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SEQUENCE AND SERIES

Contents

Syllabus

Arithmetic, geometric and harmonic progressions, arithmetic, geometric and harmonic means, sums of finite arithmetic and geometric progressions, infinite geometric series, sum of squares and cubes of the first n natural numbers.

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SEQUENCE AND PROGRESSION

KEY CONCEPTS

DEFINITION :

A sequence is a set of terms in a definite order with a rule for obtaining the terms. e.g. 1, 1/2, 1/3, ..., 1/n, ... is a sequence.

AN ARITHMETIC PROGRESSION (AP) :

AP is sequence whose terms increase or decrease by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference , then AP can be written as a, a + d, a + 2d, ... a + (n-1) d , ...

n^{th} term of this AP $t_n = a + (n-1) d$, where $d = a_n - a_{n-1}$

The sum of the first n terms of the AP is given by ; $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + \ell]$

Where ℓ is the last term .

Notes: (i) If each term of an A.P. is increased, decreased, multiplied or divided by the same nonzero number, then the resulting sequence is also an AP.

(ii) Three numbers in AP can be taken as a - d, a, a + d ; four numbers in AP can be taken as a, a - 3d, a - d, a + d, a + 3d ‘ five numbers in AP are a - 2d, a - d, a, a +d, a +2d & six terms in AP are a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d etc.

(iii) The common difference can be zero, positive or negative.

(iv) The sum of the two terms of an AP equidistant from the begining & end is constant and equal to the sum of first & last terms.

(v) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it. $a_n = 1/2\ (a_{n-k} + a_{n+k}),\ k < n$ For $k = 1,\ a_n = (1/2)\ (a_{n-1} + a_{n+1})$; For $k = 2,\ a_n = (1/2)\ (a_{n-2} + a_{n+2})$ and so on.

(vi) $t_r = S_r - S_{r-1}$

(vii) If a, b, c are in AP $\Rightarrow$ 2b = a + c.

GEOMETRIC PROGRESSION (GP) :

GP is a sequence of numbers whose first term is non-zero & each of the succeeding terms is equal to to the proceeding terms multiplied by a constant. Thus in a GP the ratio of successive term is constant. This constant factor is called the **COMMON RATIO** of the series & is obtained by dividing any term by that which immediately proceeds it. Therefore a, ar, ar^2, ar^3, ar^4 , ... is a GP with **a** as the first term & **r** as common ratio.

(i) n^{th} term $= a\, r^{\,n-1}$

(ii) Sum of the I^{st} n terms i.e. $S_n = \frac{a(r^n - 1)}{r-1}$, if $r \neq 1$,

(iii) sum of infinite GP when $|r| < 1$ when $n \to \infty$ $r^n \to 0$ if $|r| < 1$ therefore, $S_\infty = \frac{a}{1-r}$ $|r| < 1$

(iv) If each term of a GP be multiplied or divided by the same non-zero, quantity, the resulting sequence is also a GP.

(v) Any 3 cossecutive terms of a GP can be taken as a/r, a, ar ; Any 4 cossecutive

(v) Any 3 cossecutive terms of a GP can be taken as a/r, a, ar ; Any 4 cossecutive terms of a GP can be taken as a/r^3, a/r, ar, ar^3 & so on.

(vi) If a, b, c are in $GP \Rightarrow b^2 = ac$.

HARMONIC PROGRESSION (HP) :

A sequence is said to HP if the reciprocals of its terms are in AP.

If the sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ is an HP then $1/a_1, 1/a_2, \ldots\ldots, 1/a_n$ is an AP & converse. Here we do not have the formula for the sum of the n terms of an HP. For HP whose first term is a & second term is b, the n^{th} term is $t_n = \frac{ab}{b+(n-1)(a-b)}$.

If a, b, c are in HP $\Rightarrow$ b = $\frac{2ac}{a+c}$ or $\frac{a}{c} = \frac{a-b}{b-c}$

MEANS

AIRTHMATIC MEAN :

If three terms are in AP then the middle term is called the AM between the order two, so if a, b, c are in AP, b is AM of a & c.

AM for any n positive number $a_1, a_2, \ldots\ldots a_n$ is ; $A = \frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n}$

n - AIRTHMATIC MEANS BETWEEN TWO NUMBERS :

If a,b are any two given numbers & a, A_1, A_2, ..., A_n, b are in AP then $A_1, A_2, \ldots\ldots A_n$ are the n AM 's between a & b.

$A_1 = a + \frac{b-a}{n+1}$, $A_2 = a + \frac{2\ (b-a)}{n+1}$, ..., $A_n = a + \frac{n\ (b-a)}{n+1}$

= a + d , = a + 2d ,......., A_n = a + nd , where $d = \frac{b-a}{n+1}$

Note : Sum of n AM's inserted between a & b is equal to n times the single AM between a & b i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single AM between a & b.

GEOMETRIC MEANS :

If a, b, c are in GP, b is the GM between a & c.

b^2 = ac, therefore $b = \sqrt{ac}$; a > 0, c > 0.

n- GEOMETRIC MEANS BETWEEN a, b :

If a, b are two given numbers & a, G_1, G_2, ..., G_n, b are in GP. Then $G_1, G_2, G_3, \ldots\ldots G_n$ are n GMs between a & b.

$G_1 = a(b/a)^{1/n+1}$, $G_2 = a(b/a)^{2/n+1}$, ..., $G_n = a(b/a)^{n/n+1}$

= ar, = ar^2, ... = ar^n, where $r = (b/a)^{1/n+1}$

Note : The product of n GMs between a & b. is equal to n^{th} power of the single GM between a & b i.e. $\pi_{r=1}^{n} G_r = (G)^n$ where G is the single GM between a & b

HARMONIC MEAN :

If a, b, c are in HP, between a & c, then b = 2ac / [a + c] .

Relation between means :

(i) If A, G, H are respectively A.M., G.M., H.M. between a & b both being unequal & positive then, $G^2 = AH$ (i.e. A, G, H are in G.P.) and $A \geq G \geq H$.

