



arride learning

CIRCULAR MOTION

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Syllabus

Kinematics and Dynamics of Circular Motion

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CIRCULAR MOTION

CIRCULAR MOTION :

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant, then its motion is known as circular motion with respect to that fixed (or moving) point.

ANGULAR VELOCITY ω :

(i) Average Angular Velocity

$$\omega_{av} = \frac{\text{Angular displacement}}{\text{Total time taken}}$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

(ii) Instantaneous Angular Velocity

$$\vec{\omega} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

ANGULAR ACCELERATION α :

(i) Average Angular Acceleration :

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1} = \frac{\Delta \vec{\omega}}{\Delta t}$$

(ii) Instantaneous Angular Acceleration :

$$\vec{\alpha} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

$$\text{Also } \vec{\alpha} = \omega \frac{d\vec{\omega}}{d\theta}$$

Relation between linear velocity and angular velocity :

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Relation between linear Acceleration and angular Acceleration :

$$\text{In vector form } \vec{a}_t = \vec{\alpha} \times \vec{r}$$

CENTRIPETAL ACCELERATION :

$$a_{net} = a_r = \frac{v^2}{r}$$

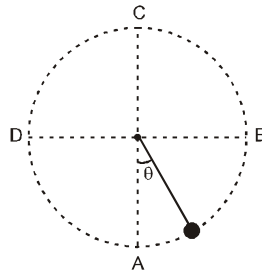
CENTRIPETAL FORCE :

$$(F_c) = ma_c = \frac{mv^2}{r} = m \omega^2 r$$

RADIUS OF CURVATURE :

$$R = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\frac{d^2y}{dx^2}}$$

MOTION IN A VERTICAL CIRCLE :



		A	B,D	C	P(general point)
1	Velocity	$\sqrt{5g\ell}$	$\sqrt{3g\ell}$	$\sqrt{g\ell}$	$\sqrt{g\ell(3+2\cos\theta)}$
2	Tension	$6mg$	$3mg$	0	$3mg(1+\cos\theta)$
3	Potential Energy	0	$mg\ell$	$2mg\ell$	$mg\ell(1-\cos\theta)$
4	Radial acceleration	$5g$	$3g$	g	$g(3+2\cos\theta)$
5	Tangential acceleration	0	g	0	$g\sin\theta$

CIRCULAR TURNING ON ROADS :

1. By Friction Only :

$$\mu \geq \frac{v^2}{rg} \text{ or } v \leq \sqrt{\mu rg}$$

2. By Banking of Roads Only :

$$v = \sqrt{rg \tan \theta}$$

3. By Friction and Banking of Road Both :

(i) Friction f will be outwards if the vehicle is at rest $v = 0$. Because in that case the component of weight $mg \sin \theta$ is balanced by f .

(ii) Friction f will be inwards if

$$v > \sqrt{rg \tan \theta}$$

(iii) Friction f will be outwards if

$$v < \sqrt{rg \tan \theta}$$

(iv) Friction f will be zero if

$$v = \sqrt{rg \tan \theta}$$

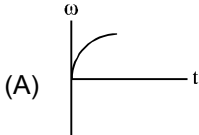
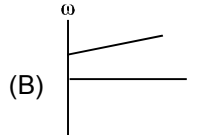
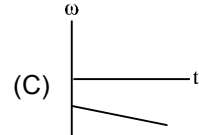
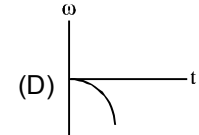
$$(v) \quad v_{\max} = \sqrt{\frac{rg(\tan \theta + \mu)}{1 - \mu \tan \theta}} \text{ and } v_{\min} = \sqrt{\frac{rg(\tan \theta - \mu)}{1 + \mu \tan \theta}}$$

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

* *Marked Questions are having more than one correct option.*

Section (A) : Kinematics of circular motion

- A-1.*** An Object follows a curved path. The following quantities may remain constant during the motion :
(A) speed (B) velocity
(C) acceleration (D) magnitude of acceleration
- A-2.** The ratio of angular speed of hours hand and seconds hand of a clock is-
(A) 1 : 1 (B) 1 : 60 (C) 1 : 720 (D) 3600 : 1
- A-3.** In uniform circular motion-
(A) both velocity and acceleration are constant
(B) acceleration and speed are constant but velocity changes
(C) both acceleration and velocity change
(D) both acceleration and speed are constant
- A-4.** If a particle moves in a circle describing equal angles in equal times, its velocity vector:
(A) remains constant (B) changes in magnitude
(C) changes in direction (D) changes both in magnitude and direction
- A-5.** A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with tangential acceleration of constant magnitude. If the speed of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is:
(A) $160 \pi \text{ m/s}^2$ (B) $40 \pi \text{ m/s}^2$ (C) 40 m/s^2 (D) $640 \pi \text{ m/s}^2$
- A-6.** Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t . The ratio of the angular speeds of the first to the second car is
(A) 1 : 1 (B) $m_1 : m_2$ (C) $r_1 : r_2$ (D) $m_1 m_2 : r_1 r_2$
- A-7.** The graphs below show angular velocity as a function of time. In which one is the magnitude of the angular acceleration constantly decreasing?
- (A)  (B)  (C)  (D) 
- A-8.** The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as
(A) $2 a \sin \omega t$ (B) $2 a \sin \frac{\omega t}{2}$ (C) $2 a \cos \omega t$ (D) $2 a \cos \frac{\omega t}{2}$

Section (B) : Radial and Tangential acceleration

B-1. A car is travelling with linear velocity v on a circular road of radius r . If it is increasing its speed at the rate of ' a ' metre/sec², then the resultant acceleration will be -

- (A) $\sqrt{\left(\frac{v^2}{r^2} - a^2\right)}$ (B) $\sqrt{\left(\frac{v^4}{r^2} + a^2\right)}$ (C) $\sqrt{\left(\frac{v^4}{r^2} - a^2\right)}$ (D) $\sqrt{\left(\frac{v^2}{r^2} + a^2\right)}$

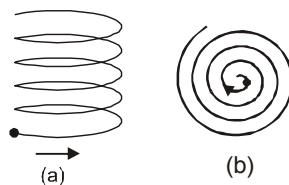
B-2. A particle begins to move with a tangential acceleration of constant magnitude 0.6 m/s^2 in a circular path. If it slips when its total acceleration becomes 1 m/s^2 , then the angle through which it would have turned before it starts to slip is :

- (A) $1/3 \text{ rad}$ (B) $2/3 \text{ rad}$ (C) $4/3 \text{ rad}$ (D) 2 rad

B-3. A particle is moving in a circle

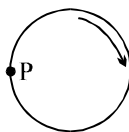
- (A) The resultant force on the particle must be towards the centre.
 (B) The resultant force may be towards the centre.
 (C) The direction of the angular acceleration and the angular velocity must be the same.
 (D) The cross product of the tangential acceleration and the angular velocity will be zero.

B-4. A particle is going with constant speed along a uniform helical and spiral path separately as shown in figure



- (A) The velocity of the particle is constant in both cases
 (B) The magnitude of acceleration of the particle is constant in both cases
 (C) The magnitude of acceleration is constant in (a) and decreasing in (b)
 (D) The magnitude of acceleration is decreasing continuously in both the cases

B-5. The diagram shows a CD rotating clockwise (as seen from above) in the CD-player. After turning it off, the CD slows down. Assuming it has not come to a stop yet, the direction of the acceleration of point P at this instance is :



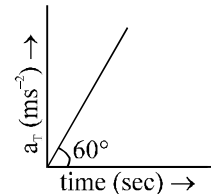
- (A) (B) (C) (D)

B-6. Two bodies having masses 10 kg and 5 kg are moving in concentric orbits of radii 4 and 8 such that their time periods are the same. Then the ratio of their centripetal accelerations is

- (A) $\frac{1}{2}$ (B) 2 (C) 8 (D) $\frac{1}{8}$

- B-7.** A particle is moving in a circle :
- (A) The resultant force on the particle must be towards the centre.
 (B) The cross product of the tangential acceleration and the angular velocity will be zero.
 (C) The direction of the angular acceleration and the angular velocity must be the same.
 (D) The resultant force may be towards the centre.
- B-8.** Tangential acceleration of a particle moving in a circle of radius 1 m varies with time t as (initial velocity of particle is zero). Time after which total acceleration of particle makes an angle of 30° with radial acceleration is :

- (A) 4 sec (B) $4/3$ sec
 (C) $2^{2/3}$ sec (D) $\sqrt{2}$ sec



- B-9.** A particle is moving along the circle $x^2 + y^2 = a^2$ in anticlockwise direction. The x - y plane is a rough horizontal stationary surface. At the point $(a \cos\theta, a \sin\theta)$, the unit vector in the direction of friction on the particle is:

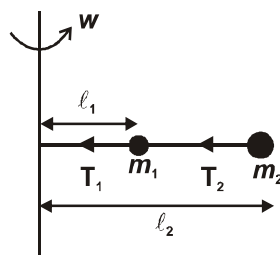
- (A) $\cos\theta \hat{i} + \sin\theta \hat{j}$ (B) $-(\cos\theta \hat{i} + \sin\theta \hat{j})$ (C) $\sin\theta \hat{i} - \cos\theta \hat{j}$ (D) $\cos\theta \hat{i} - \sin\theta \hat{j}$

- B-10.** A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}$, \vec{v} , $\vec{\alpha}$ and \vec{a}_c . Which of the following relations is/are correct ?

