}$ is also a unit vector. If pairwise angle between $\hat{a}, \hat{b}, \hat{c}$ are θ_1, θ_2 and θ_3 respectively then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$ equals

(A) 3 (B) -3 (C) 1 (D) –1

B-6. The two vectors $(x^2-1)\hat{i}+(x+2)\hat{j}+x^2\hat{k}$ & $2\hat{i}-x\hat{j}+3\hat{k}$ are orthogonal :

(A) for no real value of x (B) for x = –1

(C) for $x = \frac{1}{2}$ (D) for $x = -\frac{1}{2}$ & x = 1

B-7*. The vector $\frac{1}{3}\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ is:

(A) a unit vector

(B) makes an angle $\frac{\pi}{3}$ with the vector $2\hat{i}-4\hat{j}+3\hat{k}$

(C) parallel to the vector $-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}$

(D) perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

B-8. The area of the triangle whose vertices are A (1, -1, 2) ; B (2, 1,-1) ; C (3, -1,2) is :

(A) $\sqrt{13}$ (B) $2\sqrt{13}$ (C) 13 (D) none

B-9. Vectors $\vec{a}$ & $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. If $|\vec{a}|=1$, $|\vec{b}|=2$ then $\left\{(\vec{a}+3\vec{b})x(3\vec{a}-\vec{b})\right\}^2 =$

(A) 225 (B) 250 (C) 275 (D) 300

B-10. The coordinates of the points A, B, C, D are (4, α, 2), (5, –3, 2), (β, 1, 1) & (3, 3, –1). Line AB would be perpendicular to line CD when

(A) α = –1, β = –1 (B) α = 1, β = 2 (C) α = 2, β = 1 (D) α = 2, β = 2

B-11. A, B, C & D are four points in a plane with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ respectively such that

$\left(\vec{a}-\vec{d}\right).\left(\vec{b}-\vec{c}\right)=\left(\vec{b}-\vec{d}\right).\left(\vec{c}-\vec{a}\right)$ = 0. Then for the triangle ABC, D is its:

(A) incentre (B) circumcentre (C) orthocentre (D) centroid

B-12. Unit vector perpendicular to the plane of the triangle ABC with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ of the vertices A, B, C is:

(A) $\frac{\left(\vec{a}x\vec{b}+\vec{b}x\vec{c}+\vec{c}x\vec{a}\right)}{\Delta}$

(B) $\frac{\left(\vec{a}x\vec{b}+\vec{b}x\vec{c}+\vec{c}x\vec{a}\right)}{2\Delta}$

(C) $\frac{\left(\vec{a}x\vec{b}+\vec{b}x\vec{c}+\vec{c}x\vec{a}\right)}{4\Delta}$

(D) none of these

(where Δ is the area of the triangle ABC).

Section (C) : STP, VTP & Tetrahedron

C-1. The value of $\left[\left(\vec{a}+2\vec{b}-\vec{c}\right), \left(\vec{a}-\vec{b}\right), \left(\vec{a}-\vec{b}-\vec{c}\right)\right]$ is equal to the box product:

(A) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (B) $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (C) $3\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (D) $4\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

C-2. The set of values of m for which the vectors $\hat{i}+\hat{j}+m\hat{k}, \hat{i}+\hat{j}+(m+1)\hat{k}$ & $(\hat{i}-\hat{j}+m\hat{k})$ are non coplanar is:

(A) R (B) R - {1} (C) R - {2} (D) ϕ

C-3. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}; \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}; \vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ & $\vec{b}$. If the angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{6}$ then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 =$

(A) 0 (B) 1

(C) $\frac{1}{4}$ $(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$ (D) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)(c_1^2+c_2^2+c_3^2)$

C-4. For no-zero vectos $\vec{a}, \vec{b}, \vec{c}$, $[\,\vec{a}\ \vec{b}\ \vec{c}\,] = |\vec{a}|\ |\vec{b}|\ |\vec{c}|$ holds if and only if ;

(A) $\vec{a}\cdot\vec{b}=0,\ \vec{b}\cdot\vec{c}=0$ (B) $\vec{c}\cdot\vec{a}=0,\ \vec{a}\cdot\vec{b}=0$

(C) $\vec{a}\cdot\vec{c}=0,\ \vec{b}\cdot\vec{c}=0$ (D) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

C-5. Given unit vectors $\vec{m}, \vec{n}$ & $\vec{p}$ such that angle between $\vec{m}$ & $\vec{n}$ = angle between $\vec{p}$ and $(\vec{m}\times\vec{n})=\frac{\pi}{6}$ then $[\vec{n}\ \vec{p}\ \vec{m}]=$

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) none

C-6. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$, then the value of $\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$ is equal to :

(A) 2 (B) 4 (C) 16 (D) 64

C-7. Let $\vec{a} = x\hat{i}+12\hat{j}-\hat{k}$, $\vec{b} = 2\hat{i}+2x\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}+\hat{k}$. If the ordered set $\left[\vec{b}\ \vec{c}\ \vec{a}\right]$ is left handed, then :

(A) $x \in (2, \infty)$ (B) $x \in (-\infty, -3)$ (C) $x \in (-3, 2)$ (D) $x \in \{-3, 2\}$

C-8. The three vectors $\hat{i}+2\hat{j}+\hat{k}, a\hat{i}+\hat{j}+2\hat{k}$ & $\hat{i}+2\hat{j}+a\hat{k}$ are coplanar :

(A) for no real value of 'a' (B) for all real values of 'a'

(C) for one real value of 'a' (D) for two real values of 'a'

C-9. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three ?

(A) $\vec{u}\cdot(\vec{v}\times\vec{w})$ (B) $(\vec{v}\times\vec{w})\cdot\vec{u}$ (C) $\vec{v}\cdot(\vec{u}\times\vec{w})$ (D) $(\vec{u}\times\vec{v})\cdot\vec{w}$

C-10*. Which of the following expressions are meaningful ?

(A) $\vec{u}.(\vec{v}\times\vec{w})$ (B) $(\vec{u}.\vec{v}).\vec{w}$ (C) $(\vec{u}.\vec{v})\vec{w}$ (D) $\vec{u}\times(\vec{v}.\vec{w})$

C-11. Vector of length 3 unit which is perpendicular to $\hat{i}+\hat{j}+\hat{k}$ and lies in the plane of $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}$

(A) $\frac{3}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (B) $\frac{3}{\sqrt{6}}(2\hat{i}-\hat{j}-\hat{k})$ (C) $\frac{3}{\sqrt{114}}(8\hat{i}-7\hat{j}-\hat{k})$ (D) $\frac{3}{\sqrt{114}}(-7\hat{i}+8\hat{j}-\hat{k})$

C-12. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are linearly independent vectors, then which one of the following set of vectors is linearly dependent?

(A) $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ (B) $\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}$ (C) $\vec{a}x\vec{b}, \vec{b}x\vec{c}, \vec{c}x\vec{a}$ (D) none

C-13. Given the vertices A (2, 3, 1), B (4, 1, –2), C (6, 3, 7) & D (–5, –4, 8) of a tetrahedron. The length of the altitude drawn from the vertex D is:

(A) 7 (B) 9 (C) 11 (D) none of these

C-14. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be the unit vectors such that $\vec{b}$ is not parallel to $\vec{c}$ and $\vec{a}\ x\ (2\vec{b}\ x\ \vec{c}) = \vec{b}$, then the angle that $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$ are respectively:

(A) $\frac{\pi}{3}$ & $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ & $\frac{2\pi}{3}$ (C) $\frac{\pi}{2}$ & $\frac{2\pi}{3}$ (D) $\frac{\pi}{2}$ & $\frac{\pi}{3}$

C-15. If $\vec{A}=(1,1,1), \vec{C}=(0,1,-1)$ are given vectors, then a vector $\vec{B}$ satisfying the equation $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}\cdot\vec{B}=3$ is :

(A) (5, 2, 2) (B) $\left(\frac{5}{3},\frac{2}{3},\frac{2}{3}\right)$ (C) $\left(\frac{2}{3},\frac{5}{3},\frac{2}{3}\right)$ (D) $\left(\frac{2}{3},\frac{2}{3},\frac{5}{3}\right)$

C-16. Unit vector parallel to x – y plane and perpendicular to $4\hat{i}-3\hat{j}+\hat{k}$ is ________.

(A) $\pm\frac{4\hat{i}+3\hat{j}}{5}$ (B) $\pm\frac{3\hat{i}+4\hat{j}}{5}$ (C) $\pm\frac{\hat{i}+\hat{k}}{\sqrt{2}}$ (D) $\pm\frac{3\hat{i}-4\hat{j}}{5}$

C-17. If $\vec{b}$ and $\vec{c}$ are two non-collinear vectors such that $\vec{a} \parallel (\vec{b}\times\vec{c})$, then $(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c})$ is equal to

(A) $\vec{a}^2(\vec{b}.\vec{c})$ (B) $\vec{b}^2(\vec{a}.\vec{c})$ (C) $\vec{c}^2(\vec{a}.\vec{b})$ (D) none of these

C-18. If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are linearly independent set of vectors and $K_1\vec{a}+K_2\vec{b}+K_3\vec{c}+K_4\vec{d}=0$, then

(A) $K_1+K_2+K_3+K_4=0$ (B) $K_1+K_3=K_2+K_4=0$
(C) $K_1+K_4=K_2+K_3=0$ (D) none of these

C-19. If $\vec{a}=2\hat{i}+\lambda\hat{j}+3\hat{k}; \vec{b}=3\hat{i}+3\hat{j}+5\hat{k}; \vec{c}=\lambda\hat{i}+2\hat{j}+2\hat{k}$ are linearly dependent vectors then the number of possible values of λ is

(A) 0 (B) 1 (C) 2 (D) more then 2

C-20. Vector $\vec{x}$ satisfying the relation $\vec{A}.\vec{x}=c$ and $\vec{A}\times\vec{x}=\vec{B}$ is

(A) $\frac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|}$ (B) $\frac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|^2}$ (C) $\frac{c\vec{A}+(\vec{A}\times\vec{B})}{|\vec{A}|^2}$ (D) $\frac{c\vec{A}-2(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

C-21. Let $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{r}_1 = \vec{a} - \vec{b} + \vec{c}, \vec{r}_2 = \vec{b} + \vec{c} - \vec{a}, \vec{r}_3 = \vec{c} + \vec{a} + \vec{b}, \vec{r} = 2\vec{a} - 3\vec{b} + 4\vec{c}$

$\vec{r} = \lambda_1\vec{r}_1 + \lambda_2\vec{r}_2 + \lambda_3\vec{r}_3$, then the values of λ_1, λ_2 and λ_3 respectively are

(A) 7, 1, – 4 (B) 7/2 , 1, –1/2 (C) 5/2, 1, 1/2 (D) –1/2 , 1, 7/2

C-22 If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then the vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are:

(A) collinear (B) linearly independent
(C) perpendicular (D) parallel

Section (D) : 3-D, Plane and line

D-1. The sum of the squares of direction cosines of a straight line is

(A) zero (B) two (C) one (D) none of these

D-2. Which one of the following is best condition for the plane ax + by + cz + d = 0 to intersect the x and y axes at equal angle

(A) |a| = |b| (B) a = –b (C) a = b (D) $a^2 + b^2 = 1$

D-3. If P (2, 3, –6) and Q(3, –4, 5) are two points, the direction cosines of line PQ are

(A) $-\frac{1}{\sqrt{171}}, -\frac{7}{\sqrt{171}}, -\frac{11}{\sqrt{171}}$ (B) $\frac{1}{\sqrt{171}}, -\frac{7}{\sqrt{171}}, \frac{11}{\sqrt{171}}$

(C) $\frac{1}{\sqrt{171}}, \frac{7}{\sqrt{171}}, -\frac{11}{\sqrt{171}}$ (D) $-\frac{7}{\sqrt{171}}, -\frac{1}{\sqrt{171}}, \frac{11}{\sqrt{171}}$

D-4. The ratio in which yz-plane divide the line joining the points A(3, 1, –5) and B(1, 4, –6) is-

(A) –3 : 1 (B) 3 : 1 (C) –1: 3 (D) 1 : 3

D-5. A straight line is inclined to the axes of x and z at angles 45° and 60° respectively, then the inclination of the line to the y-axis is

(A) 30° (B) 45° (C) 60° (D) 90°

D-6. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is :

(A) $-\hat{i} + \hat{j} + 2\hat{k}$ (B) $3\hat{i} - \hat{j} + \hat{k}$ (C) $3\hat{i} + \hat{j} - \hat{k}$ (D) $\hat{i} - \hat{j} - \hat{k}$

D-7. If a line has a vector equation $\vec{r} = 2\hat{i} + 6\hat{j} + \lambda\left(\hat{i} - 3\hat{j}\right)$, then which of the following statements hold good?