(ii) **A.M. ≥ G.M. ≥ H.M.**

Let $a_1, a_2, a_3, ...a_n$ be n positive real numbers, then we define their

A.M. = $\frac{a_1 + a_2 + a_3 + ... + a_n}{n}$, their

G.M. = $(a_1 a_2 a_3 ....a_n)^{1/n}$ and their

H.M. = $\frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}}$.

It can be shown that

A.M. ≥ G.M. ≥ H.M. and equality holds at either places iff

$a_1 = a_2 = a_3 = ...= a_n$

ARITHMETICO - GEOMETRIC SERIES :

A series each term of which is formed by multiplying the corresponding term of an AP & GP is called the Arithmetico-Geometric Series , e.g. $1 + 3x + 5x^2 + 7x^3 + ...$

Here 1, 3, 5, ... are in AP & 1, x, x^2, x^3 .... are in GP..

Sum of n terms of an Arithmetico-Geometric Series :

Let $S_n = a + (a + d)r + (a + 2d)r^2 + ... + [a + (n - 1)d]\ r^{n-1}$

then $S_n = \frac{a}{1-r} + \frac{dr\ (1-r^{n-1})}{(1-r)^2} - \frac{[a+\ (n-1)d]\ r^n}{1-r}, \quad r \neq 1$

SUM TO INFINITY :

If | r | < 1 & n → ∞ then $\lim_{n\to\infty} r^n = 0. S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

SIGMA NOTATIONS

THEOREM :

(i) $\sum_{r=1}^{n} (a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$ (ii) $\sum_{r=1}^{n} k\ a_r = k \sum_{r=1}^{n} a_r$ (iii) $\sum_{r=1}^{n} k = nk$; where k is a constant

RESULTS

(i) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ (sum of the first n natural nos.)

(ii) $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(iii) $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} \quad \left[\sum_{r=1}^{n} r\right]^2$ (sum of the cubes of the first n natural numbers)

(iv) $\sum_{r=1}^{n} r^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$

Method of difference for finding nth term :

Let u_1, u_2, u_3 .... be a sequence, such that $u_2 - u_1, u_3 - u_2$, ... is either an A.P. or a G.P. then nth term u_n of this sequence is obtained as follows

$S = u_1 + u_2 + u_3 + \ldots\ldots\ldots + u_n$ ....(i)

$S = \quad u_1 + u_2 + \ldots\ldots\ldots + u_{n-1} + u_n$ ....(ii)

(i) – (ii) $\Rightarrow u_n = u_1 + (u_2 - u_1) + (u_3 - u_2) + \ldots\ldots\ldots + (u_n - u_{n-1})$

Where the series $(u_2 - u_1) + (u_3 - u_2) + \ldots\ldots\ldots + (u_n - u_{n-1})$ is

either in A.P. or in G.P. then we can find u_n.

Note : The above method can be generalised as follows :

Let u_1, u_2, u_3, ... be a given sequence.

The first differens aee $\Delta_1 u_1, \Delta_1 u_2, \Delta_1 u_3$, ... where $\Delta_1 u_1 = u_2 - u_1$, $\Delta_2 u_2 = u_2 - u_2$ etc.

The second differences are $\Delta_2 u, \Delta_2 u_2, \Delta_2 u_3$, ..., where $\Delta_2 u_1 = \Delta_1 u_2 - \Delta_1 u_1$, $\Delta_2 u_2 = \Delta_1 u_3 - \Delta_1 u_2$ etc.

This process is continued unitil the kth differences $\Delta_k u_1$, $\Delta_k u_2$, ... are obtained, where the kth difference are all equal or they form a GP with common ratio different form 1.

Case - 1 : The kth differences are all equal.

In this case the nth term, u_n is given by

$u_n = a_0 n^k + a_1 n^{k-1} + \ldots + a_k$, where $a_0, a_1, \ldots, a_k$ are calculated by using first 'k + 1' terms of the sequence.

Case - 2 : The kth differences are in GP with common ratio r ($r \neq r$)

The nth term is given by $u_n = \lambda r^n + a_0 n^{k-1} + a_1 n^{k-2} + \ldots + a_{k-1}$

Method of difference for finding s_n :

If possible express rth term as difference of two terms as $t_r = \pm (f(r) - f(r \pm 1))$. This can be explained with the help of examples given below.

$t_1 = f(1) - f(0)$,

$t_2 = f(2) - f(1)$,

....

$t_n = f(n) - f(n-1)$

$\Rightarrow \quad S_n = f(n) - f(0)$

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

* ***Marked Questions are having more than one correct option.***

Section (A) : Arithmetic Progression

A-1. Which term of the series 3 + 8 + 13 + 18 + ... is 498-
(A) 95^{th} (B) 100^{th} (C) 102^{th} (D) 101^{th}

A-2. If fourth term of an A.P. is thrice its first term and seventh term – 2 (third term) = 1, then its common difference is-
(A) 1 (B) 2 (C) – 2 (D) 3

A-3. The first term of an A.P. of consecutive integer is $p^2 + 1$. The sum of (2p + 1) terms of this series can be expressed as
(A) $(p + 1)^2$ (B) $(2p + 1)(p + 1)^2$ (C) $(p + 1)^3$ (D) $p^3 + (p + 1)^3$

A-4. The sum of integers from 1 to 100 that are divisible by 2 or 5 is
(A) 2550 (B) 1050 (C) 3050 (D) none of these

A-5. If $a_1, a_2, a_3,$........ are in A.P. such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + ... + a_{23} + a_{24}$ is equal to
(A) 909 (B) 75 (C) 750 (D) 900

A-6. The interior angles of a polygon are in A.P. If the smallest angle is 120º & the common difference is 5º, then the number of sides in the polygon is:
(A) 7 (B) 9 (C) 16 (D) none

A-7. Consider an A.P. with first term 'a' and the common difference 'd'. Let S_k denote the sum of its first K terms. If $\frac{S_{kx}}{S_x}$ is independent of x, then
(A) a = d/2 (B) a = d (C) a = 2d (D) none

A-8. There are n A.M's between 3 and 54, such that the 8th mean: $(n - 2)^{th}$ mean:: 3: 5. The value of n is.
(A) 12 (B) 16 (C) 18 (D) 20