- (A) $\vec{\omega} \cdot \vec{v} = 0$ (B) $\vec{\omega} \cdot \vec{\alpha} = 0$ (C) $\vec{\omega} \cdot \vec{a}_c = 0$ (D) $\vec{v} \cdot \vec{a}_c = 0$

Section (C) : Circular Motion in Horizontal plane

- C-1.** The whole set up shown in the figure is rotating with constant angular velocity ω on a horizontal frictionless table then the ratio of tensions $\frac{T_1}{T_2}$ is (Given $\frac{\ell_2}{\ell_1} = \frac{2}{1}$)

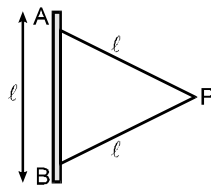


- (A) $\frac{m_1}{m_2}$ (B) $\frac{(m_1 + 2m_2)}{2m_2}$ (C) $\frac{m_2}{m_1}$ (D) $\frac{(m_2 + m_1)}{m_2}$

- C-2.** A coin placed on a rotating turn table just slips if it is at a distance of 40 cm from the centre if the angular velocity of the turntable is doubled, it will just slip at a distance of

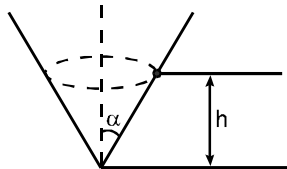
- (A) 10 cm (B) 20 cm (C) 40 cm (D) 80 cm

- C-3.*** A particle P of mass m is attached to a vertical axis by two strings AP and BP of length ℓ each. The separation $AB = \ell$. P rotates around the axis with an angular velocity ω . The tensions in the two strings are T_1 and T_2



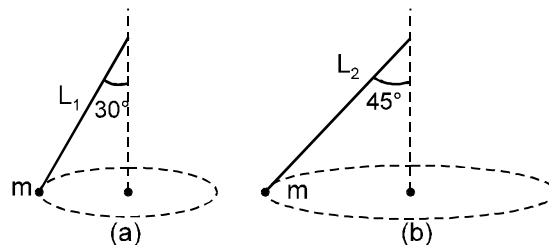
- (A) $T_1 = T_2$ (B) $T_1 + T_2 = m \omega^2 \ell$
 (C) $T_1 - T_2 = 2 mg$ (D) BP will remain taut only if $\omega \geq \sqrt{\frac{2g}{\ell}}$

- C-4.*** A particle is describing circular motion in a horizontal plane in contact with the smooth inside surface of a fixed right circular cone with its axis vertical and vertex down. The height of the plane of motion above the vertex is h and the semivertical angle of the cone is α . The period of revolution of the particle:



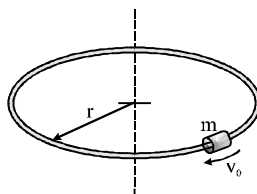
- (A) increases as h increases (B) decreases as h increases
 (C) increases as α increases (D) decreases as α increases

- C-5.** Two particles tied to different strings are whirled in a horizontal circle as shown in figure. The ratio of lengths of the strings so that they complete their circular path with equal time period is:



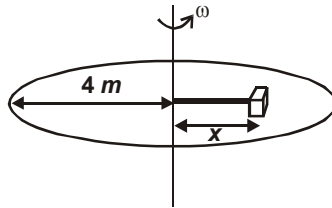
- (A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{\frac{2}{3}}$ (C) 1 (D) None of these

- C-6.** A small hoop of mass m is given an initial velocity of magnitude v_0 on the horizontal circular ring of radius ' r '. If the coefficient of kinetic friction is μ_k the tangential acceleration of the hoop immediately after its release is (assume the horizontal ring to be fixed and not in contact with any supporting surface)

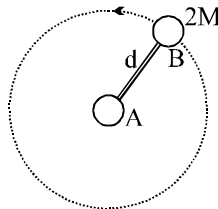


- (A) $\mu_k g$ (B) $\mu_k \frac{v_0^2}{r}$ (C) $\mu_k \sqrt{g^2 + \frac{v_0^2}{r}}$ (D) $\mu_k \sqrt{g^2 + \frac{v_0^4}{r^2}}$

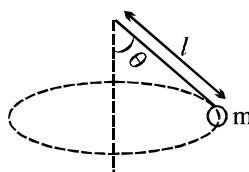
- C-7.** A disc of radius 4 m is rotating about its fixed centre with a constant angular velocity $\omega = 2 \text{ rad/s}$ (in the horizontal plane). A block is also rotating with the disc without slipping. If co-efficient of friction between the block and the disc is 0.4, then the maximum distance at which the block can rotate without slipping is ($g = 10 \text{ m/s}^2$)



- (A) 1 m (B) 2 m (C) 3 m (D) 4 m
- C-8.** A man is standing on a rough ($\mu = 0.5$) horizontal disc rotating with constant angular velocity of 5 rad/sec. At what distance from centre should he stand so that he does not slip on the disc?
 (A) $R \leq 0.2\text{m}$ (B) $R > 0.2 \text{ m}$ (C) $R > 0.5 \text{ m}$ (D) $R > 0.3 \text{ m}$
- C-9.** The dumbbell is placed on a frictionless horizontal table. Sphere A is attached to a frictionless pivot so that B can be made to rotate about A with constant angular velocity. If B makes one revolution in period P, the tension in the rod is



- (A) $\frac{4\pi^2 Md}{P^2}$ (B) $\frac{8\pi^2 Md}{P^2}$ (C) $\frac{4\pi^2 Md}{P}$ (D) $\frac{2Md}{P}$
- C-10.** A road is banked at an angle of 30° to the horizontal for negotiating a curve of radius $10\sqrt{3} \text{ m}$. At what velocity will a car experience no friction while negotiating the curve?
 (A) 54 km/hr (B) 72 km/hr (C) 36 km/hr (D) 18 km/hr
- C-11.** The ratio of period of oscillation of the conical pendulum to that of the simple pendulum is : (Assume the strings are of the same length in the two cases and θ is the angle made by the string with the vertical in case of conical pendulum)
 (A) $\cos \theta$ (B) $\sqrt{\cos \theta}$ (C) 1 (D) none of these
- C-12.** A conical pendulum is moving in a circle with angular velocity ω as shown. If tension in the string is T, which of following equations are correct ?



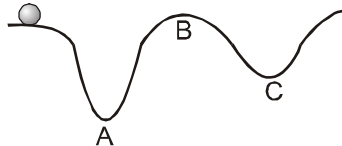
- (A) $T = m\omega^2 l$ (B) $T \sin \theta = m\omega^2 l$
 (C) $T = mg \cos \theta$ (D) $T = m\omega^2 l \sin \theta$

Section (D) : Radius of curvature

D-1. A particle is projected with a speed u at an angle θ with the horizontal. Consider a small part of its path near the highest position and take it approximately to be a circular arc. What is the radius of this circle?

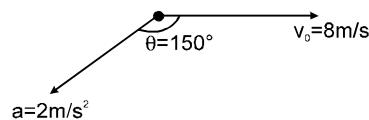
- (A) $\frac{u^2 \sin^2 \theta}{g}$ (B) $\frac{u^2 \cos^2 \theta}{g}$ (C) $\frac{u^2 \tan^2 \theta}{g}$ (D) $\frac{u^2}{g}$

D-2. A body moves along an uneven surface with constant speed at all points. The normal reaction of the road on the body is :



- (A) maximum at A (B) maximum at B
(C) minimum at C (D) the same at A, B & C

D-3. The figure shows the velocity and acceleration of a point like body at the initial moment of its motion. The acceleration vector of the body remains constant. The minimum radius of curvature of trajectory of the body is :



- (A) 2 meter (B) 4 meter (C) 8 meter (D) 16 meter.

D-4. A train is moving towards North. At one place it turns towards North-East. Here we observe that

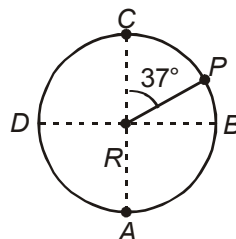
- (A) the radius of curvature of outer rail will be greater than that of the inner rail
(B) the radius of curvature of one of the rails will be greater
(C) the radius of curvature of inner rail will be greater than that of the outer rail
(D) the radius of curvature of the outer and inner rails will be the same

Section (E) : Circular motion in vertical plane

E-1. A ball tied to a string (in vertical plane) is swinging in a vertical circle. Which of the following remains constant during the motion ?

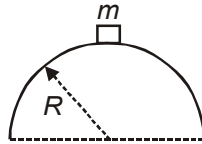
- (A) tension in the string (B) speed of the ball
(C) centripetal force (D) gravitational force on the ball

E-2. A particle is moving along a vertical circle of radius R . The velocity of particle at P will be (assume critical condition at C)



- (A) $\sqrt{\frac{7}{5}gR}$ (B) $\sqrt{2gR}$ (C) $\sqrt{\frac{3gR}{5}}$ (D) $\sqrt{\frac{3}{2}gR}$

- E-3.** A particle is projected so as to just move along a vertical circle of radius r . The ratio of the tension in the string when the particle is at the lowest and highest point on the circle is -
 (A) 1 (B) finite but large (C) zero (D) Infinite
- E-4.** If a particle of mass m resting at the top of a smooth hemisphere of radius R is displaced, then the angle with vertical at which it loses contact with the surface is



- (A) $\tan^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{3}\right)$ (C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{3}\right)$

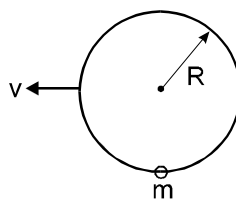
- E-5.** A bucket is whirled in a vertical circle with a string attached to it. The water in bucket does not fall down even when the bucket is inverted at the top of its path. We can say that in this position.