(A) the line is parallel to $2\hat{i} + 6\hat{j}$ (B) the line passes through the point $3\hat{i} + 3\hat{j}$

(C) the line passes through the point $\hat{i} + 9\hat{j}$ (D) the line is parallel to XY-plane

D-8. The equation of a plane which passes through (2, – 3, 1) & is normal to the line joining the points (3, 4, – 1) & (2, – 1, 5) is given by:

(A) x + 5y – 6z + 19 = 0 (B) x – 5y + 6z – 19 = 0
(C) x + 5y + 6z + 19 = 0 (D) x – 5y – 6z – 19 = 0

D-9. The coordinates of the point of intersection of the line $\frac{x+1}{1}=\frac{y+3}{3}=\frac{z+2}{-2}$ with the plane 3x + 4y + 5z = 5

(A) (5, 15, –14) (B) (3, 4, 5) (C) (1, 3, –2) (D) (3, 12, –10)

D-10. Perpendicular is drawn from the point (0, 3, 4) to the plane 2x – 2y + z = 10. The coordinates of the foot of the perpendicular are

(A) $\left(-\frac{8}{3},\frac{1}{3},\frac{16}{3}\right)$ (B) $\left(\frac{8}{3},\frac{1}{3},\frac{16}{3}\right)$ (C) $\left(\frac{8}{3},-\frac{1}{3},\frac{16}{3}\right)$ (D) $\left(\frac{8}{3},\frac{1}{3},-\frac{16}{3}\right)$

D-11. The equation of the plane through the line of intersection of the planes 2x + y – z – 4 = 0 and 3x + 5z – 4 = 0 which cuts off equal intercepts from the x-axis and y-axis is

(A) 3x + 3y –8z + 8 = 0
(B) 3x + 3y –8z – 8 = 0
(C) 3x – 3y –8z – 8 = 0
(D) x + y – 8z – 8 = 0

D-12. The symmetric form of the equation of the line x + y – z = 1, 2x – 3y + z = 2 is

(A) $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ (B) $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{5}$ (C) $\frac{x}{2}=\frac{y-1}{3}=\frac{z}{5}$ (D) $\frac{x-1}{2}=\frac{y}{3}=\frac{z}{5}$

D-13. The line $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ is parallel to the plane

(A) 2x + y + 2z + 3 = 0
(B) 2x – y – 2z = 3
(C) 21x – 12y + z = 0
(D) 2x + y – 2z = 0

D-14. Equation of straight line which passes through the point P(1, 0, –3) and Q(–2, 1, –4) is

(A) $\frac{x-2}{-3}=\frac{y+1}{1}=\frac{z-4}{-1}$
(B) $\frac{x-1}{3}=\frac{y}{1}=\frac{z+3}{1}$
(C) $\frac{x-1/2}{-3}=\frac{y-1}{1}=\frac{z+4}{-1}$
(D) $\frac{x-1}{-3}=\frac{y}{1}=\frac{z+3}{-1}$

D-15. The straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{-2}$ are

(A) Parallel lines
(B) Intersecting at 60º
(C) Skew lines
(D) Intersecting at right angle

D-16*. The equation of the line x + y + z – 1 = 0, 4x + y – 2z + 2 = 0 written in the symmetrical form is

(A) $\frac{x+1}{1}=\frac{y-2}{-2}=\frac{z-0}{1}$
(B) $\frac{x}{1}=\frac{y}{-2}=\frac{z-1}{1}$
(C) $\frac{x+1/2}{1}=\frac{y-1}{-2}=\frac{z-1/2}{1}$
(D) $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z-2}{2}$

D-17*. Let a perpendicular PQ be drawn from P(5, 7, 3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ where Q is the foot of perpendicular, then

(A) Q is (9, 13, 15)

(B) PQ = 14

(C) the equation of plane containing PQ and the given line is $9x-4y-z-14=0$

(D) none of these

D-18. The equation of the plane passing through the point (1, – 3, – 2) and perpendicular to planes $x+2y+2z=5$ and $3x+3y+2z=8$, is

(A) $2x-4y+3z-8=0$ (B) $2x-4y-3z+8=0$

(C) $2x+4y+3z+8=0$ (D) None of these

D-19. The reflection of the point (2, –1, 3) in the plane $3x-2y-z=9$ is :

(A) $\left(\frac{26}{7},\frac{15}{7},\frac{17}{7}\right)$ (B) $\left(\frac{26}{7},\frac{-15}{7},\frac{17}{7}\right)$ (C) $\left(\frac{15}{7},\frac{26}{7},\frac{-17}{7}\right)$ (D) $\left(\frac{26}{7},\frac{17}{7},\frac{-15}{7}\right)$

D-20. The distance of the point, (– 1, –5, – 10) from the point of intersection of the line, $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane, $x-y+z=5$, is:

(A) 10 (B) 11 (C) 12 (D) 13

D-21. The distance of the point (1, – 2, 3) from the plane $x-y+z=5$ measured parallel to the line, $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$, is:

(A) 1 (B) 6/7 (C) 7/6 (D) None of these

D-22. The condition that the line $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ lies in the plane $ax+by+cz+d=0$ is

(A) $ax_1+by_1+cz_1+d=0$ and $a\ell+bm+cn\neq 0$

(B) $a\ell+bm+cn=0$ and $ax_1+by_1+cz_1+d\neq 0$

(C) $ax_1+by_1+cz_1+d=0$ and $a\ell+bm+cn=0$

(D) $ax_1+by_1+cz_1=0$ and $a\ell+bm+cn=0$

D-23. The points (0, –1, –1), (–4, 4, 4), (4, 5, 1) and (3, 9, 4) are

(A) collinear (B) coplanar (C) forming a square (D) none of these

D-24. The equation of the plane bisecting the acute angle between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$

(A) $23x-13y+32z+45=0$ (B) $5x-y-4z=3$

(C) $5x-y-4z+45=0$ (D) $23x-13y+32z+3=0$

D-25. The equation of the plane containing the line $\frac{x-\alpha}{\ell}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is $a(x-\alpha)+b(y-\beta)+c(z-\gamma)=0$, where $a\ell+bm+cn$ is equal to-

(A) 1 (B) –1 (C) 2 (D) 0

D-26. The shortest distance between the two straight lines $\frac{x-4/3}{2}=\frac{y+6/5}{3}=\frac{z-3/2}{4}$ and $\frac{5y+6}{8}=\frac{2z-3}{9}=\frac{3x-4}{5}$ is

(A) $\sqrt{29}$ (B) 3 (C) 0 (D) $6\sqrt{10}$

D-27. A straight line passes through the point (2, –1, –1). It is parallel to the plane 4x + y + z + 2 = 0 and is perpendicular to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z-5}{1}$. The equations of the straight line are

(A) $\frac{x-2}{4}=\frac{y+1}{1}=\frac{z+1}{1}$ (B) $\frac{x+2}{4}=\frac{y-1}{1}=\frac{z-1}{1}$

(C) $\frac{x-2}{-1}=\frac{y+1}{1}=\frac{z+1}{3}$ (D) $\frac{x+2}{-1}=\frac{y-1}{1}=\frac{z-1}{3}$

D-28. Equation of the plane passing through A(x_1, y_1, z_1) and containing the line

$$\frac{x-x_2}{d_1}=\frac{y-y_2}{d_2}=\frac{z-z_2}{d_3} \quad \text{is}$$

(A) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ d_1 & d_2 & d_3 \end{vmatrix}=0$ (B) $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix}=0$

(C) $\begin{vmatrix} x-d_1 & y-d_2 & z-d_3 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}=0$ (D) $\begin{vmatrix} x & y & z \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix}=0$

D-29. The acute angle that the vector $2\hat{i}-2\hat{j}+\hat{k}$ makes with the plane contained by the two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $\hat{i}-\hat{j}+2\hat{k}$ is given by:

(A) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (C) $\tan^{-1}\left(\sqrt{2}\right)$ (D) $\cot^{-1}\left(\sqrt{2}\right)$

D-30. If line $\vec{r}=(\hat{i}-2\hat{j}-\hat{k})+\lambda(2\hat{i}+\hat{j}+2\hat{k})$ is parallel to the plane $\vec{r}\cdot(3\hat{i}-2\hat{j}-m\hat{k})=14$, then the value of m is

(A) 2 (B) – 2

(C) 0 (D) can not be predicted with these informations

D-31. If a plane cuts off intercepts OA = a, OB = b, OC = c from the coordinate axes, then the area of the triangle ABC =

(A) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ (B) $\frac{1}{2}$ (bc + ca + ab)

(C) $\frac{1}{2}$ abc (D) $\frac{1}{2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}$

Section (E) : Miscellaneous

E-1. The vertices of a triangle are A (1, 1, 2), B(4, 3, 1) and C(2, 3, 5). A vector representing the internal bisector of the angle A is :

(A) $\hat{i}+\hat{j}+2\hat{k}$ (B) $2\hat{i}-2\hat{j}+\hat{k}$ (C) $2\hat{i}+2\hat{j}-\hat{k}$ (D) $2\hat{i}+2\hat{j}+\hat{k}$

E-2. The vector $\vec{c}$, directed along the internal bisector of the angle between the vector $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$, is :

(A) $\frac{5}{3}\left(\hat{i}-7\hat{j}+2\hat{k}\right)$ (B) $\frac{5}{3}\left(\hat{i}+7\hat{j}-2\hat{k}\right)$ (C) $\frac{5}{3}\left(-\hat{i}+7\hat{j}+2\hat{k}\right)$ (D) $\frac{5}{3}\left(-\hat{i}-7\hat{j}+2\hat{k}\right)$

E-3. Equation of the angle bisector of the angle between the lines $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$ & $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{-1}$ is :

(A) $\frac{x-1}{2}=\frac{y-2}{2}$; $z-3=0$ (B) $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

(C) $x-1=0$; $\frac{y-2}{1}=\frac{z-3}{1}$ (D) None of these

E-4. $\hat{a}$ and $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}$, $\hat{b}$ and $\hat{a}\times\hat{b}$ will be:

(A) $-\frac{1}{\sqrt{3}}\left(\hat{a}+\hat{b}+\hat{a}\times\hat{b}\right)$ (B) $\frac{1}{\sqrt{3}}\left(\hat{a}+\hat{b}+\hat{a}\times\hat{b}\right)$

(C) $\frac{1}{\sqrt{3}}\left(\hat{a}+\hat{b}-\hat{a}\times\hat{b}\right)$ (D) $-\frac{1}{\sqrt{3}}\left(\hat{a}+\hat{b}-\hat{a}\times\hat{b}\right)$

E-5. The locus represented by $xy + yz = 0$ is

(A) A pair of perpendicular lines (B) A pair of parallel lines

(C) A pair of parallel planes (D) A pair of perpendicular planes

E-6. A variable plane passes through a fixed point (1, 2, 3). The locus of the foot of the perpendicular drawn from origin to this plane is:

(A) $x^2+y^2+z^2-x-2y-3z=0$ (B) $x^2+2y^2+3z^2-x-2y-3z=0$

(C) $x^2+4y^2+9z^2+x+2y+3=0$ (D) $x^2+y^2+z^2+x+2y+3z=0$

E-7. The radius of the circular section of the sphere $|\vec{r}|=5$ by the plane, $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=3\sqrt{3}$ is:

(A) 3 (B) 4 (C) 5 (D) none of these

PART - II : SUBJECTIVE QUESTIONS

Section (A) : Basics, section formula & Direction Ratios

A-1. (a) The position vector of two points A and B are $6\vec{a} + 2\vec{b}$ and $\vec{a} - 3\vec{b}$. If a point C divides AB in the ratio 3 : 2, show that the position vector of C is $3\vec{a} - \vec{b}$.

(b) In a ΔOAB, E is the mid-point of OB and D is a point on AB such that AD : DB = 2 : 1. If OD and AE intersect at P, then determine the ratio OP : PD using vector methods.

A-2 If ABCD is a quadrilateral, E and F are the mid-points of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$

A-3 Show that the points (0, 7, 10) , (–1, 6, 6) and (– 4, 9, 6) form an isosceles right angled triangle

A-4 Find the coordinates of the point equidistant from the point (a, 0, 0), (0, b, 0), (0, 0, c) and (0, 0, 0).

A-5 Show that the foot of the perpendicular from the origin to the join of A(–9, 4, 5) and B (11, 0, –1) is the mid point of AB.

A-6. Find the ratio in which the line joining the points (3, 5,–7) and (–2, 1, 8) is divided by the y.z-plane. Find also the point of intersection of the plane and the line

A-7. What are the direction cosines of a line that passes through the points P(6, –7, –1) and Q(2, –3, 1) and is so directed that it makes an acute angle α with the positive direction of x-axis.

Section (B) : Dot and cross product

B-1 (i) If $\vec{e}_1$ and $\vec{e}_2$ are two unit vectors such that $\vec{e}_1 - \vec{e}_2$ is also a unit vector, then find the angle θ between $\vec{e}_1$ and $\vec{e}_2$.

(ii) Prove that $\left(\frac{\vec{a}}{\vec{a}^2} - \frac{\vec{b}}{\vec{b}^2}\right)^2 = \left(\frac{\vec{a} - \vec{b}}{|\vec{a}|\,|\vec{b}|}\right)^2$

B-2. If $\vec{a}, \vec{b}$ are two unit vectors and θ is the angle between them, then show that:

(i) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ (ii) $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$

B-3. If the three successive vertices of a parallelogram have the position vectors as,
A (– 3, – 2, 0); B (3, – 3, 1) and C (5, 0, 2). Then find

(i) position vector of the fourth vertex D

(ii) a vector having the same direction as that of $\overrightarrow{AB}$ but magnitude equal to $\overrightarrow{AC}$

(iii) the angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

B-4. (i) A vector $\vec{c}$ is perpendicular to the vectors $2\hat{i}+3\hat{j}-\hat{k}$, $\hat{i}-2\hat{j}+3\hat{k}$ and satisfies the condition $\vec{c}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+6=0$. Find the vector $\vec{c}$

(ii) Given $|\vec{a}|=10, |\vec{b}|=2$ and $\vec{a}\,.\,\vec{b}=12$, then find $|\vec{a}\times\vec{b}|$.

B-5. (i) Show that the perpendicular distance of the point $\vec{c}$ from the line joining $\vec{a}$ and $\vec{b}$ is,

$$\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}.$$

(ii) Given a parallelogram ABCD with area 12 sq. units. A straight line is drawn through the mid point M of the side BC and the vertex A which cuts the diagonal BD at a point ' O '. Use vectors to determine the area of the quadrilateral OMCD.

B-6. Find the angle between the lines whose direction cosines are given by $\ell + m + n = 0$ and $\ell^2 + m^2 = n^2$.

B-7 P and Q are the points (–1, 2, 1) and (4, 3, 5). Find the projection of PQ on a line which makes angles of 120º and 135º with y and z axes respectively and an acute angle with x-axis.

Section (C) : STP, VTP & Tetrahedron

C-1. (i) Given unit vectors $\hat{m}$, $\hat{n}$ and $\hat{p}$ such that $\left(\hat{m}\,\hat{^}\,\hat{n}\right)=\hat{p}\,\hat{^}\left(\hat{m}\times\hat{n}\right)=\alpha$, then find value of $\left[\hat{n}\;\hat{p}\;\hat{m}\right]$ in terms of α.

(ii) Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three unit vectors and $\vec{a}\,.\vec{b}=\vec{a}\,.\vec{c}=0$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\,[\vec{a}\,\vec{b}\,\vec{c}]\,\right|$.

C-2. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}-\hat{j}-2\hat{k}$, then

(i) if $\vec{a}\times(\vec{b}\times\vec{c})=p\vec{a}+q\vec{b}+r\vec{c}$, then find value of p, q and r.

(ii) find the value of $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c}).\vec{d}$

C-3. Find the volume of the tetrahedron with vertices P(2, 3, 2), Q (1 ,1, 1), R (3, –2, 1) and S(7, 1, 4).

C-4. Are the following set of vectors linearly independent?

(i) $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=3\hat{i}-6\hat{j}+9\hat{k}$

(ii) $\vec{a}=-2\hat{i}-4\hat{k}$, $\vec{b}=\hat{i}-2\hat{j}-\hat{k}$, $\vec{c}=\hat{i}-4\hat{j}+3\hat{k}$

C-5. Given that $\vec{x}+\frac{1}{\vec{p}^2}(\vec{p}\,.\,\vec{x})\,\vec{p}=\vec{q}$, then show that $\vec{p}\,.\,\vec{x}=\frac{1}{2}(\vec{p}\,.\,\vec{q})$ and find $\vec{x}$ in terms of $\vec{p}$ and $\vec{q}$.

C-6. Examine for coplanarity of the following sets of points

(a) $4\hat{i}+8\hat{j}+12\hat{k},\ 2\hat{i}+4\hat{j}+6\hat{k},\ 3\hat{i}+5\hat{j}+4\hat{k}, 5\hat{i}+8\hat{j}+5\hat{k}.$

(b) $3\vec{a}+2\vec{b}-5\vec{c}, 3\vec{a}+8\vec{b}+5\vec{c},\ -3\vec{a}+2\vec{b}+\vec{c},\ \vec{a}+4\vec{b}-3\vec{c}$

Section (D) : Plane and line

D-1. Find the equation of the planes passing through points (1, 0, 0) and (0, 1, 0) and making an angle of 0.25 π radians with plane $x + y - 3 = 0$

D-2. The foot of the perpendicular drawn from the origin to the plane is (4, –2, –5), then find the vector equation of plane.

D-3. (i) If $\hat{n}$ is the unit vector normal to a plane and p be the length of the perpendicular from the origin to the plane, find the vector equation of the plane.

(ii) Find the equation of the plane which contains the origin and the line of intersection of the planes $\vec{r}\cdot\vec{a} = p$ and $\vec{r}\cdot\vec{b} = q$

D-4. Find the distance between the parallel planes $\vec{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k}) = 5$ and $\vec{r}\cdot(6\hat{i}-9\hat{j}+18\hat{k}) + 20 = 0$.

D-5. Find the angle between the plane passing through points. (1, 1, 1), (1, – 1, 1), (–7, –3, –5) & x–z plane.

D-7. Find the cartesian form of the equation of a line whose vector form is given by $\vec{r} = 2i - j + 4k + \lambda(i + j - 2k)$.

D-8. Find the distance between points of intersection of

(i) Lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ & $\dfrac{x-4}{5} = \dfrac{y-1}{2} = z$

(ii) Lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ & $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

D-9. Find the shortest distance between the lines :

$\vec{r} = (4\hat{i}-\hat{j}) + \lambda(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{r} = (\hat{i}-\hat{j}+2\hat{k}) + \mu(2\hat{i}+4\hat{j}-5\hat{k})$

D-11. The position vectors of the four angular points of a tetrahedron are : $A(\hat{j}+2\hat{k})$, $B(3\hat{i}+\hat{k})$, $C(4\hat{i}+3\hat{j}+6\hat{k})$ and $D(2\hat{i}+3\hat{j}+2\hat{k})$. Find :

(i) the volume of the tetrahedron ABCD.
(ii) the shortest distance between the lines AB and CD.

D-12. If $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \mu(\hat{i}+\hat{j}-\hat{k})$ are two lines, then find the equation of acute angle bisector of two lines.

D-13. Find the equation of image of the line $\dfrac{x-1}{9} = \dfrac{y-2}{-1} = \dfrac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$

D-14. Find the equation of the plane containing parallel lines $(x - 4) = \dfrac{3-y}{4} = \dfrac{z-2}{5}$ and $(x - 3) = \lambda(y + 2) = \mu z$

D-15. Find the equation to the line passing through the point (1, –2, –3) and parallel to the line $2x + 3y - 3z + 2 = 0 = 3x - 4y + 2z - 4$

D-16. Find the equation of the projection of the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ on the plane x + 2y + z = 12

D-17. Find the equation of the plane containing the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$ and perpendicular to the plane x - y + z + 2 = 0

D-18. Show that the line ; $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are coplanar. Find the co-ordinates of point of intersection and the equation to the plane containing them.

D-19. Find the equation of the line which is reflection of the line $\frac{x-1}{9}=\frac{y-2}{-1}=\frac{z+3}{-3}$ in the plane 3x - 3y + 10z = 26.

Section (E) : Miscellaneous

E-1. Find the equation of the sphere described on the line (2, –1, 4) and (–2, 2, –2) as diameter. Also find the area of the circle in which the sphere is intersected by the plane 2x + y – z = 3

PART - III : MISCELLANEOUS OBJECTIVE QUESTIONS

1.

