arride learning

GRAVITATION

Contents

Syllabus

Law of gravitation; Gravitational potential and field; Acceleration due to gravity; Motion of planets and satellites in circular orbits; Escape velocity.

Name : ______________________________ **Contact No.** ____________________

ARRIDE LEARNING ONLINE E-LEARNING ACADEMY

A-479 indra Vihar, Kota Rajasthan 324005

Contact No. 8033545007

GRAVITATION

KEY CONCEPT

1. Gravitation:

Gravitation is the force of attraction between any two point particles in the universe. It is given by:

$$F = \frac{Gm_1m_2}{r^2}$$

where G is universal gravitational constant. The value of $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$.

2. Variation of 'g':

(i) *Due to altitude*: Acceleration due to gravity at a height h above the surface of earth is given by:

$$g_h = \frac{GM}{(R+h)^2} = g\left[1 - \frac{2h}{R}\right] \quad \text{(for h << R)}$$

(ii) *Due to depth:* Acceleration due to gravity at a depth h below the surface of earth is given by

$$g_h' = g\left[1 - \frac{h}{R}\right] \quad \text{(for all depths)}$$

At h = R (i.e. at the centre of earth): $g_h' = 0$

(iii) *Due to rotation of earth:* Acceleration due to gravity at latitude λ is given by:

$g_\lambda = g - R\omega^2\cos^2\lambda$ where ω = angular velocity of the earth

(a) ***At poles***: $g_p = g - R\omega^2\cos^2(\pi/2) = g = g_{max}$

(b) ***At equator***: $g_{eq} = g - R\omega^2\cos^2 0 = g - R\omega^2 = g_{min}$

(iv) *Due to non-spherical shape of earth:* Due to the shape of the earth, g is maximum at poles and minimum at equator.

3. Inertial and gravitational mass :

(i) *Inertial mass:* It is defined as the ratio of the magnitude of external force applied on the body to the magnitude of acceleration produced in it, i.e., a = (F/m).

(ii) *Gravitational mass:* Mass of the material of the body, which is determined by gravitational pull acting on it, is called as gravitational mass, i.e., $m = \frac{FR^2}{GM}$

(iii) Inertial and gravitational masses are found to be equal by observation.

4. Gravitational intensity:

In case of a solid or hollow sphere of mass M and radius R:

(a) For an external point (r > R): $I_0 = (GM/r^2)$

(b) For an internal point (r < R):

(i) of a spherical shell: $I_i = 0$

(ii) of a solid sphere: $I_i = (GM/R^3)r$

5. Gravitational potential:

In case of a solid or hollow sphere:

(a) For an external point (r > R): $V_0 = -(GM/r)$

(b) For an internal point (r < R):

(i) of a spherical shell: $V_i = -GM/R$ = constant

(ii) of a solid sphere: $V_i = -\frac{GM}{2R^3}(3R^2 - r^2)$

6. Escape velocity:

(i) It is minimum speed with which a body must be projected away from the surface of the earth so that it may never return to the earth.

(ii) Escape velocity of a body from the surface of earth is given by : $v_{es} = \sqrt{2gR} = \sqrt{(2GM/R)}$

7. Geostationary satellite:

(a) A satellite which appears to be stationary for a person on the surface of the earth is called geostationary satellite.

(b) It revolves in the equatorial plane from west to east with a time period of 24 hours.

(c) Its height from the surface of the earth is nearly 35600 km and radius of the circular orbit is nearly 42000 km.

(d) The orbital velocity of this satellite is nearly 3.08 km/sec

(e) The relative velocity of geostationary satellite with respect to earth is zero.

(f) The orbit of a geostationary satellite is called as **parking orbit**.

8. Kepler's laws:

(i) All planets move around the sun in **elliptical orbits**, with the sun being at rest at one focus of the orbit.

(ii) The position vector from the sun to the planet sweeps out equal area in equal time, i.e., areal velocity of a planet around the sun always remains constant. This gives that the **angular momentum or moment of momentum remain constant**.

(iii) The square of the time period of a planet around the sun is proportional to the cube of the semi-major axis of the ellipse or mean distance of the from the sun, i.e. $T^2 \propto a^3$ where a is the semi-major axis of the ellipse.

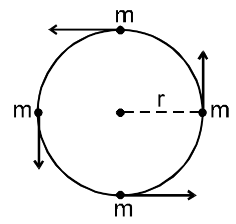
EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

** Marked Questions are having more than one correct option.*

Section (A) : Universal law of gravitation

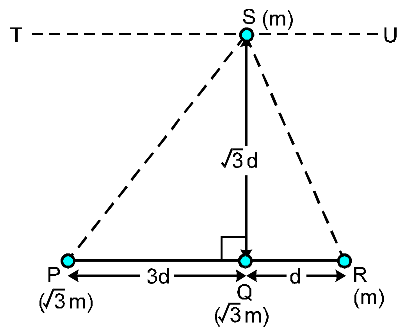
A 1. Four similar particles each of mass m are orbiting in a circle of radius r in the same sense and same speed because of their mutual gravitational attractive force as shown in the figure. Velocity of a particle is given by :