- (A) $mg = \frac{mv^2}{r}$ (B) mg is greater than $\frac{mv^2}{r}$
 (C) mg is not greater than $\frac{mv^2}{r}$ (D) mg is not less than $\frac{mv^2}{r}$

- E-6.** A simple pendulum of length L and mass M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement θ [$|\theta| < \phi$], the tension in the string and the velocity of the bob are T and v respectively. The following relation holds good under the above conditions -

- (A) $T = Mg \cos \theta$ (B) $T \cos \theta = Mg$
 (C) $T - Mg \cos \theta = Mv^2/L$ (D) $T + Mg \cos \theta = Mv^2/L$

- E-7.** A ring of radius R lies in vertical plane. A bead of mass ' m ' can move along the ring without friction. Initially the bead is at rest at the bottom most point on ring. The minimum constant horizontal speed v with which the ring must be pulled such that the bead completes the vertical circle



- (A) $\sqrt{3gR}$ (B) $\sqrt{4gR}$ (C) $\sqrt{5gR}$ (D) $\sqrt{5.5gR}$

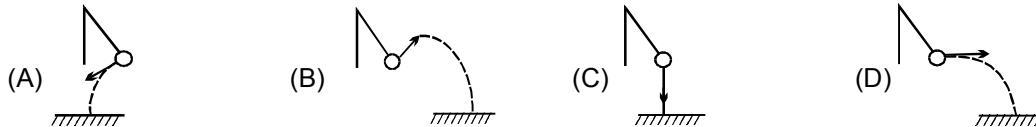
- E-8.** A particle suspended from a fixed point, by a light inextensible thread of length L is projected horizontally from its lowest position with velocity $\sqrt{\frac{7gL}{2}}$. The thread will slack after swinging through an angle θ , such that θ equal-

- (A) 30° (B) 135° (C) 120° (D) 150°

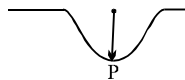
E-9. Which vector in the figures best represents the acceleration of a pendulum mass at the intermediate point in its swing?



E-10. A pendulum bob is swinging in a vertical plane such that its angular amplitude is less than 90° . At its highest point, the string is cut. Which trajectory is possible for the bob afterwards.



E-11. A car travelling on a smooth road passes through a curved portion of the road in form of an arc of circle of radius 10 m. If the mass of car is 500 kg, the reaction on car at lowest point P where its speed is 20 m/s is:

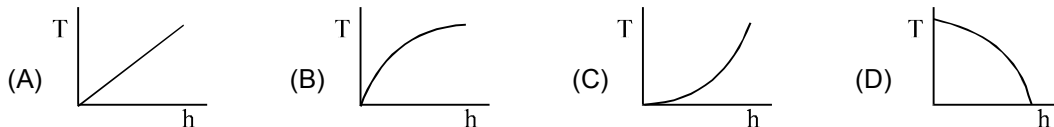
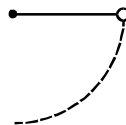


- (A) 35 kN (B) 30 kN (C) 25 kN (D) 20 kN

E-12. A particle originally at rest at the highest point of a smooth vertical circle is slightly displaced. It will leave the circle at a vertical distance h below the highest point, such that

- (A) $h = R$ (B) $h = R/3$ (C) $h = R/2$ (D) $h = 2R$

E-13. A bob attached to a string is held horizontal and released. The tension and vertical distance from point of suspension can be represented by.



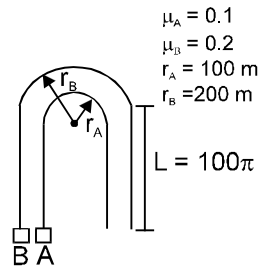
Section (F) : Motion of a vehicle, Centrifugal force and rotation of earth

- F-1.** Centrifugal force is considered as pseudo force when
 (A) An observer at the centre of circular motion
 (B) An outside observe
 (C) An observer who is moving with the particle which is experiencing the force
 (D) None of the above

F-2. If the apparent weight of the bodies at the equator is to be zero, then the earth should rotate with angular velocity

- (A) $\sqrt{\frac{g}{R}}$ rad/sec (B) $\sqrt{\frac{2g}{R}}$ rad/sec (C) $\sqrt{\frac{g}{2R}}$ rad/sec (D) $\sqrt{\frac{3g}{2R}}$ rad/sec

- F-3.** A vehicle can travel round a curve at a higher speed when the road is banked than when the road is level. This is because
 (A) banking increases the coefficient of friction
 (B) banking increases the radius,
 (C) the normal reaction has a horizontal component,
 (D) when the track is banked the weight of the car acts down the incline.
- F-4.** Two cars A and B start racing at the same time on a flat race track which consists of two straight sections each of length 100π and one circular section as in fig. The rule of the race is that each car must travel at constant speed at all times without ever skidding



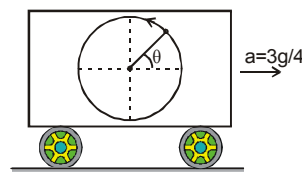
- (A) car A completes its journey before car B
 (B) both cars complete their journey in same time
 (C) velocity of car A is greater than that of car B
 (D) car B completes its journey before car A.
- F-5.** A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. Normal force by the track on train A is N_1 and that on train B is N_2 ;
 (A) $N_1 > N_2$ (B) $N_1 < N_2$ (C) $N_1 = N_2$
 (D) the information is insufficient to find the relation between N_1 and N_2 .

PART - II : MISCELLANEOUS OBJECTIVE QUESTIONS

Comprehensions Type :

COMPREHENSION # 1

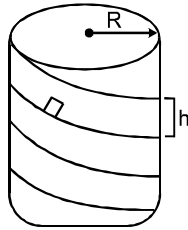
A bus is moving with a constant acceleration $a = 3g/4$ towards right. In the bus, a ball is tied with a rope of length ℓ and is rotated in vertical circle as shown.



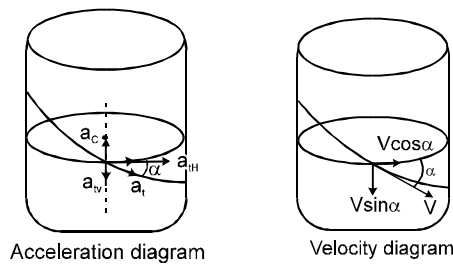
- At what value of angle θ , tension in the rope will be minimum
 (A) $\theta = 37^\circ$ (B) $\theta = 53^\circ$ (C) $\theta = 30^\circ$ (D) $\theta = 90^\circ$
- At above mentioned position, find the minimum possible speed V_{\min} during whole path to complete the circular motion :
 (A) $\sqrt{5g\ell}$ (B) $\frac{5}{2}\sqrt{g\ell}$ (C) $\frac{\sqrt{5g\ell}}{2}$ (D) $\sqrt{g\ell}$
- For above value of V_{\min} find maximum tension in the string during circular motion.
 (A) 6 mg (B) $\frac{117}{20} \text{ mg}$ (C) $\frac{15}{2} \text{ mg}$ (D) $\frac{17}{2} \text{ mg}$

COMPREHENSION # 2

In following passage we will study that when a particle moves in a helical groove then question related this low can be solved.



The motion of the body can be considered as a superposition of movement along a circumference with a radius 'R' in a horizontal plane & vertical straight line motion.



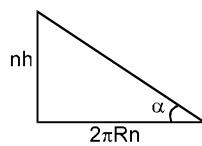
The velocity of the body 'V' at the given moment can be represented as the geometrical sum of the components:

$$\begin{aligned} V \cos \alpha & : && \text{horizontal velocity} \\ \& \ V \sin \alpha & : & \text{vertical velocity} \end{aligned}$$

Here ' α ' is the angle formed by the helical line of groove with the horizontal plane.

A component of the acceleration of the body is responsible for change in direction & other for the change in speed, i.e. centripetal acceleration & tangential acceleration. The tangential acceleration have two components : one along the circle & one in vertical direction.

The value of tangential acceleration ' a_t ' can be found by mentally developing the surface of the cylinder with the helical groove into a plane. In this case the groove will become an inclined plane with height nh & length of its base $2\pi Rn$, where ' n ' is the number of turns in the helix.



4. Distance travelled by the object when it completes one revolution along the groove is :

(A) h (B) $2\pi R$ (C) $\sqrt{h^2 + (2\pi R)^2}$ (D) $h \sin \alpha$
5. The angular acceleration of the object moving along the circle will be :

(A) $\frac{g \sin \alpha}{R}$ (B) $\frac{g \sin \alpha \cos \alpha}{R}$ (C) $\frac{g \sin^2 \alpha}{R}$ (D) zero
6. The speeds of the object at the end of 1st round, 2nd round and 3rd round are in ratio : (Assuming the body starts from rest)

(A) $1 : 2 : 3$ (B) $1 : 3 : 5$ (C) $\sqrt{1} : \sqrt{2} : \sqrt{3}$ (D) $\sqrt{1} : \sqrt{3} : \sqrt{5}$

7. The time taken by the block to complete 1st round, 2nd round and 3rd round are in the ratio:
- (A) $\sqrt{1} : \sqrt{2} : \sqrt{3}$ (B) $\sqrt{1} : \sqrt{2} - \sqrt{1} : \sqrt{3} - \sqrt{2}$
 (C) $\sqrt{2} - \sqrt{1} : \sqrt{3} - \sqrt{2} : \sqrt{3} - \sqrt{1}$ (D) 1 : 4 : 9
8. If the speed of the object is v at an instant, then the force exerted by the helical groove at the same instant is :
- (A) $\frac{mv^2}{R}$ (B) $\sqrt{\left(\frac{mv^2}{R}\right)^2 + (mg\cos\alpha)^2}$
 (C) $\sqrt{\left(\frac{mv^2}{R}\right)^2 + (mg\sin\alpha)^2}$ (D) $\sqrt{\left(\frac{mv^2 \cos^2 \alpha}{R}\right)^2 + (mg\cos\alpha)^2}$

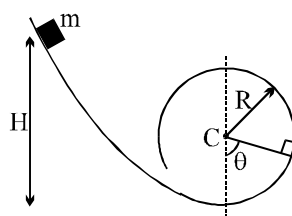
COMPREHENSION # 3

A particle is moving in the vertical plane. It is attached at one end of a string of length ℓ whose other end is fixed. The velocity at the lowest point is u . The tension in the string is \vec{T} and acceleration of the particle is \vec{a} at any position.