Column – I	**Column – II**
(A) If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ constitute the sides of a ΔABC. if length of the median bisecting the vector $\vec{c}$ is λ, then λ^2	(p) 2
(B) Let $\vec{p}$ is the position vector of the orthocentre and $\vec{g}$ is the position vector of the centroid of the triangle ABC, where circumcentre is the origin. If $\vec{p} = K\vec{g}$, then K is equal to :	(q) 3
(C) Twice of the area of the parallelogram constructed on the vectors $\vec{a} = \vec{p} + 2\vec{q}$ and $\vec{b} = 2\vec{p} + \vec{q}$, where $\vec{p}$ and $\vec{q}$ are unit vectors containing an angle of 30º, is :	(r) 6
(D) Let $\vec{u}, \vec{v}$ and $\vec{w}$ are vector such that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$. If $\|\vec{u}\|= 3, \|\vec{v}\| = 4+\|\vec{w}\|= 5$ then $\sqrt{\|\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}\|}$ is	(s) 12

2. Column – I **Column – II**

(A)	The distance of the point (1, 3, 4) from the plane $2x - y + z = 3$ measured parallel to the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{-1}$ is	(p)	0
(B)	The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is	(q)	$\frac{1}{\sqrt{6}}$
(C)	The points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (–4, 4, k) are coplanar then k =	(r)	4
(D)	The volume of tetrahedron included between the plane $2x - 3y + 4z - 12 = 0$ and three co-ordinate planes is	(s)	12

Comprehension #1

In a parallelogram OABC, vectors $\vec{a}, \vec{b}, \vec{c}$ are respectively the position vectors of vertices A, B, C with reference to) as origin. A point E is taken on the side BC which divides it in the ratio of 2 : 1 internally. Also, the line segment AE intersect the line bisecting the angle O internally in point P. If CP, when extended meets AB in point F. Then

3. The position vector of point P , is

(A) $\frac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$ (B) $\frac{|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$

(C) $\frac{2|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$ (D) none of these

4. The position vector of point F, is

(A) $\vec{a}+\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (B) $\vec{a}+\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (C) $\vec{a}+\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$ (D) $\vec{a}-\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

5. The vector $\overline{AF}$, is given by

(A) $-\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (B) $\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (C) $\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$ (D) $\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

Comprehension #2

Let $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ be two planes, where $d_1, d_2 > 0$. Then origin lies in acute angle if $a_1a_2 + b_1b_2 + c_1c_2 < 0$ and origin lies in obtuse angle if $a_1a_2 + b_1b_2 + c_1c_2 > 0$.
Further point (x_1, y_1, z_1) and origin both lie either in acute angle or in obtuse angle , if one of (x_1, y_1, z_1) and origin lie in acute angle and the other in obtuse angle, if

$$(a_1x_1 + b_1y_1 + c_1z_1 + d_1)(a_2x_1 + b_2y_1 + c_2z_1 + d_2) < 0$$

6. Given the planes $2x + 3y - 4z + 7 = 0$ and $x - 2y + 3z - 5 = 0$, if a point P is (1, – 2, 3), then
(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle
(C) O lies in acute angle, P lies in obtuse angle.
(D) O lies in obtue angle, P lies an acute angle.

7. Given the planes x + 2y – 3z + 5 = 0 and 2x + y + 3z + 1 = 0. If a point P is (2, –1, 2), then
(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle
(C) O lies in acute angle, P lies in obtuse angle.
(D) O lies in obtue angle, P lies an acute angle.

8. Given the planes x + 2y – 3z + 2 = 0 and x – 2y + 3z + 7 = 0, if the point P is (1, 2, 2), then
(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle
(C) O lies in acute angle, P lies in obtuse angle.
(D) O lies in obtue angle, P lies an acute angle.

Assertion/Reasoning

9. **Statement 1 :** If $\vec{a},\vec{b},\vec{c}$ are non–coplanar and $\vec{d}$ is any vector, then

$$[\vec{d}\ \vec{b}\ \vec{c}]\ \vec{a} + [\vec{d}\ \vec{c}\ \vec{a}]\vec{b} + [\vec{d}\ \vec{a}\ \vec{b}]\vec{c} - \vec{d}\ [\vec{a}\ \vec{b}\ \vec{c}] = 0\ .$$

Statement 2 : Any vector is 3D can be written in linear combination of three non–coplanar vectors.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10. **Statement 1 :** If $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}, \vec{a} \neq 0$, then $\vec{b} = \vec{c} + \lambda\vec{a}$.
Statement 2 : Two vectors are non–collinear and non–zero then they are linearly independent.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

11. **Statement-1** : If I is incentre of Δ ABC then $|\overrightarrow{BC}|\,\overrightarrow{IA} + |\overrightarrow{CA}|\,\overrightarrow{IB} + |\overrightarrow{AB}|\,\overrightarrow{IC} = 0$

Statement-2 : In an Δ, if position vector of vertex are $\vec{a}, \vec{b}, \vec{c}$, position vector of incentre is $\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12. **Statement 1 :** If α,β,γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
Statement 2 : If ℓ,m,n are the direction cosines of a line then $\ell^2 + m^2 + n^2 = 1$.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

13. **Statement 1 :** The locus represented by xy + yz = 0 is A pair of perpendicular planes.
Statement 2 : If $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular then $a_1a_2 + b_1b_2 + c_1c_2 = 1$.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

1. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ to $\vec{c}+\vec{a}$ and $\vec{c}$ to $\vec{a}+\vec{b}$. Then $|\vec{a}+\vec{b}+\vec{c}|$ is equal to :

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D) $5\sqrt{2}$

2. The vectors $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+\hat{j}+4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are :

(A) not coplanar (B) coplanar but cannot form a triangle
(C) coplanar but can form a triangle (D) coplanar & can form a right angled triangle

3. Four coplanar forces are applied at a point O. Each of them is equal to k and the angle between two consecutive forces equals 45º. Then the resultant has the magnitude equal to :

(A) $k\sqrt{2+2\sqrt{2}}$ (B) $k\sqrt{3+2\sqrt{2}}$ (C) $k\sqrt{4+2\sqrt{2}}$ (D) none

4. A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, $\vec{a}$ has components p + 1 and 1, then

(A) p = 0 (B) p = 1 or $p = -\frac{1}{3}$
(C) p = – 1 or $p = \frac{1}{3}$ (D) p = 1 or p = – 1

5. Taken on side $\overrightarrow{AC}$ of a triangle ABC, a point M such that $\overrightarrow{AM} = \frac{1}{3}\overrightarrow{AC}$. A point N is taken on the side $\overrightarrow{CB}$ such that $\overrightarrow{BN} = \overrightarrow{CB}$, then for the point of intersection X of $\overrightarrow{AB}$ and $\overrightarrow{MN}$ which of the following holds good?

(A) $\overrightarrow{XB} = \frac{1}{3}\overrightarrow{AB}$ (B) $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$ (C) $\overrightarrow{XN} = \frac{3}{4}\overrightarrow{MN}$ (D) $\overrightarrow{XM} = 3\overrightarrow{XN}$

6 If the unit vectors $\vec{e}_1$ and $\vec{e}_2$ are inclined at an angle 2θ and $|\vec{e}_1-\vec{e}_2| < 1$, then for $\theta \in [0, \pi]$, θ may lie in the interval :

(A) $\left[0, \frac{\pi}{6}\right)$ (B) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (C) $\left(\frac{5\pi}{6}, \pi\right]$ (D) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

7. For a non zero vector $\vec{A}$ if the equations $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$ hold simultaneously, then:

(A) $\vec{A}$ is perpendicular to $\vec{B}-\vec{C}$ (B) $\vec{A}=\vec{B}$

(C) $\vec{B}=\vec{C}$ (D) $\vec{C}=\vec{A}$

8. Given $\vec{a} = x\hat{i}+y\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=\hat{i}+2\hat{j}$; $(\vec{a}\,\hat{}\,\vec{b}) = \frac{\pi}{2}$, $\vec{a}\cdot\vec{c}=4$, then

(A) $[\vec{a}\,\vec{b}\,\vec{c}]^2 = |\vec{a}|$ (B) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|$ (C) $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ (D) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|^2$

9. The volume of the parallelopiped constructed on the diagonals of the faces of the given rectangular parallelopiped is m times the volume of the given parallelopiped. Then m is equal to:
(A) 2 (B) 3 (C) 4 (D) none

10. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ. Then $[\vec{a}\,\vec{b}\,\vec{c}]$ in terms of θ is equal to:

(A) $(1+\cos\theta)\sqrt{\cos 2\theta}$ (B) $(1+\cos\theta)\sqrt{1-2\cos 2\theta}$

(C) $(1-\cos\theta)\sqrt{1+2\cos\theta}$ (D) none of these

11. Consider a tetrahedron with faces f_1, f_2, f_3, f_4. Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 and whose directions are perpendicular to these faces in the outward direction. Then,

(A) $\vec{a}_1+\vec{a}_2+\vec{a}_3+\vec{a}_4 = 0$ (B) $\vec{a}_1+\vec{a}_3 = \vec{a}_2+\vec{a}_4$

(C) $\vec{a}_1+\vec{a}_2 = \vec{a}_3+\vec{a}_4$ (D) none

12. Let $\vec{r}$ be a vector perpendicular to $\vec{a}+\vec{b}+\vec{c}$, where $[\vec{a}\ \vec{b}\ \vec{c}] = 2$. If $\vec{r} = \ell(\vec{b}\times\vec{c}) + m(\vec{c}\times\vec{a}) + n(\vec{a}\times\vec{b})$, then $(\ell+m+n)$ is equal to

(A) 2 (B) 1 (C) 0 (D) none of these

13. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar non-zero vectors and $\vec{r}$ is any vector in space, then $(\vec{a}\times\vec{b})\times(\vec{r}\times\vec{c}) + (\vec{b}\times\vec{c})\times(\vec{r}\times\vec{a}) + (\vec{c}\times\vec{a})\times(\vec{r}\times\vec{b})$ is equal to

(A) $2[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$ (B) $3[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$ (C) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$ (D) none of these

14. A plane meets the coordinate axes in A, B, C and (α, β, γ) is the centroid of the triangle ABC. Then the equation of the plane is

(A) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$ (B) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$ (C) $\frac{3x}{\alpha}+\frac{3y}{\beta}+\frac{3z}{\gamma}=1$ (D) $\alpha x+\beta y+\gamma z=1$

15. Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is:

(A) $4x + 3y + 5z = 25$
(B) $4x + 3y + 5z = 50$
(C) $3x + 4y + 5z = 49$
(D) $x + 7y - 5z = 2$

16. The non zero value of 'a' for which the lines $2x - y + 3z + 4 = 0 = ax + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are co-planar is :

(A) – 2
(B) 4
(C) 6
(D) 0

17. If the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4}$ and $\frac{x+k}{3} = \frac{y-1}{2} = \frac{z-2}{h}$ are concurrent then

(A) h = – 2, k = – 6
(B) $h = \frac{1}{2}$, k = 2
(C) h = 6, k = 2
(D) h = 2, $k = \frac{1}{2}$

18. The coplanar points A , B , C , D are (2 – x , 2 , 2) , (2 , 2 – y , 2) , (2 , 2 , 2 – z) and (1 , 1 , 1) respectively . Then :

(A) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$
(B) x + y + z = 1
(C) $\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z} = 1$
(D) none of these

19. The co-ordinates of the centre and the radius of the circle $x + 2y + 2z = 15$, $x^2 + y^2 + z^2 - 2y - 4z = 11$ are

(A) (4, 3, 1), $\sqrt{5}$
(B) (3, 4, 1), $\sqrt{6}$
(C) (1, 3, 4), $\sqrt{7}$
(D) none of these

20. Consider the lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ the equation of the line which

(A) bisects the angle between the lines is $\frac{x}{3} = \frac{x}{3} = \frac{z}{8}$

(B) bisects the angle between the lines is $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