(A) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ (B) $\sqrt{\frac{Gm}{r}}$

(C) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ (D) zero

A-2. Three particles P, Q and R are placed as per given figure. Masses of P, Q and R are $\sqrt{3}$ m, $\sqrt{3}$ m and m respectively. The gravitational force on a fourth particle 'S' of mass m is equal to :



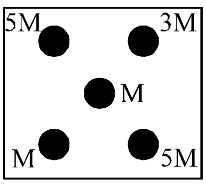
(A) $\frac{\sqrt{3}\ \ Gm^2}{2d^2}$ in ST direction only

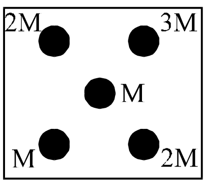
(B) $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SU direction

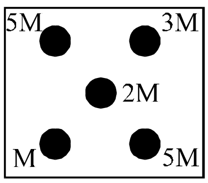
(C) $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction only

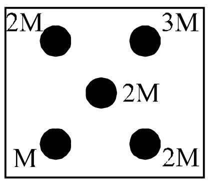
(D) $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in ST direction

A-3 A mass is at the center of a square, with four masses at the corners as shown.

(A) 

(B) 

(C) 

(D) 

Rank the choices according to the magnitude of the gravitational force on the center mass.

(A) $F_A = F_B < F_C = F_D$ (B) $F_A > F_B < F_D < F_C$

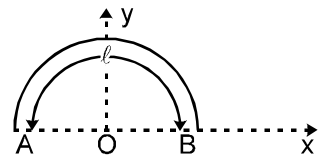
(C) $F_A = F_B > F_C = F_D$ (D) None

Section (B) : Gravitational field and potential

B-1. Let gravitation field in a space be given as E = – (k/r). If the reference point is at distance d_i where potential is V_i then relation for potential is :

(A) $V = k\,\ell n\frac{1}{V_i} + 0$ (B) $V = k\,\ell n\frac{r}{d_i} + V_i$ (C) $V = \ell n\frac{r}{d_i} + kV_i$ (D) $V = -k\ell n\frac{r}{d_i} + V_i$

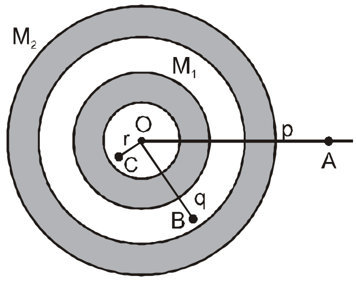
B-2. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass m and length ℓ as shown in the figure. is :



(A) $\frac{Gm}{\ell^2}$ along +x axis
(B) $\frac{Gm}{\pi\ell^2}$ along +y axis
(C) $\frac{2\pi Gm}{\ell^2}$ along + x axis
(D) $\frac{2\pi Gm}{\ell^2}$ along + y axis

B-3. A very large number of particles of same mass M are kept at horizontal distances (in metres) of 1m, 2m, 4m, 8m and so on from (0,0) point. The total gravitational potential at this point is :
(A) – 8G M
(B) – 3G M
(C) – 4G M
(D) – 2G M

B-4. Two concentric shells of uniform density of mass M_1 and M_2 are situated as shown in the figure. The forces experienced by a particle of mass m when placed at positions A, B and C respectively are (given OA = p, OB = q and OC = r).



(A) zero, $G\frac{M_1m}{q^2}$ and $G\frac{(M_1+m_2)m}{p^2}$

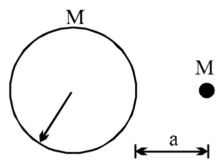
(B) $G\frac{(M_1+M_2)m}{p^2}$, $G\frac{(M_1+M_2)m}{q^2}$ and $G\frac{M_1m}{r^2}$

(C) $G\frac{M_1m}{q^2}$, $\frac{G(M_1+M_2)m}{p^2}$, $G\frac{M_1m}{q^2}$ and zero

(D) $\frac{G(M_1+M_2)m}{p^2}$, $G\frac{M_1m}{q^2}$ and zero

B-5*. In case of earth :
(A) field is zero, both at centre and infinity
(B) potential is zero, both at centre and infinity
(C) potential is same, both at centre and infinity but not zero
(D) potential is minimum at the centre

B-6. A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a.



(A) Gravitational field and potential both are zero at centre of the shell.
(B) Gravitational field is zero not only inside the shell but at a point outside the shell also.
(C) Inside the shell, gravitational field alone is zero.
(D) Neither gravitational field nor gravitational potential is zero inside the shell.

B-7. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre
(A) increases
(B) decreases
(C) remains same
(D) during the compression increases then returns at the previous value.

B-8. Select the correct choice(s):
(A) The gravitational field inside a spherical cavity, within a spherical planet must be non zero and uniform.
(B) When a body is projected horizontally at an appreciable large height above the earth, with a velocity less than for a circular orbit, it will fall to the earth along a parabolic path.
(C) A body of zero total mechanical energy placed in a gravitational field if it is travelling away from source of field will escape the field
(D) Earth's satellite must be in equatorial plane.