9. Then, $\vec{T} \cdot \vec{a}$ is zero at the highest point:
- (A) only if $u \leq \sqrt{2g\ell}$ (B) if $u \leq \sqrt{5g\ell}$ (C) only if $u = \sqrt{2g\ell}$ (D) only if $u > \sqrt{2g\ell}$
10. In the above question, $\vec{T} \cdot \vec{a}$ is non-negative at the lowest point for:
- (A) $u \leq \sqrt{2g\ell}$ (B) $u = \sqrt{2g\ell}$ (C) $u < \sqrt{2g\ell}$ (D) any value of u
11. In the above question, $\vec{T} \cdot \vec{v}$ is zero for:
- (A) $u \leq \sqrt{2g\ell}$ (B) $u = \sqrt{2g\ell}$ (C) $u \geq \sqrt{2g\ell}$ (D) any value of u

COMPREHENSION # 4

A particle of mass m is released from a height H on a smooth curved surface which ends into a vertical loop of radius R , as shown



12. Choose the correct alternative(s) if $H = 2R$
- (A) The particles reaches the top of the loop with zero velocity
 (B) The particle cannot reach the top of the loop
 (C) The particle breaks off at a height $H = R$ from the base of the loop
 (D) The particle break off at a height $R < H < 2R$

13. If θ is instantaneous angle which the line joining the particle and the centre of the loop makes with the vertical, then identify the correct statement(s) related to the normal reaction N between the block and the surface
- (A) The maximum value N occurs at $\theta = 0$
 (B) The minimum value of N occurs at $\theta = \pi$ for $H > 5R/2$
 (C) The value of N becomes negative for $\pi/2 < \theta < 3\pi/2$
 (D) The value of N becomes zero only when $\theta \geq \pi/2$
14. The minimum value of H required so that the particle makes a complete vertical circle is given by
 (A) $5R$ (B) $4R$ (C) $2.5R$ (D) $2R$

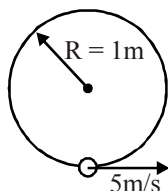
Match the Column :

15. In column-I, a situation is depicted each of which is in vertical plane. The surfaces are frictionless. Match with appropriate entries in column-II.

Column-I

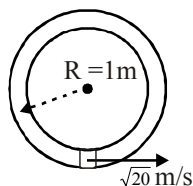
Column-II

- (A) Bead is threaded on a circular fixed wire and is projected from the lowest point.



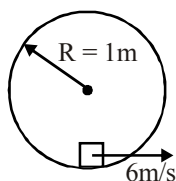
- (P) Normal force is zero at top most point of its trajectory.

- (B) Block loosely fits inside the fixed small tube and is projected from lowest point



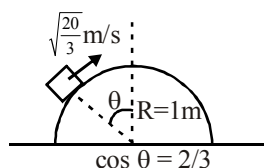
- (Q) Velocity of the body is zero at top most point of its trajectory

- (C) Block is projected horizontally from lowest point of a smooth fixed cylinder.



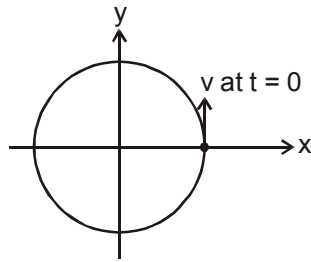
- (R) Acceleration of the body is zero at the top most point of its trajectory

- (D) Block is projected on a fixed hemisphere from angular position θ .

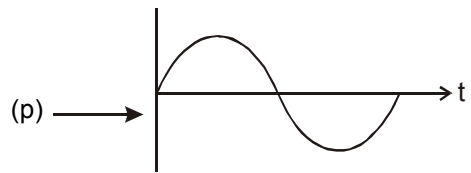


- (S) Normal force is radially outward at top most point of trajectory.

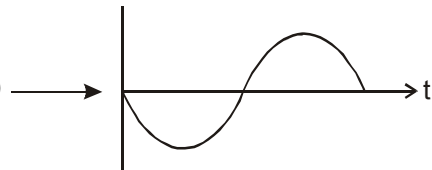
16. A particle is moving in a uniform circular motion on a horizontal surface. Particle position and velocity at time $t = 0$ are shown in the figure in the coordinate system. Which of the indicated variable on the vertical axis is/are correctly matched by the graph(s) shown alongside for particle's motion ?



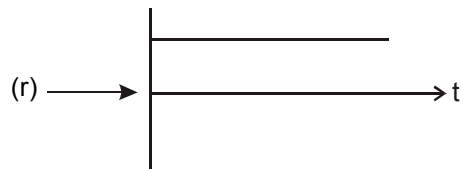
(A) x component of velocity



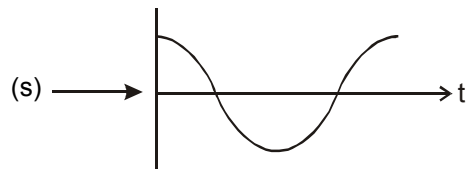
(B) y component of force keeping particle moving in a circle



(C) Angular velocity of the particle

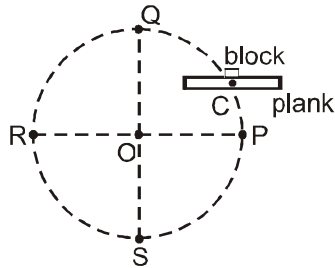


(D) x coordinate of the particle



(t) None of these

17. A small block lies on a rough horizontal platform above its centre C as shown figure. The plank is moved in vertical plane such that it always remains horizontal and its centre C moves in a vertical circle of centre O with constant angular velocity ω . There is no relative motion between block and the plank and the block does not lose contact with the plank anywhere. P, Q, R and S are four points on circular trajectory of centre C of platform. P and R lie on same horizontal level as O. Q is the highest point on the circle and S is the lowest point on the shown circle. Match the statements in column-I with points in column-II.



Column-I

Column-II

- | | |
|--|---------------------------------|
| (A) Magnitude of frictional force on block is maximum | (p) when block is at position P |
| (B) Magnitude of normal reaction on block is equal to mg | (q) when block is at position Q |
| (C) Magnitude of frictional force is zero | (r) when block is at position R |
| (D) Net contact force on the block is directed toward centre | (s) when block is at position S |
| | (t) None of these |

TRUE & FALSE :

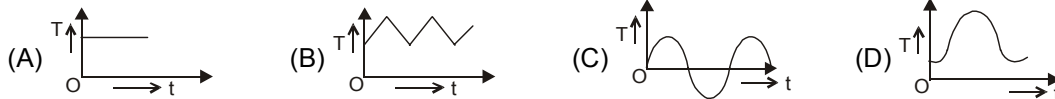
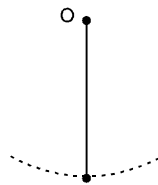
18. (i) For motion along a curved path, the velocity and acceleration vectors are never in the same direction.
(ii) The resultant force acting on a particle moving in a circular path is always directed towards the centre.
(iii) A circular curve of a highway is designed for traffic moving at 72 km/h. If the radius of the curved path is 100 m, the correct angle of banking of the road should be $\tan^{-1} \frac{1}{5}$. ($g = 10 \text{ m/s}^2$)

EXERCISE # 2

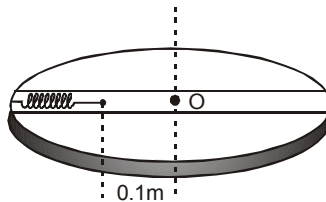
PART - I : OBJECTIVE QUESTIONS

Single Correct Answer Type

- A bottle of soda water is grasped by the neck and swing briskly in a vertical circle. Near which portion of the bottle do the bubbles collect?
 (A) near the near bottom (B) in the middle of the bottle
 (C) near the neck (D) uniformly distributed in the bottle
- A point moves along a circle with velocity $v = at$ where $a = 0.5 \text{ m/sec}^2$. Then the total acceleration of the point at the moment when it covered $(1/10)^{\text{th}}$ of the circle after beginning of motion -
 (A) 0.5 m/sec^2 (B) 0.6 m/sec^2 (C) 0.7 m/sec^2 (D) 0.8 m/sec^2
- A particle of mass m is suspended from a fixed point O by a string of length ℓ . It is displaced by angle θ ($\theta < 90^\circ$) from equilibrium position and released from there at $t = 0$. The graph, which shows the variation of the tension T in the string with time ' t ', may be :

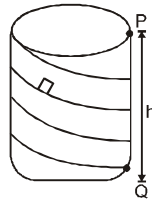


- A circular turn table of radius 0.5 m has a smooth groove as shown in fig. A ball of mass 90 g is placed inside the groove along with a spring of spring constant 10^2 N/cm . The ball is at a distance of 0.1 m from the centre when the turn table is at rest. On rotating the turn table with a constant angular velocity of $10^2 \text{ rad-sec}^{-1}$ the ball moves away from the initial position by a distance nearly equal to-

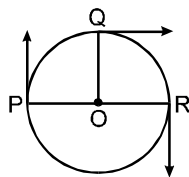


- (A) 10^{-1} m (B) 10^{-2} m (C) 10^{-3} m (D) $2 \times 10^{-1} \text{ m}$
- A ball suspended by a thread swings in a vertical plane so that its acceleration in the extreme position and lowest position are equal. The angle θ of thread deflection in the extreme position will be -
 (A) $\tan^{-1}(2)$ (B) $\tan^{-1}(\sqrt{2})$ (C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $2 \tan^{-1}\left(\frac{1}{2}\right)$