(C) passes through origin and is perpendicular to the given lines is x = y = – z

(D) none of these

21. If $\vec{z}_1 = a\hat{i} + b\hat{j}$ and $\vec{z}_2 = c\hat{i} + d\hat{j}$ are two vectors in $\hat{i}$ and $\hat{j}$ system, where $|\vec{z}_1| = |\vec{z}_2| = r$ and $\vec{z}_1 . \vec{z}_2 = 0$, then $\vec{w}_1 = a\hat{i} + c\hat{j}$ and $\vec{w}_2 = b\hat{i} + d\hat{j}$ satisfy:

(A) $|\vec{w}_1| = r$
(B) $|\vec{w}_2| = r$
(C) $\vec{w}_1 . \vec{w}_2 = 0$
(D) none of these

22. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are A(1, 0, 1), B(2,0, 0) and C(0, 1, 0), then position vectors of the vertex A_1 can be:

(A) (2, 2, 2)
(B) (0, 2, 0)
(C) (0, – 2, 2)
(D) (0, – 2, 0)

23. The direction cosines of the lines bisecting the angle between the lines whose direction cosines are ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 and the angle between theses lines is θ, are

(A) $\frac{\ell_1+\ell_2}{\cos\frac{\theta}{2}}, \frac{m_1+m_2}{\cos\frac{\theta}{2}}, \frac{n_1+n_2}{\cos\frac{\theta}{2}}$

(B) $\frac{\ell_1+\ell_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}}$

(C) $\frac{\ell_1+\ell_2}{\sin\frac{\theta}{2}}, \frac{m_1+m_2}{\sin\frac{\theta}{2}}, \frac{n_1+n_2}{\sin\frac{\theta}{2}}$

(D) $\frac{\ell_1-\ell_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}$

24. The planes $2x - 3y - 7z = 0$, $3x - 14y - 13z = 0$ and $8x - 31y - 33z = 0$

(A) pass through origin
(B) intersect in a common line
(C) from a triangular prism
(D) none of these

25. Let the points A (a, b, c) and B(a′, b′, c′) be at distances r and r′ from origin. The line AB passes through origin when

(A) $\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$
(B) $aa' + bb' + cc' = rr'$
(C) $aa' + bb' + cc' = r^2 + r'^2$
(D) none of these

26. A line passes through a point A with position vector $3\hat{i} + \hat{j} - \hat{k}$ and is parallel to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. If P is a point on this line such that AP = 15 units, then the position vector of the point P is/are

(A) $13\hat{i} + 4\hat{j} - 9\hat{k}$ (B) $13\hat{i} - 4\hat{j} + 9\hat{k}$ (C) $7\hat{i} - 6\hat{j} + 11\hat{k}$ (D) $-7\hat{i} + 6\hat{j} - 11\hat{k}$

PART - II : SUBJECTIVE QUESTIONS

1. Points X and Y are taken on the sides QR and RS, respectively of a parallelogram PQRS, so that QX = 4XR and RY = 4YS. The line XY cuts the line PR at Z. Find the ratio PZ : ZR.

2. In triangle ABC using vector method show that the distance between the circumcentre and the orthocentre is $R\sqrt{1-8\cos A\cos B\cos C}$, where R is the circumradius of the triangle ABC.

3. Given four non zero vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$. The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}, \vec{b}$ and $\vec{c}$ and $(\vec{a}\,\hat{}\,\vec{b}) = (\vec{b}\,\hat{}\,\vec{c}) = \frac{\pi}{3}$, $(\vec{d}\,\hat{}\,\vec{a}) = \alpha$ and $(\vec{d}\,\hat{}\,\vec{b}) = \beta$, prove that $(\vec{d}\,\hat{}\,\vec{c}) = \cos^{-1}(\cos\beta - \cos\alpha)$.

4. (i) Let $\vec{A} = 2\hat{i} + \hat{k}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$. Determine a vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} \cdot \vec{A} = 0$

(ii) Find a vector $\vec{v}$ which is coplanar with the vectors $\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ and is orthogonal to the vector $-2\hat{i} + \hat{j} + \hat{k}$. It is given that the projection of $\vec{v}$ along the vector $\hat{i} - \hat{j} + \hat{k}$ is equal to $6\sqrt{3}$.

5. For any two vectors $\vec{u}$ & $\vec{v}$, prove that

(A) $(\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2$ &

(B) $(1 + |\vec{u}|^2)(1 + |\vec{v}|^2) = (1 - \vec{u} \cdot \vec{v})^2 + |\vec{u} + \vec{v} + (\vec{u} \times \vec{v})|^2$

6. Show that the circumcentre of the tetrahedron OABC is given by $\frac{\vec{a}^2(\vec{b}\times\vec{c}) + \vec{b}^2(\vec{c}\times\vec{a}) + \vec{c}^2(\vec{a}\times\vec{b})}{2[\vec{a}\,\vec{b}\,\vec{c}]}$, where $\vec{a}, \vec{b}$ & $\vec{c}$ are the position vectors of the points A, B, C respectively relative to the origin 'O'.

7. Let $\vec{u}$ & $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + (\vec{w}\times\vec{u}) = \vec{v}$, then prove that $\left|(\vec{u}\times\vec{v}).\vec{w}\right| \le \frac{1}{2}$ and the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

8. The edges of a rectangular parallelepiped are a, b, c; show that the angles between the four diagonals are given by $\cos^{-1}\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}$.

9. Find the equation of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\pi/3$.

10. If 2d be the shortest distance between the lines $\frac{y}{b} + \frac{z}{c} = 1$; $x = 0$ and $\frac{x}{a} - \frac{z}{c} = 1$; $y = 0$ then prove $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.

11. Find the plane π passing through the points of intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ and is perpendicular to the plane $3x - y - 2z = 4$. Find the image of point (1, 1, 1) in plane π.

12. Find the equation of the straight line which passes through the point (2, –1, –1); is parallel to the plane $4x + y + z + 2 = 0$ and is perpendicular to the line of intersection of the planes $2x + y = 0 = x - y + z$.

13. If the distance between points $(\alpha, 5\alpha, 10\alpha)$ from the point of intersection of the lines $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 12\hat{k})$ and plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$ is 13 units. Find the possible values of α.

14. Prove that the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{1}$ lies in the plane $3x + 4y + 6z + 7 = 0$. If the plane is rotated about the line till the plane passes through the origin then find the equation of the plane in the new position.

15. A line $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z-k}{3}$ cuts the y-z plane and the x-y plane at A and B respectively. If $\angle AOB = \frac{\pi}{2}$, then find k, where O is the origin.

16. Find the point R in which the line AB cuts the plane CDE, where position vectors of points A, B, C, D, E are respectively $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = -4\hat{j} + 4\hat{k}$, $\vec{d} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{e} = 4\hat{i} + \hat{j} + 2\hat{k}$.

17. (a) Use vectors to prove that the acute angle between the plane faces of a regular tetrahedron is arc cos (1/3).
(b) Use vectors to find the circum–radius and in–radius of a regular tetrahedron in terms of the length k of each edge.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

1. The value of a for which the volume of parallelopiped formed by the vectors $\hat{i}+a\hat{j}+\hat{k}$ and $\hat{j}+a\hat{k}$ +and $a\hat{i}+\hat{k}$ is minimum is

(A) –3 (B) 3 (C) $1/\sqrt{3}$ (D) $-\sqrt{3}$

[JEE 03 (screening)]

2. If $\vec{u}, \vec{v}, \vec{w}$ are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively.

Prove that $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}] = \frac{1}{16}[\vec{u}\ \vec{v}\ \vec{w}]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$ **[JEE 03 (Mains)]**

3. The value of k such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane 2x – 4y +z = 7, is

(A) 7 (B) 6 (C) all values of k (D) –7

[JEE 03 (screening)]

4. (i) Find the equation of the plane passing through the points (2,1,0),(5,0,1) and (4,1,1)
(ii) If P is the point (2,1,6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. **[JEE 03 Main,]**

5. If for vectors $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{b}=1, \vec{a}\times\vec{b}=\hat{j}-\hat{k}, \vec{a}=\hat{i}+\hat{j}+\hat{k}$ then vector $\vec{b}$ is– **[JEE 04 (screening)]**

(A) $\hat{i}-\hat{j}+\hat{k}$ (B) $\hat{j}-\hat{k}$ (C) $\hat{i}$ (D) $\hat{j}+\hat{k}$

6. A given unit vector is orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$ and coplanar with $\hat{i}-\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ then the vector is–

(A) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (B) $\frac{3\hat{k}-\hat{j}}{\sqrt{10}}$ (C) $\frac{3\hat{j}+\hat{k}}{\sqrt{10}}$ (D) $\frac{\hat{j}+3\hat{k}}{\sqrt{10}}$

7. $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four distinct vectors satisfying the conditions $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ & $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then prove that $\vec{a}.\vec{b}+\vec{c}.\vec{d}\neq\vec{a}.\vec{c}+\vec{b}.\vec{d}$

8. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ are intersecting each other then 'k' is

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) 1 (D) $\frac{3}{2}$

[JEE 04 (screening)]

9. A plane is parallel to two lines whose direction ratios (1, 0, –1) & (–1, 1, 0) and it contains the point (1, 1, 1). If it cuts the coordinate axes at A, B, C. then find the volume of tetrahedron OABC, where O is the origin.

[JEE 04 (Mains)]

10. Two planes P_1 & P_2 pass through origin. Two lines L_1 and L_2 also passing through origin, are such that L_1 lies on P_1 but not on P_2, L_2 lies on P_2 but not on P_1. A, B, C are three points other than origin, then prove that the permutation [A', B', C'] of [A, B, C] exists such that **[JEE 04 (Mains)]**

11. T is a parallellopiped in which A, B, C and D are vertices of one face. And the face just above it has corresponding vertices A' , B', C', D'. T is now compressed to S with face ABCD remaining same and A' , B', C', D' shifted to A", B", C" and D" in S. The volume of S is reduced to 90% of T. Prove that locus of A" is a plane.

12. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and $\vec{b}_1 = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2 = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1$,

$\vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}_1.\vec{c}}{|\vec{b}_1|^2}\vec{b}_1$, $\vec{c}_3 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1$ then the set of orthogonal vectors is **[JEE 04 (Mains)]**

13. Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. **[JEE 05 (Mains)]**

14. A variable plane at a distance of 1 unit from the origin cut the coordinate axis and centroid of triangle is C (x, y, z) then the value of $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, value of k is

(A) 3 (B) 1 (C) 1/3 (D) 9

15. Find the equation of the plane containing the line 2x – y + z – 3 = 0, 3x + y + z = 5 and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1)

16. Let $\overline{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vectors $\overline{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is **[JEE 06]**

(A) $\pi/2$ (B) $\pi/4$ (C) $\pi/6$ (D) $3\pi/4$

17. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is- **[JEE 06]**

(A) $4\hat{i} - \hat{j} + 4\hat{k}$ (B) $3\hat{i} + \hat{j} - 3\hat{k}$ (C) $2\hat{i} + \hat{j} - 2\hat{k}$ (D) $4\hat{i} + \hat{j} - 4\hat{k}$

18. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is- **[JEE 07]**

(A) zero (B) one (C) two (D) three

19. Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement -1 : $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$

because

Statement -2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$.