Section (C) : Gravitational Potential Energy and Self Energy

C-1. A body starts from rest at a point, distance R_0 from the centre of the earth of mass M, radius R. The velocity acquired by the body when it reaches the surface of the earth will be

(A) $GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (B) $2\,GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (C) $\sqrt{2GM\left(\frac{1}{R}-\frac{1}{R_0}\right)}$ (D) $2GM\sqrt{\left(\frac{1}{R}-\frac{1}{R_0}\right)}$

C-2. Three equal masses each of mass 'm' are placed at the three-corners of an equilateral triangle of side 'a'.

(a) If a fourth particle of equal mass is placed at the centre of triangle, then net force acting on it, is equal to :

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(b) In above problem, if fourth particle is at the mid-point of a side, then net force acting on it, is equal to:

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(c) If above given three particles system of equilateral triangle side a is to be changed to side of 2a, then work done on the system is equal to :

(A) $\frac{3Gm^2}{a}$ (B) $\frac{3Gm^2}{2a}$ (C) $\frac{4Gm^2}{3a}$ (D) $\frac{Gm^2}{a}$

(d) In the above given three particle system, if two particles are kept fixed and third particle is released. Then speed of the particle when it reaches to the mid-point of the side connecting other two masses:

(A) $\sqrt{\frac{2Gm}{a}}$ (B) $2\sqrt{\frac{Gm}{a}}$ (C) $\sqrt{\frac{Gm}{a}}$ (D) $\sqrt{\frac{Gm}{2a}}$

C-3. A satellite of mass m, initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is

(A) $\frac{\sqrt{3}}{4}mgR$ (B) $\frac{1}{2}mgR$ (C) $\frac{1}{4}mgR$ (D) $\frac{3}{4}mgR$

Section : (D) Kepler's law for Satellites, Orbital Velocity and Escape Velocity

D-1. Periodic-time of satellite revolving around the earth is - (ρ is density of earth)

(A) Proportional to $\frac{1}{\rho}$ (B) Proportional to $\frac{1}{\sqrt{\rho}}$ (C) Proportional to ρ (D) does not depend on ρ.

D-2. An artificial satellite of the earth releases a package. If air resistance is neglected the point where the package will hit (with respect to the position at the time of release) will be

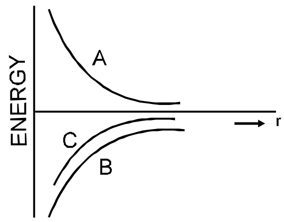
(A) ahead (B) exactly below
(C) behind (D) it will never reach the earth

D-3*. An orbiting satellite will escape if :

(A) its speed is increased by $(\sqrt{2}-1)100\%$
(B) its speed in the orbit is made $\sqrt{1.5}$ times of its initial value
(C) its KE is doubled
(D) it stops moving in the orbit

D-4*. A satellite close to the earth is in orbit above the equator with a period of revolution of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time
(A) 1.5 hours
(B) 1.6 hours if it is rotating from west to east
(C) 24/17 hours if it is rotating from east to west
(D) 24/17 hours if it is rotating from west to east

D-5. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



(A) A shows the kinetic energy, B the total energy and C the potential energy of the system
(B) C shows the total energy, B the kinetic energy and A the potential energy of the system
(C) C and A are kinetic and potential energies respectively and B is the total energy of the system
(D) A and B are the kinetic and potential energies and C is the total energy of the system.

D-6*. In case of an orbiting satellite if the radius of orbit is decreased :
(A) its Kinetic Energy decreases
(B) its Potential Energy decreases
(C) its Mechanical Energy decreases
(D) its speed decreases

D-7. A planet of mass m revolves around the sun of mass M in an elliptical orbit. The minimum and maximum distance of the planet from the sun are r_1 & r_2 respectively. If the minimum velocity of the planet is $\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$ then it's maximum velocity will be :

(A) $\sqrt{\frac{2GMr_2}{(r_1+r_2)r_1}}$
(B) $\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$
(C) $\sqrt{\frac{2Gmr_2}{(r_1+r_2)r_1}}$
(D) $\sqrt{\frac{2GM}{r_1+r_2}}$

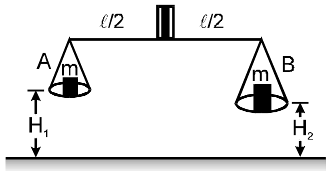
D-8. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V.
(A) $V_e = 2V$
(B) $V_e = V$
(C) $V_e = V/2$
(D) $V_e = \sqrt{3}\,V$

D-9. Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity $\frac{v_A}{v_B}$ is
(A) 2
(B) $\sqrt{2}$
(C) $1/\sqrt{2}$
(D) $1/2$

D-10. The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be
(A) v_e
(B) $\frac{v_e}{\sqrt{2}}$
(C) $\frac{v_e}{2}$
(D) zero

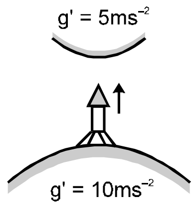
Section (E) : Earth and Other Planets Gravity