A-9. The A.M. between two numbers is A, and S is the sum of n arithmetic means between these numbers, then :
(A) S = n A (B) A = n S (C) A = S (D) none of these

A-10. If the root of the equation $x^3 - 12x^2 + 39x - 28 = 0$, are in A.P., then their common difference, will be :
(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

A-11*. For the A.P. given by $a_1, a_2,$............., a_n,..........., the equations satisfied are
(A) $a_1 + 2a_2 + a_3 = 0$ (B) $a_1 - 2a_2 + a_3 = 0$
(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$ (D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

Section (B): Geometric progeression

B-1. The third term of a G.P is 4. The product of the first five terms is
(A) 4^3 (B) 4^5 (C) 4^4 (D) none of these

B-2. The sum of the series $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + ... + \frac{1}{\log_{2^n} 4}$ is

(A) $\frac{1}{2}$ n (n + 1) (B) $\frac{1}{12}$ n (n + 1) (2n + 1)
(C) $\frac{1}{n(n+1)}$ (D) $\frac{1}{4}$ n (n + 1)

B-3. α, β be the roots of the equation $x^2 - 3x + a = 0$ and γ, δ the roots of $x^2 - 12x + b = 0$ and numbers $\alpha, \beta, \gamma, \delta$ (in this order) form an increasing G.P., then
(A) a = 3, b = 12 (B) a = 12, b = 3 (C) a = 2, b = 32 (D) a = 4, b = 16

B-4. The rational number, which equals the number $2.\overline{357}$ with recurring decimal is

(A) $\frac{2355}{1001}$ (B) $\frac{2379}{997}$ (C) $\frac{2355}{999}$ (D) none of these

B-5*. If sum of the infinite G.P., p, 1, $\frac{1}{p}, \frac{1}{p^2}, \frac{1}{p^3}$,....... is $\frac{9}{2}$, the value of p is

(A) 3 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{1}{3}$

B-6*. Indicate the correct alternative(s), for $0 < \phi < \pi/2$, if:

$x = \sum_{n=0}^{\infty} \cos^{2n}\phi$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$, $z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$ then:

(A) xyz = xz + y (B) xyz = xy + z (C) xyz = x + y + z (D) xyz = yz + x

Section (C) : Harmonic Progression, AGP, Relation between AM, GM, HM & Miscellaneous

C-1. Suppose a, b, c are in A.P. & $|a|, |b|, |c| < 1$. If $x = 1 + a + a^2 +......$ to ∞;
$y = 1 + b + b^2 +......$ to ∞ & $z = 1 + c + c^2 +......$ to ∞ then x, y, z are in:
(A) A.P. (B) G.P. (C) H.P. (D) none

C-2*. If positive numbers a, b, c are in A.P. and a^2, b^2, c^2 are in H.P., then

(A) a = b = c (B) 2b = a + c (C) $b^2 = \sqrt{\frac{ac}{8}}$ (D) none of these

C-3. If the sum of the roots of the quadratic equation, $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in

(A) A.P. (B) G.P. (C) H.P. (D) none

C-4. If $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, then a,b,c are in :

(A) A.P (B) G.P (C) H.P (D) none of these

C-5. If $a^x = b^y = c^z = d^t$ and a, b, c, d are in G.P., then x, y, z, t are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

C-6. If $3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + ... +$ upto $\infty = 8$, then the value of d is:

(A) 9 (B) 5 (C) 1 (D) none of these

C-7. The H.M. between two numbers is 4, their A.M. is A and G.M. is G. If $2A + G^2 = 27$, then the numbers are :

(A) 8, 2 (B) 8, 6 (C) 6, 3 (D) 6, 4

C-8. If A, G & H are respectively the A.M., G.M. & H.M. of three positive numbers a, b, & c, then the equation whose roots are a, b, & c is given by:

(A) $x^3 - 3Ax^2 + 3G^3x - G^3 = 0$ (B) $x^3 - 3Ax^2 + 3(G^3/H)x - G^3 = 0$

(C) $x^3 + 3Ax^2 + 3(G^3/H)x - G^3 = 0$ (D) $x^3 - 3Ax^2 - 3(G^3/H)x + G^3 = 0$

C-9*. If the arithmetic mean of two positive numbers a & b (a > b) is twice their geometric mean, then a: b is:

(A) $2 + \sqrt{3} : 2 - \sqrt{3}$ (B) $7 + 4\sqrt{3} : 1$ (C) $1 : 7 - 4\sqrt{3}$ (D) $2 : \sqrt{3}$

C-10. The sum $\sum_{r=2}^{\infty} \frac{1}{r^2 - 1}$ is equal to:

(A) 1 (B) 3/4 (C) 4/3 (D) none

C-11*. If $\sum_{r=1}^{n} r(r+1)(2r + 3) = an^4 + bn^3 + cn^2 + dn + e$, then

(A) $a + c = b + d$ (B) $e = 0$

(C) a, b – 2/3, c – 1 are in A.P. (D) c/a is an integer

C-12. The sum of the first n-terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, the sum is

(A) $\frac{n(n+1)^2}{4}$ (B) $\frac{n^2(n+2)}{4}$ (C) $\frac{n^2(n+1)}{2}$ (D) $\frac{n(n+2)^2}{4}$

PART - II : MISCELLANEOUS OBJECTIVE QUESTIONS

Comprehension # 1

We know that $1 + 2 + 3 + ... = \frac{n(n+1)}{2} = f(n)$,

$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6} = g(n)$,

$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2 = h(n)$

1. $g(n) - g(n-1)$ must be equal to
(A) n^2 (B) $(n-1)^2$ (C) $n - 1$ (D) n^3

2. Greatest even natural number which divides $g(n) - f(n)$, for every $n \geq 2$, is
(A) 2 (B) 4 (C) 6 (D) none of these

3. $f(n) + 3\,g(n) + h(n)$ is divisible by $1 + 2 + 3 + ... + n$
(A) only if n = 1 (B) only if n is odd (C) only if n is even (D) for all $n \in N$

Comprehension # 2

There are 4n + 1 terms in a sequence of which first 2n + 1 are in A. P. and last 2n + 1 are in G. P. the common difference of A. P. is 2 and common ratio of G. P. is 1/2 . The middle term of the A. P. is equal to middle term of G. P. Let middle term of the sequence is a_m and a_m is the sum of infinite G. P. Whose sum of first two terms is $\left(\frac{5}{4}\right)^2$ n and ratio of these terms is $\frac{9}{16}$.