(A) Statement -1 is true, Statement - 2 is true ; Statement - 2 is a correct explanation for Statement - 1
(B) Statement -1 is true, Statement - 2 is true ; Statement - 2 is **NOT** a correct explanation for Statement - 1
(C) Statement -1 is true, Statement - 2 is false.
(D) Statement -1 is false, Statement - 2 is true **[JEE 07,]**

20. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct ?

(A) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$
(B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$
(C) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}\neq\vec{0}$
(D) $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ are mutually perpendicular

[JEE 07]

21. Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5
Statement -1:The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.
because
Statement- 2 : The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of given planes.
(A) Statement -1 is true, Statement - 2 is true ; Statement - 2 is a correct explanation for Statement - 1
(B) Statement -1 is true, Statement - 2 is true ; Statement - 2 is **NOT** a correct explanation for Statement - 1
(C) Statement -1 is true, Statement - 2 is false.
(D) Statement -1 is false, Statement - 2 is true **[JEE 07]**

22. Match the following
Consider the following linear equations

ax + by + cz = 0
bx + cy + az = 0
cx + ay + bz = 0

Column I	Column II
(A) $a+b+c\neq 0$ and $a^2+b^2+c^2=ab+bc+ca$	(P) the equations represent planes meeting only at a single point.
(B) $a+b+c=0$ and $a^2+b^2+c^2\neq ab+bc+ca$	(Q) the equations represent the line x = y = z.
(C) $a+b+c\neq 0$ and $a^2+b^2+c^2\neq ab+bc+ca$	(R) the equations represent identical planes.
(D) $a+b+c=0$ and $a^2+b^2+c^2=ab+bc+ca$	(S) the equations represent the whole of the three dimensional space.

[JEE 07]

23. (i) The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}.\hat{b}=\hat{b}.\hat{c}=\hat{c}.\hat{a}$ = ½. Then, the volume of the parallelopiped is **[JEE 08]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

24. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t+\hat{b}\sin t$. When P is farthest from origin O, Let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,-

(A) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+\hat{a}.\hat{b})^{\frac{1}{2}}$
(B) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+\hat{a}.\hat{b})^{\frac{1}{2}}$
(C) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+2\hat{a}.\hat{b})^{\frac{1}{2}}$
(D) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+2\hat{a}.\hat{b})^{\frac{1}{2}}$ **[JEE 08]**

25. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

STATEMENT - 1 : At least two of the lines L_1, L_2 and L_3 are non parallel.

and

STATEMENT - 2 : The three planes do not have a common point.

(A) Statement -1 is true, Statement - 2 is true ; Statement - 2 is a correct explanation for Statement - 1

(B) Statement -1 is true, Statement - 2 is true ; Statement - 2 is **NOT** a correct explanation for Statement - 1

(C) Statement -1 is true, Statement - 2 is false.

(D) Statement -1 is false, Statement - 2 is true. **[JEE 08]**

Comprehension (26-28)

Consider the lines

$$L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \qquad L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

26. The unit vector perpendicular to both L_1 and L_2 is

(A) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (C) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

27. The shortest distance between L_1 and L_2 is-

(A) 0 (B) $\frac{17}{\sqrt{3}}$ (C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

28. The distance of the point (1, 1, 1) from the plane passing through thepoint (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is-

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$ (C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

[JEE 08]

29. Match the statements/expression given in **Column–I** with the values given in **Column–II**.

Column–I		**Column–II**
(A) Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p)	$\pi/6$
(B) Points of discontinuity of the function $f(x) = [6x/\pi]\cos[3x/\pi]$,	(q)	$\pi/4$
where [y] denotes the largest integer less than or equal to y.	(r)	$\pi/3$
(C) Volume of the parallelopiped with its edges represented by	(s)	$\pi/2$
the vector $\hat{i}+\hat{j}$, $\hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(t)	π
(D) Angle between vector $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vecotrs satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$		

[JEE 09]

30. A line with positive direction cosines passes through the point P(2, –1, 2) and makes equals angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals:

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2 **[JEE 09]**

31. Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r} = (\hat{i}+\hat{j}+2\hat{k}) + \mu(-3\hat{i}+\hat{j}+5\hat{k})$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x -4y + 3z = 1 is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

32. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}\ \frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}\ \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is **[JEE 2010]**

33. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by **[JEE 2010]**

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

34. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straigth lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is **[IIT-JEE-2010, Paper-1]**

35. If the distance between the plane Ax - 2y + z = d and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$ then |d| is **[JEE 2010, (3, 0) Out of 84]**

36. If the distance of the point P(1, – 2, 1) from the plane x + 2y – 2z = α, where α > 0, is 5, then the foot of the perpendicular from P to the plane is **[JEE 2010, (5, -2) Out of 79]**

(A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ (C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

37. Match the statements in **Column-I** with those in **Column-II.**

Column-I	**Column-II**
(A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p) –4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b}=0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then possible value of μ are	(r) 4
(D) Let f be the function on [–π, π] given by $f(0)=9$ and $f(x)=\frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)}$ for $x\neq0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi}f(x)dx$ is	(s) 5
	(t) 6

[JEE 2010, (8, 0) Out of 79]

38. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors . A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}-\hat{k}$ (C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

[JEE 2011]

39. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is / are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

[JEE 2011]

40. Let $\vec{a}=-\hat{i}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}+\hat{j}$ and $\vec{c}=-\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\times\vec{a}=0$, then the value of $\vec{r}\,.\,\vec{b}$ is **[IIT-JEE 2011, Paper-2]**

41. The point P is the intersection of the straight line joining the points Q (2, 3, 5) and R (1 – 1, 4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T (2, 1, 4) to QR, then the length of the line segment PS is **[IIT-JEE 2012, Paper-1]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

42. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

[IIT-JEE 2012, Paper-1]

43. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is **[IIT-JEE 2012, Paper-2]**

(A) 5x - 11y + z = 17 (B) $\sqrt{2}x+y=3\sqrt{2}-1$ (C) x + y + z = $\sqrt{3}$ (D) $x-\sqrt{2y}=1-\sqrt{2}$

44. If $\vec{a}$ and $\vec{b}$ are vector such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is **[IIT-JEE 2012, Paper-2]**

(A) 0 (B) 3 (C) 4 (D) 8

45. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, the the plane(s) containing these two liens is (are) **[IIT-JEE 2012, Paper-2]**

(A) y + 2z = –1 (B) y + z = –1 (C) y – z = –1 (D) y – 2z = – 1

46. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line **[JEE-Advanced-2013]**

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

(C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

47. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}$, $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **[JEE-Advanced-2013]**

(A) 5 (B) 20 (C) 10 (D) 30

48. A line l passing through the origin is perpendicular to the lines

l_1: $(3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}, -\infty < t < \infty$ l_2: $(3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, -\infty < s < \infty$

Then, the coordinate(s) of the point(s) l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are)

[JEE-Advanced-2013]

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (B) (–1, –1, 0) (C) (1, 1, 1) (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

49. Consider the set of eight vectors V = $\left\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right\}$. Three noncoplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE-Advanced-2013]**

50. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

[JEE-Advanced-2013]

(A) 1 (B) 2 (C) 3 (D) 4

51. Match List-I with List-II and select the correct answer using the code given below the lists :

[JEE-Advanced-2013]

	List-I		List-II
P.	Volume of parallelepiped determined by vector $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $(\vec{c} \times \vec{a})$ is	1.	100
Q.	Volume of parallelepiped determined by vector $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), (\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	2.	30
R.	Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is	3.	24
S.	Area of a parallelogram with triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	4.	60

Codes:

	(P)	(Q)	(R)	(S)
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

52. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, P_2 : $3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2.
Match List I with List II and select the correct answer using the code given below the lists:

[JEE-Advanced-2013]

List-I	List-II
(P) a =	1. 13
(Q) b =	2. – 3
(R) c =	3. 1
(S) d =	4. – 2

Codes:

	(P)	(Q)	(R)	(S)
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is equal to : **[AIEEE 2003]**

(1) 0 (2) – 7 (3) 7 (4) 1

2. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}).[(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})]$ equals :

(1) 0 (2) $\vec{u}.\vec{v} \times \vec{w}$ (3) $\vec{u}.\vec{w} \times \vec{v}$ (4) $3\,\vec{u}.\vec{v} \times \vec{w}$ **[AIEEE 2003]**

3. The vectors $\vec{AB} = 3\hat{i} + 4\hat{k}$ and $\vec{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC. The length of the median through A is : **[AIEEE 2003]**

(1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{288}$

4. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$, then $|\vec{w}.\hat{n}|$ is equal to : **[AIEEE 2003]**

(1) 0 (2) 1 (3) 2 (4) 3

5. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled, then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled, then $P^2 : Q^2 : R^2$ is : **[AIEEE 2003]**

(1) 3 : 1 : 1 (2) 2 : 3 : 2 (3) 1 : 2 : 3 (4) 2 : 3 : 1

6. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a : **[AIEEE 2003]**

(1) square (2) rhombus
(3) rectangle (4) none of these

7. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces in standard unit is : **[AIEEE 2003,04]**

(1) 20 unit (2) 30 unit (3) 40 unit (4) 50 unit

8. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$, is : **[AIEEE 2003]**

(1) 1 (2) 2 (3) 3 (4) 4

9. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, if : **[AIEEE 2003]**

(1) k = 0 or – 1 (2) k = 1 or – 1 (3) k = 0 or – 3 (4) k = 3 or – 3

10. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be perpendicular, if and only if :

(1) $aa' + bb' + cc' + 1 = 0$ (2) $aa' + bb' + cc' = 0$ **[AIEEE 2003]**
(3) $(a + a')(b + b') + (c + c') = 0$ (4) $aa' + cc' + 1 = 0$

11. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is : **[AIEEE 2003]**

(1) 26 (2) $11\frac{4}{13}$ (3) 13 (4) 39

12. Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a′, b′, c′ from the origin, then : **[AIEEE 2003]**

(1) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$ (2) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(3) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$ (4) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

13. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar), then $\vec{a}+2\vec{b}+6\vec{c}$ equals:

(1) $\lambda\vec{a}$ (2) $\lambda\vec{b}$ (3) $\lambda\vec{c}$ (4) $\vec{0}$ **[AIEEE 2004]**

14. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for : **[AIEEE 2004]**

(1) all values of λ. (2) all except one value of λ.
(3) all except two values of λ. (4) no value of λ.

15. Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals : **[AIEEE 2004]**

(1) 2 (2) $\sqrt{7}$ (3) $\sqrt{14}$ (4) 14

16. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals : **[AIEEE 2004]**

(1) $\frac{1}{3}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{2\sqrt{2}}{3}$

17. A line makes the same angle θ with each of the x and z axes. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals : **[AIEEE 2004]**

(1) $\frac{2}{3}$ (2) $\frac{1}{5}$ (3) $\frac{3}{5}$ (4) $\frac{2}{5}$

18. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is : **[AIEEE 2004]**

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

19. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points of intersection are given by : **[AIEEE 2004]**

(1) (3a, 3a, 3a), (a, a, a) (2) (3a, 2a, 3a), (a, a, a)
(3) (3a, 2a, 3a), (a, a, 2a) (4) (2a, 3a, 3a), (2a, a, a)

20. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parametres s and t respectively, are coplanar, then λ equals : **[AIEEE 2004]**

(1) -2 (2) -1 (3) $-\frac{1}{2}$ (4) 0

21. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane : **[AIEEE 2004]**

(1) $x - y - z = 1$ (2) $x - 2y - z = 1$ (3) $x - y - 2z = 1$ (4) $2x - y - z = 1$

22. If C is the mid point of AB and P is any point outside AB, then : **[AIEEE 2005]**

(1) $\vec{PA} + \vec{PB} + \vec{PC} = \vec{0}$ (2) $\vec{PA} + \vec{PB} + 2\vec{PC} = \vec{0}$

(3) $\vec{PA} + \vec{PB} = \vec{PC}$ (4) $\vec{PA} + \vec{PB} = 2\vec{PC}$

23. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to : **[AIEEE 2005]**

(1) $4\vec{a}^2$ (2) $2\vec{a}^2$ (3) $\vec{a}^2$ (4) $3\vec{a}^2$

24. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$. Then, $[\vec{a} \ \vec{b} \ \vec{c}]$ depends on :
(1) neither x nor y (2) both x and y (3) only x (4) only y **[AIEEE 2005]**

25. Let a, b, and c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is : **[AIEEE 2005]**

(1) the harmonic mean of a and b. (2) equal to zero.
(3) the arithmetic mean of a and b. (4) the geometric mean of a and b.

26. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then

$[\lambda(\vec{a} + \vec{b}) \;\; \lambda^2 \vec{b} \;\; \lambda \vec{c}] = [\vec{a} \;\; \vec{b} + \vec{c} \;\; \vec{b}]$ for : **[AIEEE 2005]**

(1) exactly two values of λ (2) exactly three values of λ
(3) no value of λ (4) exactly one value of λ

27. The plane x + 2y – z = 4 cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius : **[AIEEE 2005]**

(1) $\sqrt{2}$ (2) 2 (3) 1 (4) 3

28. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$. The value of λ is : **[AIEEE 2005]**

(1) $-\frac{4}{3}$ (2) $\frac{3}{4}$ (3) $-\frac{3}{5}$ (4)

29. The angle between the lines 2x = 3y = – z and 6x = – y = – 4z is : **[AIEEE 2005]**

(1) 30º (2) 45º (3) 90º (4) 0º

30. If the plane 2ax – 3ay + 4az + 6 = 0 passes through the mid–point of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a equals :

(1) 2 (2) –2 (3) 1 (4) –1 **[AIEEE 2005]**

31. The distance between the line and the plane is : **[AIEEE 2005]**

(1) (2) (3) (4)

32. If (×) × = × (×), where , and are any three vectors such that . ≠ 0, . ≠ 0, then and are : **[AIEEE 2006]**

(1) Inclined at an angle of between them (2) perpendicular
(3) parallel (4) Inclined at an angle of between them

33. ABC is triangle, right angled at A. The resultant of the forces acting along , with magnitudes and respectively is the force along , where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is : **[AIEEE 2006]**

(1) (2) + (3) (4)

34. The value of a, for which the points A, B, C with position vectors 2 – + , – 3 – 5 and a – 3 + respectively are the vertices of a right angled triangle with C = are : **[AIEEE 2006]**

(1) – 2 and – 1 (2) – 2 and 1 (3) 2 and – 1 (4) 2 and 1

35. The two lines x = ay + b, z = cy + d and x = a′ y + b′ , z = c′y + d′ are perpendicular to each other, if : **[AIEEE 2003, 06]**

(1) aa′ + cc′ = 1 (2) + = – 1 (3) + = 1 (4) aa′ + cc′ = – 1

36. The image of the point (– 1, 3, 4) in the plane x – 2y = 0 is : **[AIEEE 2006]**

(1) (15, 11, 4) (2) (3) (8, 4, 4) (4)

37. If and are unit vectors and θ is the acute angle between them, then 2 × 3 is a unit vector for : **[AIEEE 2007]**

(1) exactly two values of θ (2) more than two values of θ
(3) no value of θ (4) exactly one value of θ

38. Let = + + , = – + 2 and = x + (x – 2) – . If the vector lies in the plane of and , then x equals : **[AIEEE 2007]**

(1) 0 (2) 1 (3) – 4 (4) – 2

39. Let L be the line of intersection of the planes 2x + 3y + z = 1 and x + 3y + 2z = 2. If L makes an angle α with the positive x-axis, then cos α equals : **[AIEEE 2007]**

(1) 1/ (2) 1/2 (3) 1 (4) 1/

40. If a line makes an angle of with the positive directions of each of x-axis & y-axis then the angle that the line makes with the positive direction of the z-axis is : **[AIEEE 2007]**

(1) (2) (3) (4)

41. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 – 6x – 12y – 2z + 20 = 0$, then the coordinates of the other end of the diameter are- **[AIEEE 2007]**

(1) (4, 9, – 3) (2) (4, –3, 3) (3) (4, 3, 5) (4) (4, 3, – 3)

42. The vector = α + 2 + β lies in the plane of the vectors = + and = + and bisects the angle between and . Then, which one of the following gives possible values of α and β ? **[AIEEE 2008]**

(1) α = 2, β = 2 (2) α = 1, β = 2 (3) α = 2, β = 1 (4) α = 1, β = 1

43. The non-zero vectors , and are related by = 8 and = – 7 . Then, the angle between and is : **[AIEEE 2008]**

(1) 0 (2) (3) (4) π

44. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point . Then : **[AIEEE 2008]**

(1) a = 2, b = 8 (2) a = 4, b = 6 (3) a = 6, b = 4 (4) a = 8, b = 2

45. If the straight lines = = and = = intersect at a point, then the integer k is equal to : **[AIEEE 2008]**

(1) – 5 (2) 5 (3) 2 (4) – 2

46. If , , are non-coplanar vectors and p, q are real numbers, then the equality

[3 p p] – [p q] – [2 q q] = 0 holds for : **[AIEEE 2009]**

(1) exactly two values of (p, q).
(2) more than two but not all values of (p, q).
(3) all values of (p, q).
(4) exactly one value of (p, q).

47. Let the line = = lies in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals : **[AIEEE 2009]**

(1) (6, – 17) (2) (– 6, 7) (3) (5, – 15) (4) (– 5, 15)

48. The projections of a vector on the three coordinate axes are 6, –3, 2 respectively. The direction cosines of the vector are : **[AIEEE 2009]**

(1) 6, –3, 2 (2) , – , (3) , – , (4) – , – ,

49. Let and . Then the vector satisfying and is : **[AIEEE 2010]**

(1) (2) (3) (4)

50. If the vectors , and are mutually orthogonal, then (λ, μ) =

(1) (2, – 3) (2) (–2, 3) (3) (3, – 2) (4) (–3, 2) **[AIEEE 2010]**

51. **Statement -1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.
Statement -2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1,6) and B(1, 3, 4).

(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
(2) Statement-1 is true, Statement-2 is false. **[AIEEE 2010]**
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

52. A line AB in three-dimensional space makes angles 45º and 120º with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equal : **[AIEEE 2010]**

(1) 45º (2) 60º (3) 75º (4) 30º

53. If = and = , then the value of . is :

[AIEEE 2011]

(1) – 3 (2) 5 (3) 3 (4) – 5

54. The vector and are not perpendicular and and are two vectors satisfying : = and = 0. Then the vector is equal to : **[AIEEE 2011]**

(1) + (2) + (3) – (4) –

55. If the vector are coplanar, then the value of pqr – (p + q + r) is -

(1) 2 (2) 0 (3) – 1 (4) – 2 **[AIEEE 2011]**

56. Let be three non-zero vectors which are pairwise non-collinear. If is collinear with and + is collinear with , then is : **[AIEEE 2011]**

(1) (2) (3) (4)

57. If the angle between the line x = = and the plane x + 2y + 3z = 4 is $\cos^{-1}$, then λ equals : **[AIEEE 2011]**

(1) (2) (3) (4)

58. Statement-1 : The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line = = .

Statement-2 : The line = = bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).

[AIEEE 2011]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

59. The distance of the point (1, –5, 9) from the plane x – y + z = 5 measured along a straight line x = y = z is :

(1) (2) (3) (4) **[AIEEE 2011]**

60. The length of the perpendicular drawn from the point (3, – 1, 11) to the line is :

(1) (2) (3) (4) **[AIEEE 2011]**

61. Let and be two unit vectors. If the vectors and are perpendicular to each other, then the angle between and is : **[AIEEE 2012]**

(1) (2) (3) (4)

62. An equation of a plane parallel to the plane x – 2y + 2z – 5 = 0 and at a unit distance from the origin is : **[AIEEE 2012]**

(1) x – 2y + 2z – 3 = 0 (2) x – 2y + 2z + 1 = 0

(3) x – 2y + 2z – 1 = 0 (4) x – 2y + 2z + 5 = 0

63. If the lines = = and = = intersect, then k is equal to : **[AIEEE 2012]**

(1) –1 (2) (3) (4) 0

64. Let ABCD be a parallelogram such that and ∠BAD be and acute angle . If is the vector that coincides with the altitude directed from the vertex B to the side AD, then is given by : **[AIEEE 2012]**

(1) (2) (3) (4)

65. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is : **[AIEEE 2013]**

(1) (2) (3) (4)

66. If the lines and are coplanar, then k can have : **[AIEEE 2013]**

(1) any value (2) exactly one value

(3) exactly two values (4) exactly three values

67. If the vector and are the sides of triangle ABC, then the length of the median through A is : **[AIEEE 2013]**

(1) (2) (3) (4)

NCERT BOARD QUESTIONS

1. Find the unit vector in the direction of sum of vectors and .

2. If and , find the unit vector in the direction of

(A) (B)

3. Find a unit vector in the direction of , where P and Q have co-ordinates (5, 0, 8) and (3, 3, 2), respectively.

4. If are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that BC = 1.5 BA.

5. Using vector, find the value of k such that the points (k, – 10, 3), (1 – 1, 3) and (3, 5, 3) are collinear.

6. A vector is inclined at equal angles to the three axes. If the magnitude of is units, find

7. A vector has magnitude 14 and direction ratios 2, 3, – 6. Find the direction cosines and components of . given that makes an acute angle with x-axis.

8. Find a vector of magnitude 6, which is perpendicular to both the vector and .

9. Find the angle between the vectors and .

10. If = 0 , show that . Interpret the result geometrically ?

11. Find the sine of the angle between the vectors and

12. If A, B, C, D are the points with position vectors , , respectively, find the projection of along .

13. Using vectors, find the area of the triangle ABC with vertices A(1, 2, 3), B(2, –1, 4) and C(4, 5, –1).

14. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

15. Prove that in any triangle ABC, , where a, b, c are the magnitudes of the sides opposite to the vertices A, B, C, respectively.

16. If determine the vertices of a triangle, show that gives the vector area of the triangle. Hence deduce the condition that the three points are collinear. Also find the unit vector normal to the plane of the triangle.

17. Show that area of the parallelogram whose diagonals are given by and is . Also find the area of the parallelogram whose diagonals are and .

18. If and , find a vector such that and

19. Find the position vector of a point A in space such that is inclined at 60º to OX and at 45º to OY and = 10 units.

20. Find the vector equation of the line which is parallel to the vector and which passes through the point (1, – 2, 3).

21. Show that the lines

 and intersect. Also, find their point of intersection.

(–1, –1,–1)

22. Find the angle between the lines

23. Prove that the line through A(0, –1, –1) and B (4, 5, 1) intersects the line through C(3, 9, 4) and D(–4, 4, 4).

24. Prove that the lines x = py + q, z = ry + s and x = p'y + q', z = r'y + s' are perpendicular if pp' + rr' + 1 = 0.

25. Find the equation of a plane which bisects perpendicularly the line joining the points A (2, 3, 4) and B(4, 5, 8) at right angles.