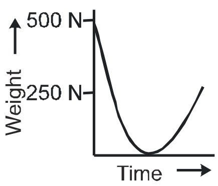
E-1. Two blocks of masses m each are hung from a balance as shown in the figure. The scale pan A is at height H_1 whereas scale pan B is at height H_2. Net torque acting on the rod of pan, will be (length of the rod is ℓ and H_1 & H_2 are << R) ($H_1 > H_2$)

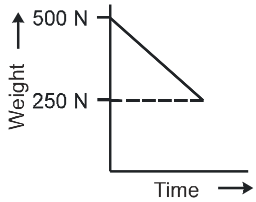


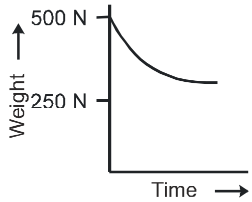
(A) $mg\left(\frac{1-2H_1}{R}\right)\ell$ (B) $\frac{mg}{R}(H_1-H_2)\,\ell$ (C) $\frac{2mg}{R}(H_1+H_2)\,\ell$ (D) $2mg\frac{H_2H_1}{H_1+H_2}\,\ell$

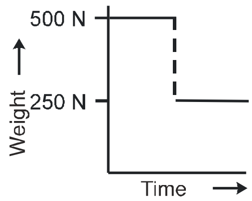
E-2. If acceleration due to gravity is 10 ms^{-2} then let acceleration due to gravitational acceleration at another planet of our solar system be 5 ms^{-2}. An astronaut weighing 50 kg on earth goes to this planet in a spaceship with a constant velocity. The weight of the astronaut with time of flight is roughly given by



(A) 

(B) 

(C) 

(D) 

E-3. At what altitude will the acceleration due to gravity be 25% of that at the earth's surface (given radius of earth is R)?
(A) R/4 (B) R (C) 3R/8 (D) R/2

E-4. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole (d<<R) The value of d is

(A) $\frac{\omega^2R^2}{g}$ (B) $\frac{\omega^2R^2}{2g}$ (C) $\frac{2\omega^2R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$

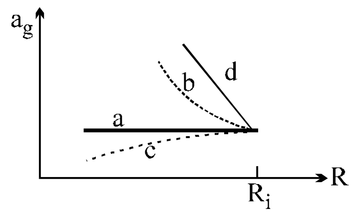
E-5. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same?

(A) 1/25 (B) 1/5 (C) $1/\sqrt{5}$ (D) 5

E-6. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth?

(A) $\sqrt{2}$ second (B) $2\sqrt{2}$ seconds (C) $\frac{1}{\sqrt{2}}$ second (D) $\frac{1}{2\sqrt{2}}$ second

E-7. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse?



(A) a
(B) b
(C) c
(D) d

E-8. In older times, people used to think that the Earth was flat. Imagine that the Earth is indeed not a sphere of radius R, but an infinite plate of thickness H. What value of H is needed to allow the same gravitational acceleration to be experienced as on the surface of the actual Earth ? (Assume that the Earth's density is uniform and equal in the two models.)

(A) $\frac{2R}{3}$ (B) $\frac{4R}{3}$ (C) $\frac{8R}{3}$ (D) $\frac{R}{3}$

PART - II : MISCELLANEOUS QUESTIONS

Comprehension # 1:

Two uniform spherical stars made of same material have radii R and 2R. Mass of the smaller planet is m. They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution 1/2.

1. Kinetic energy of the system just after the collision is:

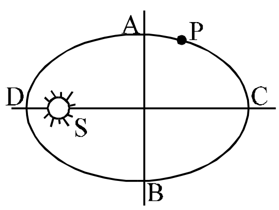
(A) $\frac{8Gm^2}{3R}$ (B) $\frac{2Gm^2}{3R}$ (C) $\frac{4Gm^2}{3R}$ (D) cannot be determined

2. The maximum separation between their centres after their first collision is :

(A) 4 R (B) 6 R (C) 8 R (D) 12 R

Comprehension # 2 :

Figure shows the orbit of a planet P round the sun S. AB and CD are the minor and major axes of the ellipse.



3. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then

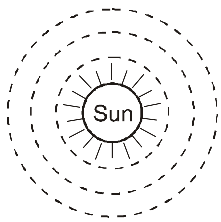
(A) $t_1 = t_2$ (B) $t_1 > t_2$ (C) $t_1 < t_2$ (D) nothing can be concluded

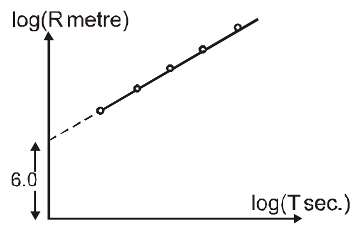
4. If U is the potential energy and K kinetic energy then |U| > |K| at

(A) Only D (B) Only C (C) both D & C (D) neither D nor C

Comprehension # 3

Many planets are revolving around the fixed sun, in circular orbits of different radius (R) and different time period (T). To estimate the mass of the sun, the orbital radius (R) and time period (T) of planets were noted.
Then $\log_{10} T$ v/s $\log_{10} R$ curve was plotted.
The curve was found to be approximately straight line (as shown in figure) having y intercept = 6.0 (Neglect the gravitational interaction among the planets