4. First term of given infinite G. P. is equal to :
(A) n/2 (B) n (C) – n (D) 9/16

5. Number of terms in the given sequence is equal to :
(A) 9 (B) 17 (C) 13 (D) none of these

6. Middle term of the given sequence, i.e. a_m is equal to :
(A) 16/7 (B) 32/7 (C) 48/7 (D) 16/9

7. First term of given sequence is equal to :
(A) –8/7, –20/7 (B) –36/7 (C) 36/7 (D) 48/7

8. Middle term of given A. P. is equal to :
(A) 6/7 (B) 10/7 (C) 78/7 (D) 11

Match the column

9. **Column – I** | **Column – II**

	Column – I		Column – II
(A)	If $\log_5 2$, $\log_5(2^x - 5)$ and $\log_5(2^x - 7/2)$ are in A.P., then value of 2x is equal to	(p)	6
(B)	Let S_n denote sum of first n terms of an A.P. If $S_{2n} = 3S_n$, then $\frac{S_{3n}}{S_n}$ is	(q)	9
(C)	Sum of infinite series $4 + \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3} + \text{- - -}$ is	(r)	2
(D)	The value of $2^{\frac{1}{4}} \cdot 4^{\frac{1}{8}} \cdot 8^{\frac{1}{16}} \text{- - -} \infty =$	(s)	1

10.

	Column - I		Column - II
(A)	If a_i's are in A.P. and $a_1 + a_3 + a_4 + a_5 + a_7 = 20$, a_4 is equal to	(p)	21
(B)	Sum of an infinite G.P. is 6 and it's first term is 3. then harmonic mean of first and third terms of G.P. is	(q)	4
(C)	$\sum_{r=2}^{\infty} \frac{2}{r(r^2 - 1)}$ is equal to	(r)	1/2
(D)	If roots of the equation $x^3 - ax^2 + bx + 27 = 0$, are in G.P. with common ratio 2, then a + b is	(s)	6/5

ASSERSION-REASON TYPE

This section contains 1 questions numbered QNo.(1-3) to Each question contains Statement–1 (Assertion) and Statement–2 (Reason). For the following questions 4 answers (A), (B), (C) and (D) are given below, of which only one is correct.

(A) Statement -1 is true, Statement - 2 is true ; Statement - 2 is correct explanation for Statement - 1

(B) Statement -1 is true, Statement - 2 is true ; Statement - 2 is **NOT** correct explanation for Statement - 1

(C) Statement -1 is true, Statement - 2 is false.

(D) Statement -1 is false, Statement - 2 is true.

11. **STATEMENT-1 :** 1, 2, 4, 8, ... is a G.P., 4, 8, 16, 32 is a G.P. and 1 + 4, 2 + 8, 4 + 16, 8 + 32, ... is also a G.P.

STATEMENT-2 : Let general term of a G.P. with common ratio r be T_{k+1} and general term of another G.P. with common ratio r be T'_{k+1}, then the series whose general term $T''_{k+1} = T_{k+1} + T'_{k+1}$ is also a G.P. with common ratio r.

12. **Statement 1 :** 3,6,12 are in G.P., then 9,12,18 are in H.P.

Statement 2 : If middle term is added in three consecutive terms of a G.P, resultant will be in H.P.

13. **Statement -1 :** In the set of natural numbers sum of first 'n' prime numbers is even or odd according as n is odd or even respectively.

Statement - 2 : Since all prime numbers are odd, sum is even when number of primes are even and odd when number of primes are odd.

EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

1. The sum of n terms of an A.P. is an(n – 1). The sum of the squares of these terms is :

(A) $a^2n^2 (n-1)^2$
(B) $\frac{a^2}{6}n\ (n-1)\ (2n-1)$
(C) $\frac{2a^2}{3}n\ (n-1)\ (2n-1)$
(D) $\frac{2a^2}{3}n\ (n+1)\ (2n+1)$

2. If $a_1, a_2, a_3, ...$ are the terms of an AP such that $\frac{a_4}{a_7} = \frac{2}{3}$ then $\frac{a_6}{a_8}$ is :

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{1}{2}$

3. If $a_1, a_2, ... a_n$ are in A.P. with common difference $d \neq 0$, then the sum of the series
$(\sin d)\ [\text{cosec } a_1 \text{ cosec } a_2 + \text{cosec } a_2 \text{ cosec } a_3 + ... + \text{cosec } a_{n-1} \text{ cosec } a_n]$
(A) $\sec a_1 - \sec a_n$
(B) $\text{cosec } a_1 - \text{cosec } a_n$
(C) $\cot a_1 - \cot a_n$
(D) $\tan a_1 - \tan a_n$

4. In a G.P. of positive terms, any term is equal to the sum of the next two terms. The common ratio of the G.P. is
(A) 2 cos 18° (B) sin 18° (C) cos 18° (D) 2 sin 18°

5. If $a, a_1, a_2, a_3, ..., a_{2n}$, b are in A.P. and $a, g_1, g_2, g_3, ...g_{2n}$, b are in G.P. and h is the harmonic mean of a and b, then $\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + ... + \frac{a_n + a_{n+1}}{g_n g_{n+1}}$ is equal to

(A) $\frac{2n}{h}$ (B) 2nh (C) nh (D) $\frac{n}{h}$

6. One side of an equilateral triangle is 24 cm. The mid–points of its sides are joined to form another triangle whose mid – points are in turn joined to form still another triangle. This process continues indefinitely. Then the sum of the perimeters of all the triangles is
(A) 144 cm (B) 212 cm (C) 288 cm (D) none of these

7. If the sum of n terms of a G.P. (with common ratio r) beginning with the p^{th} term is k times the sum of an equal number of terms of the same series beginning with the q^{th} term, then the value of k is:

(A) $r^{p/q}$ (B) $r^{q/p}$ (C) r^{p-q} (D) r^{p+q}

8. If A_1, A_2 be two A.M.s and G_1, G_2 be two G.M.s between a and b, then $\frac{A_1 + A_2}{G_1 G_2}$ is equal to

(A) $\frac{a+b}{2ab}$ (B) $\frac{2ab}{a+b}$ (C) $\frac{a+b}{ab}$ (D) $\frac{a+b}{\sqrt{ab}}$

9. If P, Q be the A.M., G.M. respectively between any two rational numbers a and b, then P – Q is equal to

(A) $\frac{a-b}{a}$ (B) $\frac{a+b}{2}$ (C) $\frac{2ab}{a+b}$ (D) $\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{2}}\right)^2$

10. If $x_i > 0$, i = 1, 2, ..., 50 and $x_1 + x_2 + .. + x_{50} = 50$, then the minimum value of

$\frac{1}{x_1} + \frac{1}{x_2} + ...+ \frac{1}{x_{50}}$ equal to

(A) 50 (B) $(50)^2$ (C) $(50)^3$ (D) $(50)^4$

11. If there are n H.M. between 1 and $\frac{1}{31}$ and the ratio of 7^{th} H.M. to $(n - 1)^{th}$ H.M. is 9 : 5, then n will be :

(A) 12 (B) 13 (C) 14 (D) 15

12. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... =$

(A) $\pi^2/12$ (B) $\pi^2/24$ (C) $\pi^2/8$ (D) none of these

13. Sum of the series
$S = 1^2 - 2^2 + 3^2 - 4^2 + ... - 2002^2 + 2003^2$ is
(A) 2007006 (B) 1005004 (C) 2000506 (D) none of these

14. If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + ...+ \frac{1}{n}$, then value of $1 + \frac{3}{2} + \frac{5}{3} + ... + \frac{2n-1}{n}$ is

(A) $2n - H_n$ (B) $2n + H_n$ (C) $H_n - 2n$ (D) $H_n + n$

15. If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, their cubes respectively, then $\frac{S_3(1+8S_1)}{S_2^2}$ is equal to
(A) 1 (B) 3 (C) 9 (D) 10

Multiple choice

16. The sides of a right triangle form a G.P. The tangent of the smallest angle is

(A) $\sqrt{\frac{\sqrt{5}+1}{2}}$ (B) $\sqrt{\frac{\sqrt{5}-1}{2}}$ (C) $\sqrt{\frac{2}{\sqrt{5}+1}}$ (D) $\sqrt{\frac{2}{\sqrt{5}-1}}$

17. If b_1, b_2, b_3 ($b_i > 0$) are three successive terms of a G.P. with common ratio r, the value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds is given by
(A) $r > 3$ (B) $0 < r < 1$ (C) $r = 3.5$ (D) $r = 5.2$

PART - II : SUBJECTIVE QUESTIONS

1. In an A.P. the third term is four times the first term, and the sixth term is 17 ; find the series.

2. The third term of an A.P. is 18, and the seventh term is 30 ; find the sum of 17 terms.

3. How many terms of the series – 9, – 6, – 3, ... must be taken that the sum may be 66 ?

4. Find the number of integers between 100 & 1000 that are
(i) divisible by 7 (ii) not divisible by 7

5. Find the sum of all those integers between 100 and 800 each of which on division by 16 leaves the remainder 7.

6. Find the sum of 35 terms of the series whose p^{th} term is $\frac{p}{7} + 2$.

7. The sum of three numbers in A.P. is 27, and their product is 504, find them.

8. If a, b, c are in A.P., then show that:

(i) $a^2 (b + c), b^2 (c + a), c^2 (a + b)$ are also in A.P.

(ii) $b + c - a, c + a - b, a + b - c$ are in A.P.

9. Show that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ cannot be the terms of a single A.P.

10. If the sum of m terms of an A.P. is equal to the sum of the n terms and the next p terms, then prove that

$$(m + n)\left(\frac{1}{m}-\frac{1}{p}\right) = (m + p)\left(\frac{1}{m}-\frac{1}{n}\right).$$

11. The third term of a G.P. is the square of the first term. If the second term is 8, find its sixth term.

12. The continued product of three numbers in G.P. is 216, and the sum of the products of them in pairs is 156; find the numbers

13. If the p^{th} , q^{th} , r^{th} terms of a G.P. be a, b, c respectively, prove that $a^{q-r}\, b^{r-p}\, c^{p-q} = 1$.

14. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 & the third is increased by 1, we obtain three consecutive terms of a G.P., find the numbers.

15. If the p^{th}, q^{th} & r^{th} terms of an AP are in GP. Find the common ratio of the GP.

16. The sum of infinite number of terms of a G.P. is 4 and the sum of their cubes is 192. Find the series.

17. If a, b, c, d are in G.P., prove that :

(i) $(a^2 - b^2), (b^2 - c^2), (c^2 - d^2)$ are in G.P.

(ii) $\frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2}, \frac{1}{c^2 + d^2}$ are in G.P.

18. In a circle of radius R a square is inscribed, then a circle is inscribed in the square, a new square in the circle and so on for n times. Find the limit of the sum of areas of all the circles and the limit of the sum of areas of all the squares as $n \to \infty$.

19. The sum of the first ten terms of an AP is 155 & the sum of first two terms of a GP is 9. The first term of the AP is equal to the common ratio of the GP & the first term of the GP is equal to the common difference of the AP. Find the two progressions.

20. If $0 < x < \pi$ and the expression

$$\exp\{(1 + |\cos x| + \cos^2 x + |\cos^3 x| + \cos^4 x + \ldots\ldots. \text{ upto } \infty) \log_e 4\}$$

satisfies the quadratic equation $y^2 - 20y + 64 = 0$ the find the value of x.

21. Find the 4^{th} term of an H.P. whose 7^{th} term is $\frac{1}{20}$ and 13^{th} term is $\frac{1}{38}$.

22. Given that α, γ are roots of the equation, $A x^2 - 4 x + 1 = 0$ and β, δ the roots of the equation, $B x^2 - 6 x + 1 = 0$, find values of A and B, such that α, β, γ & δ are in H.P.

23. Sum the following series

(i) $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots\ldots..$ to n terms.