6. A hollow vertical cylinder of radius R and height h has smooth internal surface. A small particle is placed in contact with the inner side of the upper rim at a point P . It is given a horizontal speed v_0 tangential to rim. It leaves the lower rim at point Q , vertically below P . The number of revolution made by the particle will be -

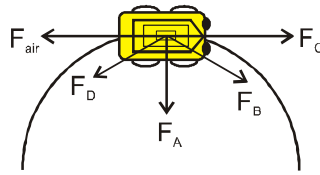


- (A) $\frac{h}{2\pi R}$ (B) $\frac{v_0}{\sqrt{2gh}}$ (C) $\frac{2\pi R}{h}$ (D) $\frac{v_0}{2\pi R} \left(\sqrt{\frac{2h}{g}} \right)$
7. If mass, speed and radius of rotation of a body moving in a circular path are all increased by 50%, the necessary force required to maintain the body moving in the circular path will have to be increased by-
 (A) 225% (B) 125% (C) 150% (D) 100 %
8. Three small balls each of mass 100 gm are attached at distance of 1 m, 2 m and 3 m from end D of a 3 m length of string. The string is rotated with uniform angular velocity in a horizontal plane about D. If the outside ball is moving at a speed of 6 m/s, the ratio of tension in the three parts of the string from inside -
 (A) 6 : 5 : 4 (B) 3 : 2 : 1 (C) 3 : 5 : 6 (D) 6 : 5 : 3
9. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 r t^2$, where k is a constant, the power delivered to the particle by the forces acting on it is-
 (A) $2 \pi m k^2 r^2 t$ (B) $m k^2 r^2 t$ (C) $(m k^4 r^2 t^5)/3$ (D) 0
10. Three point particles P, Q, R move in a circle of radius ' r ' with different but constant speeds. They start moving at $t = 0$ from their initial positions as shown in the figure. The angular velocities (in rad/sec) of P, Q and R are 5π , 2π & 3π respectively, in the same sense. The time interval after which they all meet is:



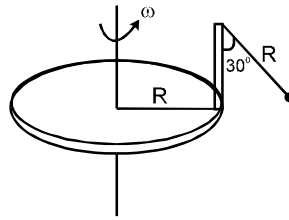
- (A) $2/3$ sec (B) $1/6$ sec (C) $1/2$ sec (D) $3/2$ sec
11. A ring of mass 2π kg and of radius 0.25 m is making 300 rpm about an axis through its centre perpendicular to its plane. The tension (in newton's) developed in the ring is (take $\pi^2 = 10$)
 (A) 50 (B) 100 (C) 175 (D) 250

12. A car travels with constant speed on a circular road on level ground. In the figure shown, F_{air} is the force of air resistance on the car. Which of the other forces best represent the horizontal force of the road on the car's tires ?



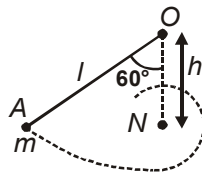
- (A) F_A (B) F_B (C) F_C (D) F_D

13. A disc of radius R has a light pole fixed perpendicular to the disc at the circumference which in turn has a pendulum of length R attached to its other end as shown in figure. The disc is rotated with a constant angular velocity ω . The string is making an angle 30° with the rod. Then the angular velocity ω of disc is:



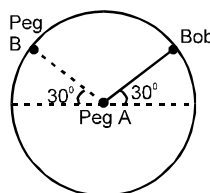
- (A) $\left(\frac{\sqrt{3}g}{R}\right)^{1/2}$ (B) $\left(\frac{\sqrt{3}g}{2R}\right)^{1/2}$ (C) $\left(\frac{g}{\sqrt{3}R}\right)^{1/2}$ (D) $\left(\frac{2g}{3\sqrt{3}R}\right)^{1/2}$

14. A particle of mass m attached to the end of string of length l is released from the initial position A as shown in the figure. The particle moves in a vertical circular path about O . When it is vertically below O , the string makes contact with nail N placed directly below O at distance h and rotates around it. If the particle just complete the vertical circle about N , then



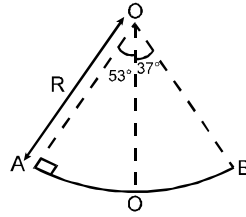
- (A) $h = \frac{3l}{5}$ (B) $h = \frac{2l}{5}$ (C) $h = \frac{l}{5}$ (D) $h = \frac{4l}{5}$

15. A bob is attached to one end of a string other end of which is fixed at peg A. The bob is taken to a position where string makes an angle of 30° with the horizontal. On the circular path of the bob in vertical plane there is a peg 'B' at a symmetrical position with respect to the position of release as shown in the figure. If V_c and V_a be the minimum speeds in clockwise and anticlockwise directions respectively, given to the bob in order to hit the peg 'B' then ratio $V_c : V_a$ is equal to :

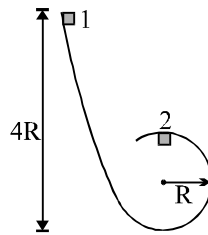


- (A) 1 : 1 (B) 1 : $\sqrt{2}$ (C) 1 : 2 (D) 1 : 4

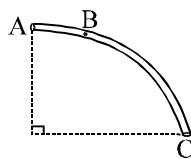
16. A section of fixed smooth circular track of radius R in vertical plane is shown in the figure. A block is released from position A and leaves the track at B . The radius of curvature of its trajectory when it just leaves the track at B is -



- (A) R (B) $\frac{R}{4}$ (C) $\frac{R}{2}$ (D) none of these
17. A body of mass 1 kg starts moving from rest at $t = 0$, in a circular path of radius 8 m . Its kinetic energy varies as a function of time as : $\text{K.E.} = 2t^2 \text{ Joules}$, where t is in seconds. Then
 (A) tangential acceleration = 4 m/s^2 (B) power of all forces at $t = 2 \text{ sec}$ is 8 watt
 (C) first round is completed in 2 sec . (D) tangential force at $t = 2 \text{ sec}$ is 4 newton .
18. A small cube with mass M starts at rest at point 1 at a height $4R$, where R is the radius of the circular part of the track. The cube slides down the frictionless track and around the loop. The force that the track exerts on the cube at point 2 is nearly _____ times the cube's weight Mg .



- (A) 1 (B) 2 (C) 3 (D) 4
19. The tube AC forms a quarter circle in a vertical plane. The ball B has an area of cross-section slightly smaller than that of the tube, and can move without friction through it. B is placed at A and displaced slightly. It will



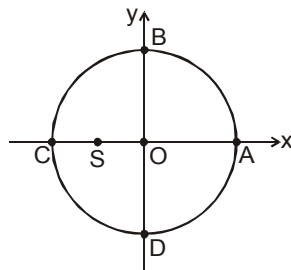
- (A) always be in contact with the inner wall of the tube
 (B) always be in contact with the outer wall of the tube
 (C) initially be in contact with the inner wall and later with the outer wall
 (D) initially be in contact with the outer wall and later with the inner wall
20. A stone is tied to a string of length l is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u . The magnitude of the change in its velocity at it reaches a position where the string is horizontal is

- (A) $\sqrt{(u^2 - 2gl)}$ (B) $\sqrt{2gl}$ (C) $\sqrt{(u^2 - gl)}$ (D) $\sqrt{2(u^2 - gl)}$

21. A cart moves with a constant speed along a horizontal circular path. From the cart, a particle is thrown up vertically with respect to the cart
- (A) The particle will land somewhere on the circular path
 (B) The particle will land outside the circular path
 (C) The particle will follow an elliptical path
 (D) The particle will follow a parabolic path

Multiple Correct Answers Type

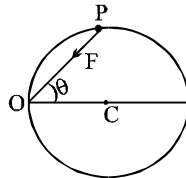
22. A stone is projected from level ground at $t = 0$ sec such that its horizontal and vertical components of initial velocity are 10 m/s and 20 m/s respectively. Then the instant of time at which magnitude of tangential and magnitude of normal components of acceleration of stone are same is : (neglect air resistance) $g = 10 \text{ m/s}^2$.
- (A) $\frac{1}{2}$ sec (B) 1 sec (C) 3 sec (D) 4 sec.
23. A particle starts from rest & moves in a circle of radius 3 m. Angular acceleration of the particle varies as $\alpha = 4t \text{ rad/sec}^2$. Then:
- (A) particle is moving in a uniform circular motion
 (B) particle will travel a distance of 2 m after 1 sec
 (C) angle between the velocity & the acceleration vector will be 45° after 1 sec
 (D) after 1 sec, speed of the particle will be 6 m/sec.
24. Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then
- (A) they cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed
 (B) they cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed
 (C) they cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed
 (D) they cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.
25. A small object moves counter clockwise along the circular path whose centre is at origin as shown in figure. As it moves along the path, its acceleration vector continuously points towards point S. Then the object



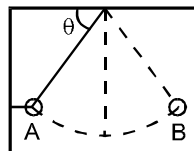
- (A) Speed up as it moves from A to C via B. (B) Slows down as it moves from A to C via B.
 (C) Slows down as it moves from C to A via D. (D) Speed up as it moves from C to A via D.
26. The position vector of a particle in a circular motion about the origin sweeps out equal area in equal time. Its
- (A) velocity remains constant (B) speed remains constant
 (C) acceleration remains constant (D) tangential acceleration remains constant
27. A body moves with constant angular velocity on a circle. Magnitude of angular acceleration is :
- (A) $r\omega^2$ (B) constant (C) zero (D) None of the above

PART - II : SUBJECTIVE QUESTIONS

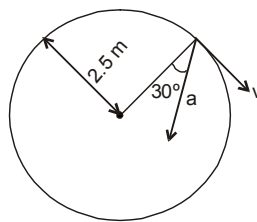
- A particle is moving with constant angular acceleration (α) in a circular path of radius $\sqrt{3}$ m. At $t = 0$, it was at rest and at $t = 1$ sec, the magnitude of its acceleration becomes $\sqrt{6}$ m/s², then α (in rad/s²) is -
- A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference. Find ratio of $\frac{d^2\theta}{dt^2}$ & $\left(\frac{d\theta}{dt}\right)^2$ when $\theta = 45^\circ$



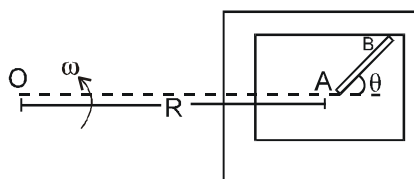
- A ball is held at rest in position A by two light cords (as in figure). The horizontal cord is cut and the ball starts swinging as pendulum. The ratio of the tension in the supporting cord in position B (after cut) to that in position A (before cut) will be _____.