[Take $G = \frac{20}{3} \times 10^{-11}$ in MKS, $\pi^2 = 10$]

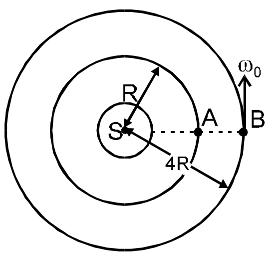
5. The slope of the line should be :

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{19}{4}$

6. Estimate the mass of the sun :

(A) 6×10^{29} kg (B) 5×10^{20} kg (C) 8×10^{25} kg (D) 3×10^{35} kg

7. Two planets A and B, having orbital radius R and 4R are initially at the closest position and rotating in the same direction. If angular velocity of planet B is ω_0, then after how much time will both the planets be again in the closest position ? (Neglect the interaction between planets).



(A) $\frac{2\pi}{7\omega_0}$ (B) $\frac{2\pi}{9\omega_0}$ (C) $\frac{2\pi}{\omega_0}$ (D) $\frac{2\pi}{5\omega_0}$

Comprehension # 4

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the surface of earth. R is the radius of earth and g is acceleration due to gravity at the surface of earth. (R = 6400 km)

8. Then the distance of satellite from the surface of earth is

(A) 3200 km (B) 6400 km (C) 12800 km (D) 4800 km

9. The time period of revolution of satellite in the given orbit is

(A) $2\pi\sqrt{\frac{2R}{g}}$ (B) $2\pi\sqrt{\frac{4R}{g}}$ (C) $2\pi\sqrt{\frac{8R}{g}}$ (D) $2\pi\sqrt{\frac{6R}{g}}$

10. If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, the speed with which it hits the surface of the earth.

(A) $\sqrt{gR}$ (B) $\sqrt{1.5gR}$ (C) $\sqrt{\frac{gR}{2}}$ (D) $\sqrt{\frac{gR}{\sqrt{2}}}$

Comprehension # 5

A pair of stars rotates about a common center of mass. One of the stars has a mass M and the other has mass m such that M = 2m. The distance between the centres of the stars is d (d being large compared to the size of either star).

11. The period of rotation of the stars about their common centre of mass (in terms of d, m, G.) is

(A) $\sqrt{\frac{4\pi^2}{Gm}d^3}$ (B) $\sqrt{\frac{8\pi^2}{Gm}d^3}$ (C) $\sqrt{\frac{2\pi^2}{3Gm}d^3}$ (D) $\sqrt{\frac{4\pi^2}{3Gm}d^3}$

12. The ratio of the angular momentum of the two stars about their common centre of mass (L_m/ L_M) is
(A) 1 (B) 2 (C) 4 (D) 9

13. The ratio of kinetic energies of the two stars (K_m/K_M.) is
(A) 1 (B) 2 (C) 4 (D) 9

Assertion/Reason Type

14. **Statement-1 :** Moon revolving around earth does not come closer despite earth's gravitational attraction.
Statement-2 : A radially outward force balances earth's force of attraction during revolution of moon.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

15. **Statement-1 :** Time period of simple pendulum in an orbiting geostationary satellite is infinite.
Statement-2 : Earth's gravitational field becomes negligible at large distance from it.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

16. **Statement-1 :** Geostationary satellites may be setup in equatorial plane in orbits of any radius more than earth's radius.
Statement-2 : Geostationary satellites have period of revolution of 24 hrs.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

17. **Statement-1 :** For the calculation of gravitational force between any two uniform spherical shells, they can always be replaced by particles of same mass placed at respective centres.
Statement-2 : Gravitational field of a uniform spherical shell out side it is same as that of particle of same mass placed at its centre of mass.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

18. **Statement-1 :** It takes more fuel for a spacecraft to travel from the earth to moon than for the return trip.
Statement-2 : Potential energy of spacecraft at moon's surface is greater than that at earth surface.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Match the column

19. A particle is taken to a distance r (> R) from centre of the earth. R is radius of the earth. It is given velocity V which is perpendicular to $\vec{r}$. With the given values of V in column I you have to match the values of total energy of particle in column II and the resultant path of particle in column III. Here 'G' is the universal gravitational constant and 'M' is the mass of the earth.

Column I (Velocity)	Column II (Total energy)	Column III (Path)
(A) $V = \sqrt{GM/r}$	(p) Negative	(t) Elliptical
(B) $V = \sqrt{2GM/r}$	(q) Positive	(u) Parabolic
(C) $V > \sqrt{2GM/r}$	(r) Zero	(v) Hyperbolic
(D) $\sqrt{GM/r} < V < \sqrt{2GM/r}$	(s) Infinite	(w) Circular

20. Let V and E denote the gravitational potential and gravitational field respectively at a point due to certain uniform mass distribution described in four different situations of column-I. Assume the gravitational potential at infinity to be zero.The value of E and V are given in column-II. Match the statement in column-I with results in column-II.