(ii) $1 + \frac{3}{4} + \frac{7}{16} + \frac{15}{64} + \frac{31}{256} + \ldots..$ to infinity.

24. Find the sum of n terms of the series the r^{th} term of which is $(2r + 1)2^r$.

25. Find the sum of the series $\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \ldots$ up to ∞

26. The arithmetic mean of two numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$. Find the two numbers.

27. If between any two quantities there be inserted two arithmetic means A_1, A_2; two geometric means G_1, G_2; and two harmonic means H_1, H_2 then prove that $G_1 G_2 : H_1 H_2 = A_1 + A_2 : H_1 + H_2$.

28. If 9 AMs and again 9 HMs are inserted between 2 and 3 then prove that $A + \frac{6}{H} = 5$, A is any AM and H the corresponding HM.

29. Using the relation A.M. $\geq$ G.M. prove that

(i) $\tan\theta + \cot\theta \geq 2$; if $0 < \theta < \frac{\pi}{2}$

(ii) $(x^2y + y^2z + z^2x)(xy^2 + yz^2 + zx^2) \geq 9x^2y^2z^2$. (x, y, z are positive real number)

(iii) $(a + b) . (b + c) . (c + a) \geq abc$; if a, b, c are positive real numbers

30. If a, b, c are positive real numbers then prove that
$b^2c^2 + c^2a^2 + a^2b^2 \geq abc(a + b + c)$.

31. Find the sum of the n terms of the series whose nth term is
(i) $n(n + 2)$ (ii) $3^n - 2^n$

32. Find the sum to n-terms of the sequence.
(i) 1 + 5 + 13 + 29 + 61 + ... to n-terms.
(ii) 3 + 33 + 333 + 3333 + ... to n terms.

33. Find the sum in the n^{th} group of sequence,
(i) 1, (2, 3); (4, 5, 6, 7); (8, 9,........, 15); ....
(ii) (1), (2, 3, 4), (5, 6, 7, 8, 9),........

34. Find the sum to n-terms of the sequence.

(i) $\frac{1}{1.3.5} + \frac{1}{3.5.7} + \frac{1}{5.7.9} + ...$

(ii) $1 . 3 . 2^2 + 2 . 4 . 3^2 + 3 . 5 . 4^2 + ...$

35. Sum the following series to n terms.

(i) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$

(ii) $\frac{n}{1.2.3} + \frac{n-1}{2.3.4} + ... + \frac{1}{n(n+1)(n+2)}$.

36. Sum of the following series

(i) $1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + ... \infty$.

(ii) $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + ...\infty$

37. If a, b, c are positive real numbers and sides of the triangle then prove that
$(a + b + c)^3 \geq 27(a + b - c)(c + a - b)(b + c - a)$

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

1. If $a_1, a_2, a_3, ..., a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + a_3 + ... + a_{n-1} + 2a_n$ is **[IIT - 2002, 3]**

(A) $n(2c)^{1/n}$ (B) $(n + 1)\, c^{1/n}$ (C) $2nc^{1/n}$ (D) $(n + 1)(2c)^{1/n}$

2. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. if $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is **[IIT - 2002, 3]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

3. Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression, show that

$$\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a + b)\ (a + 2b)}{9ab}.$$ **[IIT – 2002, 5]**

4. If $\alpha \in \left(0, \frac{\pi}{2}\right)$ then $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to: **[IIT- 2003, 3]**

(A) $2 \tan \alpha$ (B) 1 (C) 2 (D) $\sec^2 \alpha$

5. If a, b & c are in arithmetic progression and a^2, b^2 & c^2 are in harmonic progression, then prove that either $a = b = c$ or a, b & $-\frac{c}{2}$ are in geometric progression. **[IIT – 2003, 4]**

6. An infinite G.P. has first term as x and sum upto infinity as 5. Then the range of values of 'x' is: **[IIT - 2004, 3]**

(A) $x \le -10$ (B) $x \ge 10$ (C) $0 < x < 10$ (D) $-10 \le x \le 10$

7. In the quadratic equation $ax^2 + bx + c = 0$, $\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$, are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then **[IIT - 2005]**

(A) $\Delta \neq 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

8. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ...(-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$, then find the minimum natural number n_0 such that $b_n > a_n\ \forall\ n > n_0$ **[IIT – 2006, 6]**

Comprehension **[IIT - 2007]**

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r – 1). Let

$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ...

9. The sum $V_1 + V_2 + ... + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

10. T_r is always

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

11. Which one of the following is a correct statement ?

(A) Q_1, Q_2, Q_3,...... are in A.P. with common difference 5

(B) Q_1, Q_2, Q_3,...... are in A.P. with common difference 6

(C) Q_1, Q_2, Q_3,..... are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = ...$

Comprehension **[IIT - 2007]**

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

12. Which one of the following statements is correct ?

(A) $G_1 > G_2 > G_3 > ....$ (B) $G_1 < G_2 < G_3 < ......$

(C) $G_1 = G_2 = G_3 = ...$ (D) $G_1 < G_3 < G_5 < ......$ and $G_2 > G_4 > G_6 > ....$

13. Which one of the following statements is correct ?

(A) $A_1 > A_2 > A_3 > ......$ (B) $A_1 < A_2 < A_3 < ......$

(C) $A_1 > A_3 > A_5 > ....$ and $A_2 < A_4 < A_6 < ....$ (D) $A_1 < A_3 < A_5 < ....$ and $A_2 > A_4 > A_6 > ....$

14. Which one of the following statements is correct ?

(A) $H_1 > H_2 > H_3 > .....$ (B) $H_1 < H_2 < H_3 < ......$

(C) $H_1 > H_3 > H_5 > ...$ and $H_2 < H_4 < H_6 < ......$ (D) $H_1 < H_3 < H_5 < ....$ and $H_2 > H_4 > H_6 > ......$

15. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Statement -1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

Statement -2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True **[IIT-JEE 2008, Paper-2, (3, –1), 81]**

16. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is :

[IIT-JEE - 2009, Paper-2, (3, –1), 80]

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

17.* For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is(are) :

[IIT-JEE - 2009, Paper-2, (4, –1), 80]

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

18. Let S_k, k = 1, 2,...., 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} \left|(k^2-3k+1)\ S_k\right|$ is. **[IIT-JEE - 2010, Paper-1, (3, 0), 84]**

19. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ..., 11.
If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to.