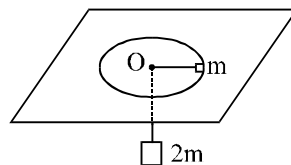
- A particle is placed on the surface of the earth such that its position vector w.r.t. centre of the earth makes an angle of 30° with the equator (i.e. its latitude is 30°). What should be the time period of the rotation of earth about its own axis so that the weight of the object measured by a weighing machine at that position is half of its real weight.
- Figure shows the direction of total acceleration and velocity of a particle moving clockwise in a circle of radius 2.5 m at a given instant of time. At this instant if magnitude of net acceleration is 25 m/sec², find :
(a) the radial acceleration, (b) the speed of the particle and (c) its tangential acceleration



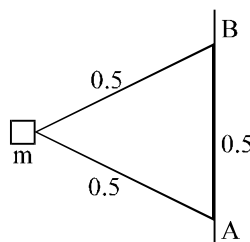
- A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius R (figure). A smooth groove AB of length $L (< R)$ is made on the surface of the table. The groove makes an angle θ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.



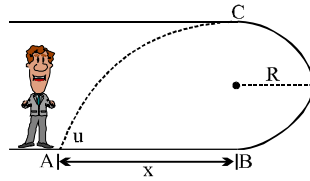
7. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37° from the direction to the centre of the circle as seen from the particle
 (a) At what rate is the speed of the particle increasing?
 (b) What is the magnitude of the acceleration?
8. A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At $t = 0$, it was subjected to a constant angular acceleration α and its angular speed increased to $(480/\pi)$ rpm in 2 sec. Particle then continues to move with attained speed. Calculate
 (a) angular acceleration of the particle,
 (b) tangential velocity of the particle as a function of time.
 (c) acceleration of the particle at $t = 0.5$ second and at $t = 3$ second
 (d) angular displacement at $t = 3$ second.
9. A stone is thrown horizontally with the velocity 15m/s. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.
10. A particle moves in a circle of radius R with a constant speed v. Then, find the magnitude of average acceleration during a time interval $\frac{\pi R}{2v}$.
11. A particle moves in the x-y plane with the velocity $\vec{v} = a\hat{i} + bt\hat{j}$. At the instant $t = a\sqrt{3}/b$ the magnitude of tangential, normal and total acceleration are _____, _____, & _____.
12. A particle is moving in a circle of radius 2m such that its centripetal acceleration is given by $a_c = 2t^2$. Find the angle (in rad.) traversed by the particle in the first two seconds.
13. A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



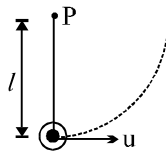
14. Two strings of length $l = 0.5$ m each are connected to a block of mass $m = 2$ kg at one end and their ends are attached to the point A and B 0.5 m apart on a vertical pole which rotates with a constant angular velocity $\omega = 7$ rad/sec. Find the ratio $\frac{T_1}{T_2}$ of tension in the upper string (T_1) and the lower string (T_2). [Use $g = 9.8$ m/s²]



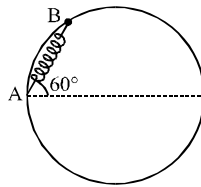
15. A person rolls a small ball with speed u along the floor from point A. If $x = 3R$, determine the required speed u so that the ball returns to A after rolling on the circular surface in the vertical plane from B to C and becoming a projectile at C. What is the minimum value of x for which the game could be played if contact must be maintained to point C? Neglect friction.



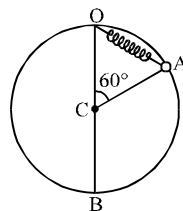
16. Consider the shown arrangement when a bob of mass ' m ' is suspended by means of a string connected to peg P. If the bob is given a horizontal velocity \vec{u} having magnitude $\sqrt{3gl}$, find the minimum speed of the bob in subsequent motion.



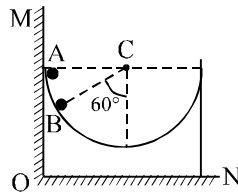
17. A bead of mass m is attached to one end of a spring of natural length $\sqrt{3}R$ and spring constant $k = \frac{(\sqrt{3}+1)mg}{R}$. The other end of the spring is fixed at point A on a smooth fixed vertical ring of radius R as shown in the figure. What is the normal reaction at B just after the bead is released?



18. A body of mass 2 kg is moving under the influence of a central force whose potential energy is given by $U(r) = 2r^3$ Joule. If the body is moving in a circular orbit of 5m, then find its energy.
19. A particle of mass 5 kg is free to slide on a smooth ring of radius $r = 20$ cm fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle OCA = 60^\circ$, C being the centre of the ring. The natural length of the spring is also equal to $r = 20$ cm. After the particle is released and slides down the ring the contact force between the particle & the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.



20. A ball of mass 1 kg is released from position A inside a wedge with a hemispherical cut of radius 0.5 m as shown in the figure. Find the force exerted by the vertical wall OM on wedge, when the ball is in position B. (neglect friction everywhere). Take ($g = 10 \text{ m/s}^2$)



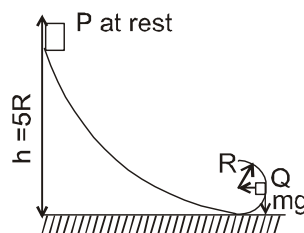
21. Two blocks of mass $m_1 = 10 \text{ kg}$ and $m_2 = 5 \text{ kg}$ connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.
- Calculate the frictional force on m_1 .
 - What should be the minimum angular speed of the turn table so that the masses will slip from this position.
 - How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1 .
22. A stone is launched upward at 45° with speed v_0 . A bee follows the trajectory of the stone at a constant speed equal to the initial speed of the stone.
- Find the radius of curvature at the top point of the trajectory.
 - What is the acceleration of the bee at the top point of the trajectory? For the stone, neglect the air resistance.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

* Marked Questions are having more than one correct option.

1. Two blocks of mass $m_1 = 10 \text{ Kg}$ and $m_2 = 5 \text{ Kg}$ connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/s about a vertical axis passing through its centre O. The masses are placed along the diameter of the table on either side of the centre O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table. [JEE 1997, 5/100]
- Calculate the frictional force on m_1 .
 - What should be the minimum angular speed of the turn table so that the masses will slip from this position ?
 - How should the masses be placed with the string remaining taut so that there is no frictional force acting on the mass m_1 ?
2. A small block of mass m slides along a smooth track as shown in figure. (i) If it starts from rest at P, what is the resultant force acting on it at Q ? (ii) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight ? R is radius of circular loop. [REE 1997]

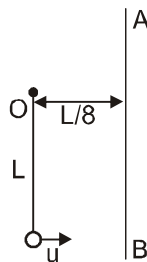


3. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u . The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is : **[JEE 1998, 2/200]**

(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$ (C) $\sqrt{u^2 - gL}$ (D) $\sqrt{2(u^2 - gL)}$

4. A particle at rest starts moving from the top of a large frictionless sphere of radius R . The sphere is fixed on the ground. Calculate that height from the ground at which the particle leaves the surface of the sphere. **[REE 1998]**

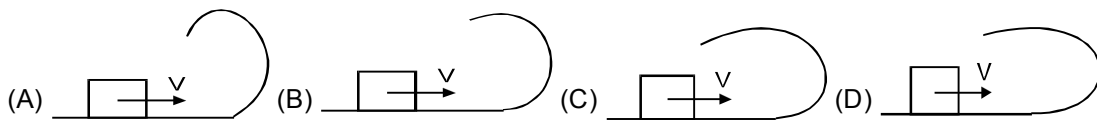
5. A particle is suspended vertically from a point O by an inextensible massless string of length L . A vertical line AB is at a distance $L/8$ from O as shown. The object is given a horizontal velocity u . At some point, its motion ceases to be circular and eventually the object passes through the line AB . At the instant of crossing AB , its velocity is horizontal. Find u . **[JEE 1999, 10/200]**



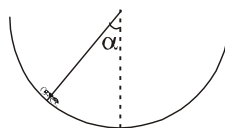
6. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α . If the coefficient of friction between the rod and the bead is μ , and gravity is neglected, then the time after which the bead starts slipping is- **[JEE (Scr) 2000, 3/105]**

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) Infinitesimal

7. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum is – **[JEE (Scr) 2001, 3/105]**

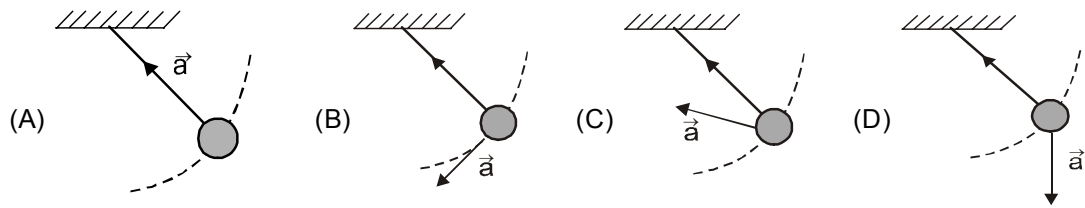


8. An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is $1/3$. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by- **[JEE (Scr) 2001, 3/105]**



(A) $\cot \alpha = 3$ (B) $\tan \alpha = 3$ (C) $\sec \alpha = 3$ (D) $\operatorname{cosec} \alpha = 3$

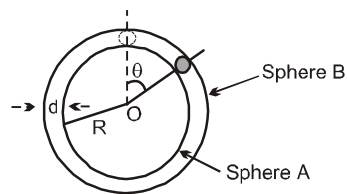
9. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector \vec{a} is correctly shown in [JEE 2002 (Scr), 3/90]



10. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d . The ball has a diameter very slightly less than d . All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is represented by θ (shown in figure) [JEE 2002 , 5/60]

(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ .

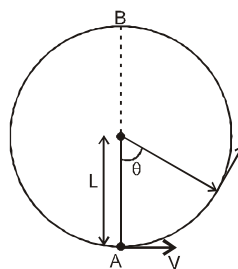
(b) Let N_A and N_B denote the magnitudes of the normal reaction force on the ball exerted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \leq \theta \leq \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axis



11. A bob of mass M is suspended by a massless string of length L . The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies

Figure :

[JEE 2008, 3/163]



(A) $\theta = \frac{\pi}{4}$

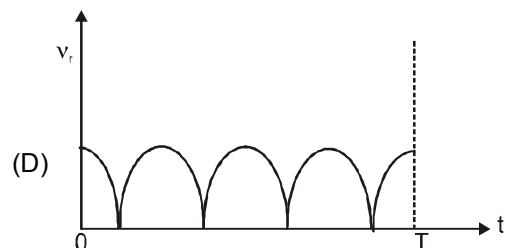
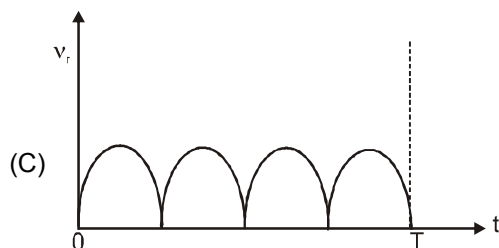
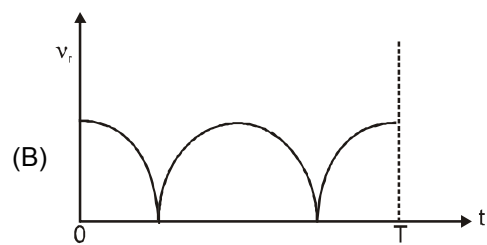
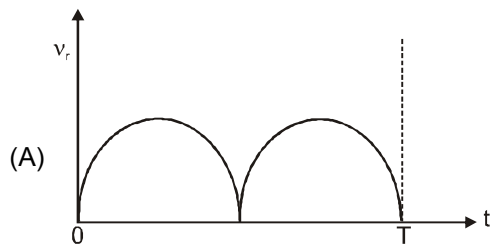
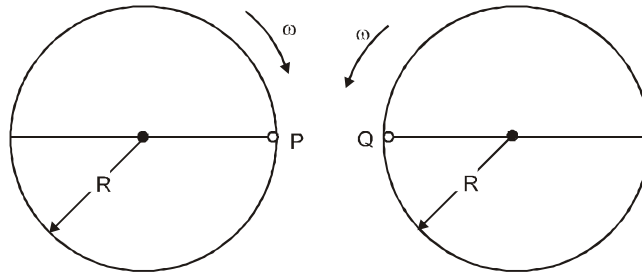
(B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$

(D) $\frac{3\pi}{4} < \theta < \pi$

12. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω . The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r . In one time period (T) of rotation of the discs v_r as a function of time is best represented by :

[JEE 2012, 4/136] [conducted by IIT Kanpur]
[CIRCULAR+RELATIVE MOTION, GRAPH, MODERATE]



13. The work done on a particle of mass m by a force, $\mathbf{K} \left[\frac{x}{(x^2 + y^2)^{3/2}} \hat{i} + \frac{y}{(x^2 + y^2)^{3/2}} \hat{j} \right]$ (K being a constant

of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x - y plane is :

[JEE (Advanced) 2013_P-1]

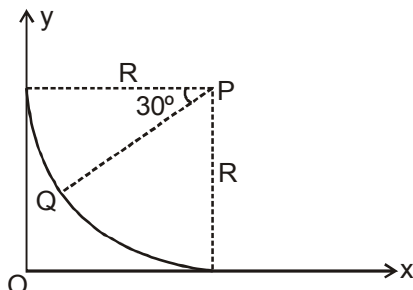
- (A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

Paragraph for Questions 14 and 15

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J.

(Take the acceleration due to gravity, $g = 10\text{ms}^{-2}$)

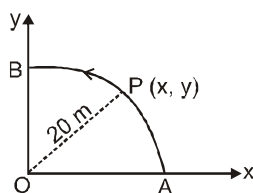
[JEE (Advanced) 2013_P-2]



14. The magnitude of the normal reaction that acts on the block at the point Q is : [JEE (Advanced) 2013_P-2]
 (A) 7.5 N (B) 8.6 N (C) 11.5 N (D) 22.5 N
15. The speed of the block when it reaches the point Q is : [JEE (Advanced) 2013_P-2]
 (A) 5ms^{-1} (B) 10ms^{-1} (C) $10\sqrt{3}\text{ms}^{-1}$ (D) 20ms^{-1}

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is : [AIEEE-2002, 4/300]
 (1) 60 (2) 30 (3) 15 (4) 25
2. Which of the following statements is false for a particle moving in a circle with a constant angular speed ? [AIEEE - 2004, 4/300]
 (1) The velocity vector is tangent to the circle
 (2) The acceleration vector is tangent to the circle
 (3) The acceleration vector point to the center of the circle
 (4) The velocity and acceleration vectors are perpendicular to each other
3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that : [AIEEE - 2004, 4/300]
 (1) its velocity is constant (2) its acceleration is constant
 (3) its kinetic energy is constant (4) it moves in a straight line
4. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2\text{ s}$ is nearly. [AIEEE - 2010, 4/144]



- (1) 13 m/s^2 (2) 12 m/s^2 (3) 7.2 m/s^2 (4) 14 m/s^2

5. For a particle in uniform circular motion, the acceleration \vec{a} at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x-axis) [AIEEE - 2010, 4/144]

(1) $-\frac{v^2}{R} \cos \theta \hat{i} + \frac{v^2}{R} \sin \theta \hat{j}$

(2) $-\frac{v^2}{R} \sin \theta \hat{i} + \frac{v^2}{R} \cos \theta \hat{j}$

(3) $-\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j}$

(4) $\frac{v^2}{R} \hat{i} + \frac{v^2}{R} \hat{j}$

6. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 , respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :

(1) $m_1 r_1 : m_2 r_2$

(2) $m_1 : m_2$

[AIEEE - 2012, 4/120]

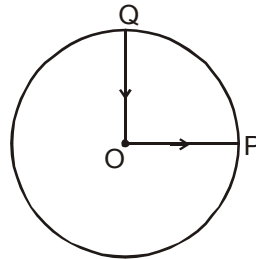
(3) $r_1 : r_2$

(4) 1 : 1

EXERCISE # 4

NCERT BOARD PATTERN QUESTIONS

- An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector? What is its magnitude?
- A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along OQ as shown in figure. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



- A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?
- A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.5 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?
- A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?
- A circular recetrack of radius 300 m is banked at an angle of 15° . If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the racecar to avoid wear and tear on its tyres, and (b) ,maximum permissible speed to avoid slipping?

7. One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v the net force on the particle (directed towards the centre) is.

(i) T , (ii) $T - \frac{mv^2}{l}$ (iii) $T + \frac{mv^2}{l}$ (iv) 0

T is the tension in the string. [Choose the correct alternative].

8. A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 m s⁻¹. What is the trajectory of the bob if the string is cut when the bob is (a) at one of its extreme positions, (b) at its mean position.
9. A stone of mass m tied to the end of a string is revolved in a vertical circle of radius R . The net forces at the lowest and highest points of the circle directed vertically downwards are : [Choose the correct alternative].

Lowest Point

Highest Point

(a) $mg - T_1$

$mg + T_2$

(b) $mg + T_1$

$mg - T_2$

(c) $mg + T_1 - (mv_1^2)/R$

$mg - T_2 + (mv_1^2)/R$

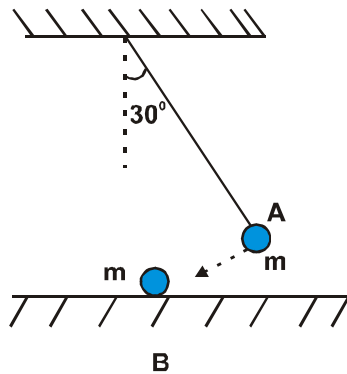
(d) $mg - T_1 - (mv_1^2)/R$

$mg + T_2 + (mv_1^2)/R$

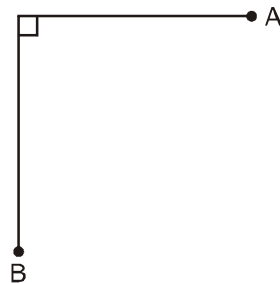
Here T_1, T_2 (and v_1, v_2) denote the tension in the string (and the speed of the stone) at the lowest and the highest point respectively.

10. A disc revolves with a speed of $33 \frac{1}{3}$ rev/min, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the co-efficient of friction between the coins and the record is 0.15 , which of the coins will revolve with the record ?
11. You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m ?
12. A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis. The co-efficient of friction between the wall and his clothing is 0.15 . What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed ?
13. A thin circular wire of radius R rotates about its vertical diameter with an angular frequency ω . Show that a small bead on the wire remains at its lower most point for bead with the vertical downward direction for $\omega \leq \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

14. The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in figure. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.