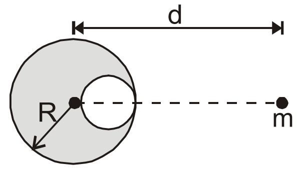
Column-I	Column-II
(A) At centre of thin spherical shell	(p) $E = 0$
(B) At centre of solid sphere	(q) $E \neq 0$
(C) A solid sphere has a non-concentric spherical cavity. At the centre of the spherical cavity	(r) $V \neq 0$
(D) At centre of line joining two point masses of equal magnitude	(s) $V = 0$

EXERCISE # 2

PART - I : MIXED OBJECTIVE

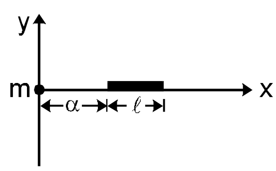
Single Choice type

1. A spherical hollow cavity is made in a lead sphere of radius R, such that its surface touches the outside surface of the lead sphere and passes through its centre. The mass of the sphere before hollowing was M. With what gravitational force will the hollowed-out lead sphere attract a small sphere of mass 'm', which lies at a distance d from the centre of the lead sphere on the straight line connecting the centres of the spheres and that of the hollow, if d = 2R :



(A) $\frac{7GMm}{18R^2}$ (B) $\frac{7GMm}{36R^2}$ (C) $\frac{7GMm}{9R^2}$ (D) $\frac{7GMm}{72R^2}$

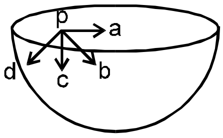
2. A straight rod of length ℓ extends from $x = \alpha$ to $x = \ell + \alpha$. as shown in the figure. If the mass per unit length is $(a + bx^2)$. The gravitational force it exerts on a point mass m placed at x = 0 is given by



(A) $Gm\left(a\left(\frac{1}{\alpha}-\frac{1}{\alpha+\ell}\right)+b\ell\right)$ (B) $\frac{Gm(a+bx^2)}{\ell^2}$

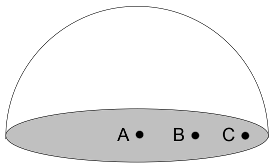
(C) $Gm\left(\alpha\left(\frac{1}{a}-\frac{1}{a+\ell}\right)+b\ell\right)$ (D) $Gm\left(a\left(\frac{1}{\alpha+\ell}-\frac{1}{\alpha}\right)+b\ell\right)$

3. Figure show a hemispherical shell having uniform mass density. The direction of gravitational field intensity at point P will be along:



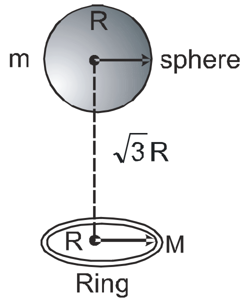
(A) a　　(B) b　　(C) c　　(D) d

4. Mass M is uniformly distributed only on curved surface of a thin hemispherical shell. *A*, *B* and *C* are three points on the circular base of hemisphere, such that *A* is the centre. Let the gravitational potential at points A, B and C be V_A, V_B, V_C respectively. Then



(A) $V_A > V_B > V_C$　　(B) $V_C > V_B > V_A$　　(C) $V_B > V_A$ and $V_B > V_C$　　(D) $V_A = V_B = V_C$

5. A uniform ring of mass M is lying at a distance $\sqrt{3}$ R from the centre of a uniform sphere of mass m just below the sphere as shown in the figure where R is the radius of the ring as well as that of the sphere. Then gravitational force exerted by the ring on the sphere is :



(A) $\frac{GMm}{8R^2}$　　(B) $\frac{GMm}{3R^2}$　　(C) $\sqrt{3}\frac{GMm}{R^2}$　　(D) $\sqrt{3}\frac{GMm}{8R^2}$

6. The gravitational potential of two homogeneous spherical shells A and B (separated by large distance) of same surface mass density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into single one such that surface mass density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :

(A) 3 : 2　　(B) 4 : 3　　(C) 5 : 3　　(D) 3 : 5

7. If a tunnel is cut at any orientation through earth, then a ball released from one end will reach the other end in time(neglect earth rotation)

(A) 84.6 minutes　　(B) 42.3 minutes　　(C) 8 minutes　　(D) depends on orientation

8. A satellite of the earth is revolving in circular orbit with a uniform velocity V. If the gravitational force suddenly disappears, the satellite will

(A) continue to move with the same velocity in the same orbit.
(B) move tangentially to the original orbit with velocity V.
(C) fall down with increasing velocity.
(D) come to a stop somewhere in its original orbit.

9. A satellite revolves in the geostationary orbit but in a direction east to west. The time interval between its successive passing about a point on the equator is :

(A) 48 hrs　　(B) 24 hrs　　(C) 12 hrs　　(D) never

10. Two point masses of mass 4*m* and *m* respectively separated by *d* distance are revolving under mutual force of attraction. Ratio of their kinetic energies will be :
(A) 1 : 4 (B) 1 : 5 (C) 1 : 1 (D) 1 : 2

11. A satellite of mass 5M orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass 4M. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass 4M is :
(A) In a circular orbit (B) unbound (C) elliptical orbit
(D) data is insufficient to determine the nature of the orbit.

12. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

13. A planet of mass m is in an elliptical orbit about the sun ($m << M_{sun}$) with an orbital period T. If A be the area of orbit, then its angular momentum would be:

(A) $\frac{2mA}{T}$ (B) mAT (C) $\frac{mA}{2T}$ (D) 2mAT

14. Satellites A and B are orbiting around the earth in orbits of ratio R and 4R respectively. The ratio of their areal velocities is:
(A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 16

15. A planet revolves about the sun in elliptical orbit. The arial velocity $\left(\frac{dA}{dt}\right)$ of the planet is 4.0×10^{16} m^2/s. The least distance between planet and the sun is 2×10^{12} m. Then the maximum speed of the planet in km/s is :
(A) 10 (B) 20 (C) 40 (D) None of these

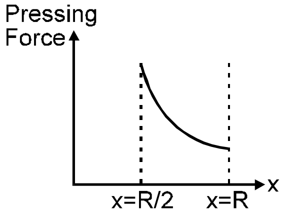
More than one choice type

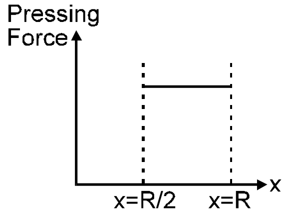
16. For a satellite to appear stationary to an observer on earth
(A) It must be rotating about the earth's axis.
(B) It must be rotating in the equatorial plane.
(C) Its angular velocity must be from west to east.
(D) Its time period must be 24 hours.

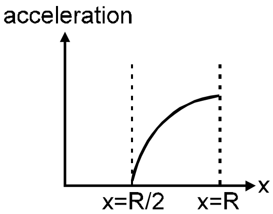
17. Which of the following are correcrt ?
(A) An astronant going from the earth to the Moon will experience weightlessness once.
(B) When a thin uniform spherical shell gradually shrinks maintaining its shape, the gravitational potential at its centre decreases.
(C) In the case of a spherical shell, the plot of V versus r is contiunous.
(D) In the case of a spherical shell, the plot of gravitational field intensity I versus r is continuous.

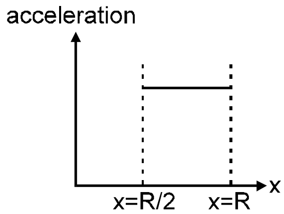
18. Which of the following statements are correct about a planet rotating around the sun in an elliptical orbit:
(A) its mechanical energy is constant
(B) its angular momentum about the sun is constant
(C) its areal velocity about the sun is constant
(D) its time period is proportional to r^3

19. A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall and the acceleration of the particle varies with x (distance of the particle from the centre) according to :

(A)


(B)


(C)


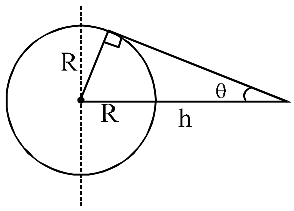
(D)


20. Assuming the earth to be a sphere of uniform density the acceleration due to gravity
(A) at a point outside the earth is inversely proportional to the square of its distance from the centre
(B) at a point outside the earth is inversely proportional to its distance from the centre
(C) at a point inside is zero
(D) at a point inside is proportional to its distance from the centre.

21. Two masses m_1 and m_2 ($m_1 < m_2$) are released from rest from a finite distance. They start under their mutual gravitational attraction
(A) acceleration of m_1 is more than that of m_2
(B) acceleration of m_2 is more than that of m_1
(C) centre of mass of system will remain at rest in all the references frame
(D) total energy of system remains constant

22. In side a hollow isolated spherical shell
(A) everywhere gravitational potential is zero.
(B) everywhere gravitational field is zero.
(C) everywhere gravitational potential is same.
(D) everywhere gravitational field is same.

23. A geostationary satellite is at a height h above the surface of earth. If earth radius is R :



(A) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1}\left(R/R+h\right)$.
(B) The maximum colatitudes on earth upto which the satellite can be used for communication is $\sin^{-1}\left(R/R+h\right)$.
(C) The area on earth escaped from this satellite is given as $2\pi R^2$ $(1 + \sin\theta)$
(D) The area on earth escaped from this satellite is given as $2\pi R^2$ $(1 + \cos\theta)$

24. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then
(A) its kinetic energy increases
(B) its kinetic energy decreases
(C) its angular momentum about the earth decreases
(D) its period of revolution around the earth increases

25. A communications Earth satellite
(A) goes round the earth from east to west
(B) can be in the equatorial plane only
(C) can be vertically above any place on the earth
(D) goes round the earth from west to east

26. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change
(A) gravitational potential energy (B) angular vleocity
(C) linear orbital velocity (D) centripetal acceleration

27. A geostationary satellite S is stationed above a point P on the equator. A particle is fired from S directly towards P.
(A) With respect to axis of rotation of the earth, P and S have the same angular velocity but different linear velocities.
(B) The particle will hit P.
(C) The particle will hit the equator east of P.
(D) The particle will hit the equator west of P.