[IIT-JEE - 2010, Paper-2, (3, 0), 79]

20. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$.

For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n , then a_2 is.

[IIT-JEE - 2011, Paper-1, (3, 0), 80]

21. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is.

[IIT-JEE - 2011, Paper-1, (3, 0), 80]

22. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is :

(A) 22 (B) 23 (C) 24 (D) 25

[IIT-JEE 2012, Paper-2, (3, –1), 66]

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. The sum of the series $1^3 - 2^3 + 3^3 - .... + 9^3 =$ **[AIEEE 2002]**

(1) 300 (2) 125 (3) 425 (4) 0

2. If the sum of an infinite GP is 20 and sum of their square is 100 then common ratio will be = **[AIEEE 2002]**

(1) 1/2 (2) 1/4 (3) 3/5 (4) 1

3. If the third term of an A.P. is 7 and its 7th term is 2 more than three times of its 3rd term, then sum of its first 20 terms is : **[AIEEE 2002]**

(1) 228 (2) 74 (3) 740 (4) 1090

4. If x_1 , x_2 , x_3 and y_1 ,y_2 , y_3 are both in GP with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) : **[AIEEE 2003]**

(1) lie on a straight line (2) lie on an elipse (3) lie on a circle (4) are vertices of a triangle

5. Let T_r be the rth term of an AP whose first term is a and common difference is d. If for some positive integers m & n, m ≠ n, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals : **[AIEEE 2004]**

(1) 0 (2) 1 (3) $\frac{1}{mn}$ (4) $\frac{1}{m} + \frac{1}{n}$

6. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a,b,c are in AP and |a| < 1, |b| < 1, |c| < 1, then x,y,z are in : **[AIEEE 2005]**

(1) HP (2) Arithmetico–Geometric Progression

(3) AP (4) GP

7. If in a △ABC, the altiudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in : **[AIEEE 2005]**

(1) G.P. (2) A.P.

(3) Arithmetico-Geometirc progression (4) H.P.

8. Let a_1, a_2, a_3,..... be terms of an AP. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, p ≠ q, then $\frac{a_6}{a_{21}}$ equals : **[AIEEE 2006]**

(1) $\frac{7}{2}$ (2) $\frac{2}{7}$ (3) $\frac{11}{41}$ (4) $\frac{41}{11}$

9. If a_1, a_2, ..., a_n are in HP, then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}\, a_n$ is equal to : **[AIEEE 2006]**

(1) $(n - 1)(a_1 - a_n)$ (2) na_1a_n (3) $(n - 1) a_1a_n$ (4) $n (a_1 - a_n)$

10. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals : **[AIEEE 2007]**

(1) $\frac{1}{2}(1-\sqrt{5})$ (2) $\frac{1}{2}\sqrt{5}$ (3) $\sqrt{5}$ (4) $\frac{1}{2}(\sqrt{5}-1)$

11. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and a_{10}, a_{11},....are in an AP with common difference –2, then the time taken by him to count all notes is : **[AIEEE 2010]**

(1) 34 minutes (2) 125 minutes (3) 135 minutes (4) 24 minutes

12. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is : **[JEE Mains 2013]**

(1) $\frac{7}{81}(179 - 10^{-20})$ (2) $\frac{7}{9}(99 - 10^{-20})$ (3) $\frac{7}{81}(179 + 10^{-20})$ (4) $\frac{7}{9}(99 + 10^{-20})$

EXERCISE # 4

NCERT BOARD QUESTIONS

1. Find the number of terms in the series 101 + 99 + 97 + ...+ 47

2. The sum of three consecutive terms of an increasing A.P. is 51. If the product of the first and third of these terms be 273, then find third term

3. If we divide 20 into four parts which are in A.P. such that product of the first and the fourth is to the product of the second and third is the same as 2 : 3, then find the smallest part

4. In any G.P. the first term is 2 and last term is 512 and common ratio is 2, then find 5^{th} term from end

5. Break the numbers 155 into three parts so that the obtained numbers form a G.P., the first term being less than the third one by 120-

6. A ball falls from a height of 100 mts. on a floor. If in each rebound it describes 4/5 height of the previous falling height, then find the total distance travelled by the ball before coming to rest

7. Find the sum of 10 terms of the series. 0.7 + .77 + .777 + ...

8. In a potato race, 8 potatoes are placed 6 meters apart on a straight line, the first being 6 meters from the basket which is also placed in the same line. A contestant starts from the basket and puts one potato at a time into the basket. Find the total distance he must run in order to finish the race :

9. The sum of n terms of two airthmetic series in the ratio of (7n + 1) : (4n + 27). Find the ratio of their nth term.

10. Sum of the series to n terms and to infinity $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \ldots\ldots\ldots\ldots$

11. The first term of an A.P. is a, and the sum of the first p terms is zero, show that the sum of its next q terms is $\frac{-a(p+q)q}{p-1}$.

12. A man saved Rs 66000 in 20 years. In each succeeding year after the first year he saved Rs 200 more than what he saved in the previous year. How much did he save in the first year?

13. A man accepts a position with an initial salary of Rs 5200 per month. It is understood that he will receive an automatic increase of Rs 320 in the very next month and each month and each month thereafter.

(a) Find his salary for the tenth month

(b) What is his total earnings during the first year?

14. If the pth and qth terms of a G.P. are q and p respectively, show that its $(p + q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$

15. A carpenter was hired to build 192 window frames. The first day he made five frames and each day, thereafter he made two more frames than he made the day before. How many days did it take him to finish the job?

16. We know the sum of the interior angles of a triangle is 180º. Show that the sums of the interior angles of polygons with 3, 4, 5, 6, ... sides form an arithmetic progression. Find the sum of the interior angles for a 21 sided polygon.

17. A side of an equilateral triangle is 20cm long. A second equilateral triangle is inscribed in it by joining the mid points of the sides of the first triangle is inscribed in it by joining the mid points of the sides of the first triangle. The process is continued as shown in the accompanying diagram. Find the perimeter of the sixth inscribed equilateral triangle.