26. Find the equation of a plane which is at a distance units from origin and the normal to which is equally inclined to coordinate axis.

27. If the line drawn from the point (–2, –1, –3) meets a plane at right angle at the point (1, –3, 3), find the equation of the plane.

28. Find the equation of the plane through the points (2, 1, 0), (3, –2, –2) and (3, 1, 7).

29. Find the equations of the two lines thought the origin which intersect the line at angles of each.

30. Find the angle between the lines whose direction cosines are given by the equation
$l + m + n = 0, \quad l^2 + m^2 - n^2 = 0$.

31. If a variable line in two adjacent positions has direction cosines l, m, n and l + δl, m + δm, n + δn, show that the small angle δθ between the two positions is given by

$$\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$$

32. O is the origin and A is (a, b , c). Find the direction cosines of the line OA and the equation of plane through A at right angle of OA.

33. Two systems of rectangular axis have the same origin. If a plane cuts them at distances a, b, c and a', b', c', respectively, from the origin, prove that

34. Find the foot of perpendicular from the point (2, 3, –8) to the line

. Also, find the perpendicular distance from the given point to the line.

35. Find the distance of a point (2, 4, –1) from the line

36. Find the length and the foot of perpendicular from the point to the plane 2x – 2y + 4z + 5 = 0.

37. Find the equation of the line passing through the point (3, 0, 1) and parallel to the planes x + 2y = 0 and 3y – z = 0.

38. Find the equation of the plane through the points (2, 1, –1) and (–1, 3, 4), and perpendicular to the plane x – 2y + 4z = 10.

39. Find the shortest distance between the lines given by

.

40. Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z + 8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0.

41. The plane ax + by = 0 is rotated about its line of intersection with the plane z = 0 through an angle α. Prove that the equation of the plane in its new position is ax + by ± .

42. Find the equation of the plane through the intersection of the planes and , whose perpendicular distance from origin is unity.

43. Show that the points and are equidistant from the plane +9=0 and lies on opposite side of it.

44. and are two vectors. The position vectors of the points A and C are and , respectively. Find the position vector of a point P on the line AB and point Q on the line CD such that is perpendicular to and both.

45. Show that the straight lines whose direction cosines are given by 2l + 2m – n = 0 and mn + nl + lm = 0 are at right angles.

46. If l_1, m_1 n_1 ; l_2, m_2, n_2;l_3,m_3,n_3 are the direction cosines of three mutually perpendicular lines, prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3$, m_2+ m_2+m_3, $n_1 + n_2 + n_3$ makes equal angles with them.

EXERCISE # 1

PART # I

A-1.	(C)	**A-2.**	(C)	**A-3.**	(B)	**A-4.**	(B)	**A-5.**	(A)	**B-1.**	(B)	**B-2.**	(D)
B-3.	(A)	**B-4.**	(B)	**B-5.**	(D)	**B-6.**	(D)	**B-7*.**	(A,C,D)	**B-8.**	(A)	**B-9.**	(D)
B-10.	(A)	**B-11.**	(C)	**B-12.**	(B)	**C-1.**	(C)	**C-2.**	(A)	**C-3.**	(C)	**C-4.**	(D)
C-5.	(A)	**C-6.**	(C)	**C-7.**	(C)	**C-8.**	(D)	**C-9.**	(C)	**C-10*.**	(A,C)	**C-11**	(D)
C-12.	(B)	**C-13.**	(C)	**C-14.**	(D)	**C-15.**	(B)	**C-16.**	(B)	**C-17.**	(A)	**C-18.**	(A,B,C)
C-19.	(C)	**C-20.**	(B)	**C-21.**	(B)	**C-22**	(A,D)	**D-1.**	(C)	**D-2.**	(A)	**D-3.**	(B)
D-4.	(A)	**D-5.**	(C)	**D-6.**	(C)	**D-7*.**	(B,C,D)	**D-8.**	(A)	**D-9.**	(A)	**D-10.**	(B)
D-11.	(B)	**D-12.**	(D)	**D-13.**	(C)	**D-14.**	(D)	**D-15.**	(D)	**D-16*.**	(A,B,C)	**D-17*.**	(A, B,C)
D-18.	(A)	**D-19.**	(B)	**D-20.**	(D)	**D-21.**	(A)	**D-22.**	(C)	**D-23.**	(B)	**D-24.**	(A)
D-25.	(D)	**D-26.**	(C)	**D-27.**	(C)	**D-28.**	(A,B)	**D-29.**	(B,D)	**D-30.**	(A)	**D-31.**	(A)
E-1.	(D)	**E-2.**	(A)	**E-3.**	(A)	**E-4.**	(A)(B)	**E-5.**	(D)	**E-6.**	(A)	**E-7.**	(B)

PART # II

A-1. OP : PD = 3 : 2 **A-4** (a/2, b/2, c/2) **A-6.** 3 : 2, (0,13/5, 1) **A-7.** (2/3, –2/3, –1/3)

B-1 (i) 60º **B-3.** (i) – + + (ii) (6 – +) (iii)

B-4. (i) 3 (ii) 16 **B-5.** (ii) 5 unit sq. **B-6.** 60º **B-7** 2 – 2

C-1. (i) $\sin\alpha\cos\alpha$ (ii) **C-2.** (i) p = 0; q = 10; r = – 3 (ii) –100

C-4. (i) No (ii) Yes **C-5.** **D-1.** x+y ± z = 1

D-2. . = 45 **D-3.** (i) r . = p (ii) . (q – p) = 0 **D-4.** unit

D-5. $\pi/2$ **D-7.** **D-8.** **D-9.** unit

D-11. (i) 6 unit^3 (ii) unit **D-12.** =

D-13. = = **D-14.** 11x – y – 3z = 35 **D-15.**

D-16. **D-17.** $2x + 3y + z + 4 = 0$ **D-18.** (2, 1, -3) ; $x + y + z = 0$

D-19. **E-1.** $x^2 + y^2 + z^2 - y - 2z - 14 = 0$,

PART # III

1. (A) → (r), (B) → (q), (C) → (q), (D) → (s)\ **2.** (A) → (p), (B) → (q), (C) → (r), (D) → (s)

3. (A) **4.** (A) **5.** (D) **6.** (B) **7.** (C) **8.** (A) **9.** (A)

10. (B) **11.** (C) **12.** (A) **13.** (C)

EXERCISE # 2

PART # I

1. (D) **2.** (B) **3.** (C) **4.** (B) **5.** (C) **6** (A) **7.** (C)

8. (D) **9.** (A) **10.** (C) **11.** (A) **12.** (C) **13.** (A) **14.** (A)

15. (B) **16.** (A) **17.** (D) **18.** (A) **19.** (C) **20.** (B,C) **21.** (A, B,C)

22. (A) **23.** (B, D) **24.** (A,B) **25.** (A) **26.** (B,D)

PART # II

1. 21 : 4 **4.** (i) = – – 8 + 2 (ii) **= 9**

9. = = , = = **11.** $7x + 13y + 4z - 9 = 0$;

12. = = **13.** $\alpha = -1$, **14.** $x + y + z = 0$

15. **16.** **17.** (b)

EXERCISE # 3

PART # I

1. (C) **3.** (A) **5.** (C) **6.** (A) **8.** (B) **13.** = –2 (.)

14. (D) **15.** $2x - y + z - 3 = 0$, $62x + 29y + 19z - 105 = 0$ **16.** (B,D) **17.** (A)

18. (C) **19.** (C) **20.** (B) **21.** (D) **22.** (A-r) (B-q) (C-p) (D-s) **23.** (A)

24. (A) **25.** (D) **26.** (B) **27.** (D) **28.** (C) **29.** (A-q,s) (B-prst), (C-t), (D-r)

30. (C) **31.** (A) **32.** 5 **33.** (B) **35.** 6 **36.** (A)

37. (A-t) (B-p) (C-q) (D-r) **38.** (C) **39.** (A,D) **41.** (A) **42.** (3) **43.** (A)

44. (C) **45.** (B,C) **46.** (D) **47.** (C) **48.** (D) **49.** (5) **50.** (A, D)

51. **(C)** **52.** **(A)**

PART # II

1.	(2)	**2.**	(2)	**3.**	(3)	**4.**	(4)	**5.**	(2)	**6.**	(4)	**7.**	(3)
8.	(3)	**9.**	(3)	**10.**	(4)	**11.**	(3)	**12.**	(4)	**13.**	(3)	**14.**	(3)
15.	(3)	**16.**	(4)	**17.**	(3)	**18.**	(3)	**19.**	(2)	**20.**	(1)	**21.**	(4)
22.	(4)	**23.**	(2)	**24.**	(1)	**25.**	(4)	**26.**	(3)	**27.**	(3)	**28.**	(4)
29.	(3)	**30.**	(2)	**31.**	(3)	**32.**	(3)	**33.**	(3)	**34.**	(4)	**35.**	(4)
36.	(4)	**37.**	(4)	**38.**	(4)	**39.**	(1)	**40.**	(4)	**41.**	(1)	**42.**	(4)
43.	(4)	**44.**	(3)	**45.**	(1)	**46.**	(4)	**47.**	(2)	**48.**	(3)	**49.**	(4)
50.	(4)	**51.**	(1)	**52.**	(2)	**53.**	(4)	**54.**	(3)	**55.**	(4)	**56.**	(3)
57.	(4)	**58.**	(1)	**59.**	(1)	**60.**	(3)	**61.**	(3)	**62.**	(1)	**63.**	(3)
64.	(2)	**65.**	**(3)**	**66.**	**(3)**	**67.**	**(3)**						

EXERCISE # 4

NCERT BOARD QUESTIONS

1. **2.** (i) (ii) **3.**

4. **5.** k = – 2 **6.** **7.**

8. **9.**

10. Area of the parallelograms formed by taking any two sides represented by and as adjacent are equal

11. **12.** **13.** **16.** **17.**

18. $\frac{1}{3}\left(5\vec{i}+2\vec{j}+2\vec{k}\right)$ **19.** $5\hat{i}+5\sqrt{2}\hat{j}+5\hat{k}$ **20.** $(x-1)\hat{i}+(y+2)\hat{j}+(z-3)\,\hat{k}=\lambda(3\hat{j}-2\hat{j}+6\hat{k}$ **21.** (–1, –1,–1)

22. $\cos^{-1}\left(\frac{19}{21}\right)$ **25.** x + y + 2z = 19 **26.** x + y + z = 9 **27.** 3x – 2y + 6z – 27 = 0

28. 21 x + 9y – 3z – 51 = 0 **29.** $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ and $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ **30.** 60º **32.** (1, 1)

33. 15º or 75º **34.** $(2,6,-2)3\sqrt{5}$ **35.** 7 **36.** $\sqrt{6}$

37. $(x-3)\hat{j}+y\hat{j}+(z-1)\,\hat{k}=\lambda(-2\hat{i}+\hat{j}+3\hat{k})$ **38.** 18x + 17y + 4z = 49 **39.** 14

40. 51x + 15y – 50z + 173 = 0 **42.** 4x + 2y – 4z – 6 = 0 and – 2x + 4 y + 4z – 6 = 0

44. $3\hat{i}+8\hat{j}+3\hat{k},\ -3\hat{i}-y\hat{j}+6\hat{k}$