15. The bob of a pendulum is released from a horizontal position A as shown in figure. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipated 5% of its initial energy against air resistance?



ANSWERS

Exercise # 1

PART-I

- A-1.*** (ACD) **A-2.** (C) **A-3.** (C) **A-4.** (C) **A-5.** (C) **A-6.** (A) **A-7.** (A)
A-8. (B) **B-1.** (B) **B-2.** (B) **B-3.** (B) **B-4.** (C) **B-5.** (A) **B-6.** (A)
B-7. (D) **B-8.** (C) **B-9.** (C) **B-10.** (A) **C-1.** (B) **C-2.** (A) **C-3.*** (BCD)
C-4.* (AC) **C-5.** (B) **C-6.** (D) **C-7.** (A) **C-8.** (A) **C-9.** (B) **C-10.** (C)
C-11. (B) **C-12.** (A) **D-1.** (B) **D-2.** (A) **D-3.** (C) **D-4.** (A) **E-1.** (D)
E-2. (A) **E-3.** (D) **E-4.** (D) **E-5.** (C) **E-6.** (C) **E-7.** (B) **E-8.** (C)
E-9. (B) **E-10.** (C) **E-11.** (C) **E-12.** (B) **E-13.** (A) **F-1.** (C) **F-2.** (A)
F-3. (C) **F-4.** (D) **F-5.** (A)

PART-II

- 1.** (B) **2.** (C) **3.** (C) **4.** (C) **5.** (B) **6.** (C) **7.** (B)
8. (D) **9.** (B) **10.** (D) **11.** (D) **12.** (BD) **13.** (ABD) **14.** (C)
15. (A) - q, s (B) - p, q (C) - p (D) - q, r, s **16.** (A) - r ; (B) - r ; (C) - p ; (D) - q
17. (A) - p, r ; (B) - p, r ; (C) - q, s ; (D) - s **18.** (i) True (ii) False (iii) **False**

Exercise # 2

PART-I

- 1.** (C) **2.** (D) **3.** (D) **4.** (B) **5.** (D) **6.** (D) **7.** (B)
8. (D) **9.** (B) **10.** (D) **11.** (D) **12.** (B) **13.** (D) **14.** (D)
15. (C) **16.** (C) **17.** (B) **18.** (C) **19.** (C) **20.** (D) **21.** (B)
22.* (BC) **23.** (BCD) **24.** (BCD) **25.** (AC) **26.** (BD) **27.** (BC) **28.** (CD)
29. (ABCD) **30.** (BC) **31.** (BCDE) **32.** (BC)

PART-II

- 1.** 1 **2.** 2 **3.** $\sin^2\theta$ **4.** $T' = 2\pi\sqrt{\frac{3R}{2g}}$
5. (a) $25\frac{\sqrt{3}}{2} \text{ m/s}^2$ (b) $\left(125\frac{\sqrt{3}}{4}\right)^{1/2} \text{ m/s}$ (c) $\frac{25}{2} \text{ m/s}^2$ **6.** $\sqrt{\frac{2L}{\omega^2 R \cos\theta}}$
7. (a) 75m/s^2 , (b) 125m/s^2
8. (a) 2 rad/s^2 , (b) $12+2t$ for $t \leq 2\text{s}$, 16 for $t > 2\text{s}$, (c) $\sqrt{28565} \approx 169, 256 \text{ m/s}^2$ (d) 44 rad
9. $a_t = \frac{2g}{\sqrt{13}}$, $a_n = \frac{3g}{\sqrt{13}}$ **10.** $\frac{2\sqrt{2} v^2}{\pi R}$ **11.** $\sqrt{3}b/2$, $b/2$, b

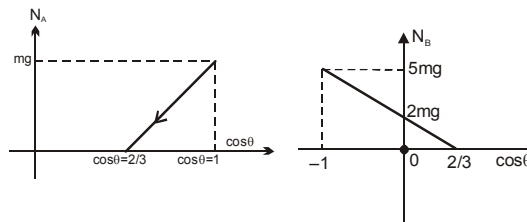
12. 2 rad. 13. $\sqrt{2g}$ rad/s 14. 9 15. $5/2\sqrt{gR}$, $x_{\min}=2R$
16. $\frac{1}{3}\sqrt{\frac{gl}{3}}$ 17. $(1-\sqrt{3}/2)mg$ 18. 625 J 19. 500N/m
20. $\frac{15\sqrt{3}}{2}$ N 21. (i) 36N, (ii) 11.66rad/sec, (iii) 0.1m, 0.2m 22. (a) $\frac{V_0^2}{2g}$, (b) 2g

Exercise # 3

PART-I

1. (i) 36 N (ii) = 11.67 rad/s (iii) $r_1 = 0.1$ m and $r_2 = 0.2$ m 2. $(\sqrt{65})mg$, $3R$
3. (D) 4. $H = (5/3)R$ 5. $u = \sqrt{gL\left(\frac{3\sqrt{3}}{2} + 2\right)}$
6. (A) 7. (A) 8. (A)
9. (C)

10. (a) $N_A = 3mg \cos\theta - 2mg$, $N_B = 2mg - 3mg \cos\theta$ (b)



11. (D) 12. (A) 13. (D) 14. (A) 15. (B)

PART-II

1. (2) 2. (2) 3. (3) 4. (4) 5. (3) 6. (3)

Exercise # 4

1. This is an example of uniform circular motion. Here $R = 12$ cm. The angular speed ω is given by $\omega = 2\pi/T = 2\pi \times 7/100 = 0.44$ rad/s
The linear speed v is :
The direction of velocity v is along the tangent to the circle at every point. The acceleration is directed towards the centre of the circle. Since this direction changes continuously, acceleration here is not a constant vector. However, the magnitude of acceleration is constant :
 $a = \omega^2 R = (0.44 \text{ s}^{-1}) (12 \text{ cm})$
 $= 2.3 \text{ cm s}^{-2}$
2. (a) O; (b) O; (c) 21.4 km h^{-1}
3. 9.9 m s^{-2} , along the radius at every point towards the centre.
4. 0.86 m s^{-2} , 54.5° with the direction of velocity.
5. On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by Eq. :
 $v^2 \leq \mu_s R g$
Now, $R = 3$ m, $g = 9.8 \text{ m s}^{-2}$, $\mu_s = 0.1$. That is, $\mu_s R g = 2.94 \text{ m}^2 \text{ s}^{-2}$. $v = 18 \text{ km/h} = 5 \text{ m s}^{-1}$; i.e., $v^2 = 25 \text{ m}^2 \text{ s}^{-2}$.
If the condition is not obeyed. Then cyclist will slip while taking the circular turn.

6. On a banked road, the horizontal component of the normal force and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the normal reaction's component is enough to provide the needed centripetal force, and the frictional force is not needed. The optimum speed v_0 is given by Eq.

$$v_0 = (R g \tan \theta)^{1/2}$$

Here $R = 300\text{m}$, $\theta = 15^\circ$, $g = 9.8 \text{ m s}^{-2}$; we have

$$v_0 = 28.1 \text{ m s}^{-1}.$$

The maximum permissible speed v_{\max} is given by Eq.

$$v_{\max} = \left(\frac{\mu_s \times \tan \theta}{1 - \mu_s \tan \theta} \right)^{1/2} = 38.1 \text{ m s}^{-1}$$

7. (a) T

8. (a) At the extreme position, the speed of the bob is zero. If the string is cut, it will fall vertically downwards.

(b) At the mean position, the bob has a horizontal velocity. If the string is cut, it will fall along a parabolic path.

9. Alternative (a) is correct. Note

$$mg + T_2 = mv_2^2/R; \quad T_1 - mg = mv_1/R$$

The moral is : do not confuse the actual material forces on a body (tension, gravitational force, etc) with the effects they produce : centripetal acceleration v_2^2/R or v_1^2/R in this example.

10. For the coin to revolve with the disc, the force of friction should be enough to provide the necessary

centripetal force, i.e. $\frac{mv^2}{R} \leq \mu mg$. Now $r = r\omega$, where $\omega = \frac{2\pi}{T}$ is the angular frequency of the disc. For

a given μ and ω , the condition is $r \leq \mu g/\omega^2$. The condition is satisfied by the nearer coin (4 cm from the centre).

11. At the uppermost point, $N + mg = \frac{mv^2}{R}$, where N is the normal force (downwards) on the motorcyclist by the ceiling of the chamber. The minimum possible speed at the uppermost point corresponds to $N = 0$.

$$\text{i.e. } v_{\min} = \sqrt{Rg} = \sqrt{25 \times 10} = 15.8 \text{ m s}^{-1}$$

12. The horizontal force N by the wall on the man provides the needed centripetal force : $N = m R \omega^2$. The frictional force f (vertically upwards) opposes the weight mg . The man remains stuck to the wall after the floor is removed if $mg = f < \mu N$ i.e. $mg < \mu m R \omega^2$. The minimum angular speed of rotation of the cylinder is $\omega_{\min} = \sqrt{g/\mu R} = 4.7 \text{ s}^{-1}$.

13. Consider the free-body diagram of the bead when the radius vector joining the centre of the wire makes an angle θ with the vertical downward direction. We have $mg = N \cos \theta$ and $m R \sin \theta \omega^2 = N \sin \theta$. These

equations give $\cos \theta = g/R\omega^2$. since $\cos \theta \leq 1$ the bead remains at its lowermost point for $\omega \leq \sqrt{\frac{g}{R}}$.

$$\text{For } \omega = \sqrt{\frac{g}{R}}, \cos \theta = \frac{1}{2} \quad \text{i.e. } \theta = 60^\circ$$

14. It transfers its entire momentum to the ball on the table, and does not rise at all.

15. 5.3 m s^{-1}