18. In a cricket tournament 16 school teams participated. A sum of Rs 8000 is to be awarded among themselves as prize money. If the last placed team is awarded Rs 275 in prize money and the award increases by the same amount for successive finishing places, how much amount will the first place team receive?

19. If $a_1, a_2, a_3, \ldots a_n$ are in A.P., where $a_i > 0$ for all i, show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots..+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

20. Find the sum of the series

$(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots..$ to (i) n terms (ii) 10 terms

21. Find the r^{th} term of an A.P. sum of whose first n terms is $2n + 3n^2$.

22. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers, then prove that

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

23. If $\theta_1, \theta_2, \theta_3 \ldots.., \theta_n$ are in A.P., whose common difference is d. show that $\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots.. +$ $\sec\theta_{n-1}\sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}$

24. If the sum of p terms of an A.P. is q and the sum of q terms is p. show that the sum of p + q terms is – (p + q). Also, find the sum of first p – q terms (p > q).

25. If p^{th}, q^{th}, and r^{th} terms of an A.P. and G.P. are both a, b and c respectively, show that $a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$

ANSWERS

EXERCISE # 1

PART # I

A-1.	(B)	**A-2.**	(B)	**A-3.**	(D)	**A-4.**	(C)	**A-5.**	(D)	**A-6.**	(B)	**A-7.**	(A)
A-8.	(B)	**A-9.**	(A)	**A-10.**	(C)	**A-11*.**	(B, D)	**B-1.**	(B)	**B-2.**	(D)	**B-3.**	(C)
B-4.	(C)	**B-5*.**	(A, C)	**B-6*.**	(B, C)	**C-1.**	(C)	**C-2*.**	(A, B)	**C-3.**	(C)	**C-4.**	(C)
C-5.	(C)	**C-6.**	(A)	**C-7.**	(C)	**C-8.**	(B)	**C-9*.**	(A,B,C)			**C-10.**	(B)
C-11*.	(A, B, C, D)			**C-12.**	(C)								

PART # II

1.	(A)	**2.**	(A)	**3.**	(D)	**4.**	(B)	**5.**	(C)	**6.**	(C)	**7.**	(B)
8.	(A)	**9.**	(A- p),	(B - p),	(C - q),	(D - r)		**10.**	(A- q),	(B - s),	(C - r),	(D - p)	
11.	(A)	**12.**	(A)	**13.**	(C)								

EXERCISE # 2

PART # I

1.	(C)	**2.**	(C)	**3.**	(C)	**4.**	(D)	**5.**	(A)	**6.**	(A)	**7.**	(C)
8.	(C)	**9.**	(D)	**10.**	(A)	**11.**	(C)	**12.**	(C)	**13.**	(A)	**14.**	(A)
15.	(C)	**16*.**	(B, C)	**17.**	(A, B, C, D)								

PART # II

1. 2, 5, 8,..... **2.** 612 **3.** 11 **4.** 128, 771 **5.** 19668 **6.** 160

7. 4, 9, 14 **11.** 128 **12.** 2, 6, 18

14. 3, 7, 11 or 12, 7, 2 **15.** $\frac{q-r}{p-q}$ **16.** 6, –3, 3/2, ... **18.** $2\pi R^2$; $4R^2$

19. (3 + 6 + 12 +......); (2/3 + 25/3 + 625/6 +......) **20.** $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{\pi}{3}$ **21.** $\frac{1}{11}$

22. A = 3; B = 8 **23.** (i) $4 - \frac{2+n}{2^{n-1}}$ (ii) $\frac{8}{3}$ **24.** $n.2^{n+2} - 2^{n+1} + 2$. **25.** $\frac{65}{36}$

26. a = 4, b = 8

31. (i) $\frac{1}{6}$ n (n + 1) (2n + 7) (ii) $\frac{1}{2}$ $(3^{n+1} + 1) - 2^{n+1}$

32. (i) $2^{n+2} - 3n - 4$ (ii) $\frac{1}{27}$ $(10^{n+1} - 9n - 10)$

33. (i) $2^{n-2}(2^n + 2^{n-1} - 1)$ (ii) $(n - 1)^3 + n^3$

34. (i) $\frac{1}{12} - \frac{1}{4(2n+1)(2n+3)}$ (ii) $\frac{n}{10}$ (n + 1) (n + 2) (n + 3) (2n + 3)

35. (i) (1/5) n (n + 1) (n + 2) (n + 3) (n + 4) (ii) $\frac{n\,(n+1)}{4\,(n+2)}$

36. (i) $\frac{25}{54}$ (ii) $\frac{n\,(n+1)}{2\,(n^2+n+1)}$; $s_\infty = \frac{1}{2}$

EXERCISE # 3

PART # I

1.	(A)	**2.**	(D)	**4.**	(A)	**6.**	(C)	**7.**	(C)	**8.** minimum natural number $n_0 = 6$			
9.	(B)	**10.**	(D)	**11.**	(B)	**12.**	(C)	**13.**	(A)	**14.**	(B)	**15.**	(C)
16.	(C)	**17.***	(C, D)	**18.**	3	**19.**	0	**20.**	3, 9	**21.**	8	**22.**	(D)

PART # II

1.	(3)	**2.**	(3)	**3.**	(3)	**4.**	(1)	**5.**	(1)	**6.**	(1)	**7.**	(2)
8.	(3)	**9.**	(3)	**10.**	(4)	**11.**	(3)						

EXERCISE # 4

1. 28 **2.** 21 **3.** 2 **4.** 32 **5.** 5,25,125 **6.** 900 mts

7. $\frac{7}{81}\left(89 + \frac{1}{10^{10}}\right)$ **8.** 420 **9.** (14n – 6) / (8n + 23)

10. $\frac{1}{24} - \frac{1}{6(3n+1)\,(3n+4)}$, $\frac{1}{24}$ **12.** Rs1400 **13.** Rs 8080, Rs 83520 **15.** 12 days

16. 3420° **17.** $\frac{15}{8}$cm **18.** Rs 725 **20.** (i) $4n^3 + 9n^2 + 6n$ (ii) 4960 **21.** $T_r = 6r - 1$