28. If a satellite orbits as close to the earth's surface as possible,
(A) its speed is maximum
(B) time period of its rotation is minimum
(C) the total energy of the 'earth plus satellite' system is minimum
(D) the total energy of the 'earth plus satellite'system is maximum

29. For a satellite to orbit around the earth, which of the following must be true?
(A) It must be above the equator at some time
(B) It cannot pass over the poles at any time
(C) Its height above the surface cannot exceed 36,000 km
(D) Its period of rotation must be > $2\pi\sqrt{R/g}$ where R is radius of earth

30. Two satellites s_1 & s_2 of equal masses revolve in the same sense around a heavy planet in coplanar circular orbit of radii R & 4R
(A) the ratio of period of revolution s_1 & s_2 is 1 : 8.
(B) their velocities are in the ratio 2 : 1
(C) their angular momentum about the planet are in the ratio 2 : 1
(D) the ratio of angular velocities of s_2 w.r.t. s_1 when all three are in the same line is 9 : 5.

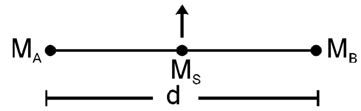
PART - II : SUBJECTIVE QUESTIONS

1. Two uniform solid spheres of same material and same radius 'r' are touching each other. If the density is 'ρ' then find out gravitational force between them.

2. The gravitational potential in a region is given by V = (20x + 40y) J/kg. Find out the gravitational field (in newton / kg) at a point having co-ordinates (2, 4). Also find out the magnitude of the gravitational force on a particle of 0.250 kg placed at the point (2, 4).

3. The gravitational field in a region is given by $\vec{E} = (3\hat{i} - 4\hat{j})$ N/kg. Find out the work done (in joule) in displacing a particle of mass 1 kg by 1 m along the line 4y = 3x + 9.

4. Two planets A and B are fixed at a distance d from each other as shown in the figure. If the mass of A is M_A and that of B is M_B, then find out the minimum velocity of a satellite of mass M_S projected from the mid point of two planets to infinity.



5. A satellite is established in a circular orbit of radius r and another in a circular orbit of radius 1.01 r. How much percentage the time period of second-satellite will be larger than the first satellite nearly.

6. Two identical stars of mass M orbit around their centre of mass. Each orbit is circular and has radius R, so that the two stars are always on opposite sides on a diameter.

(a) Find the gravitational force of one star on the other.

(b) Find the orbital speed of each star and the period of the orbit.

(c) Find their common angular speed.

(d) Find the minimum energy that would be required to separate the two stars to infinity.

(e) If a meteorite passes through this centre of mass perpendicular to the orbital plane of the stars. What value must its speed exceed at that point if it escapes to infinity from the star system.

7. Two stars of mass M_1 & M_2 are in circular orbits around their centre of mass. The star of mass M_1 has an orbit of radius R_1, the star of mass M_2 has an orbit of radius R_2. (assume that their centre of mass is not accelerating and distance between stars is fixed)

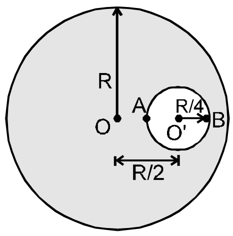
(a) Show that the ratio of the orbital radii of the two stars equals the reciprocal of the ratio of their masses, that is $R_1/R_2 = M_2/M_1$.

(b) Explain why the two stars have the same orbital period and show that the period,

$$T = 2\pi \frac{(R_1+R_2)^{3/2}}{\sqrt{G(M_1+M_2)}}.$$

(c) The two stars in a certain binary star system move in circular orbits. The first star, α moves in an orbit of radius 1.00×10^9 km. The other star, β moves in an orbit of radius 5.00×10^8 km. The orbital period is 44.5 year. What are the masses of each of the two stars ?

8. In a solid sphere of radius 'R' and density 'ρ' there is a spherical cavity of radius R/4 as shown in figure. A particle of mass 'm' is released from rest from point 'B' (inside the cavity). Find out -



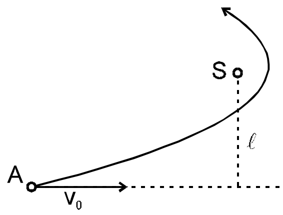
(a) The position where this particle strikes the cavity.

(b) Velocity of the particle at this instant.

9. (a) What is the escape speed for an object in the same orbit as that of Earth around sun (Take orbital radius R) but far from the earth ? (mass of the sun = M_s)

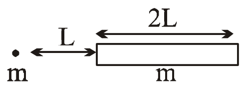
(b) If an object already has a speed equal to the earth's orbital speed, what minimum additional speed must it be given to escape as in (a) ?

10. A cosmic body A moves towards the Sun with velocity v_0 (when far from the Sun) and aiming parameter ℓ, the direction of the vector v_0 relative to the centre of the Sun as shown in the figure. Find the minimum distance by which this body will get to the Sun. (Mass of Sun = M_S)



11. If a pendulum has a period of exactly 1.00 sec. at the equator, what would be its period at the south pole ? Assume the earth to be spherical and rotational effect of the Earth is to be taken.

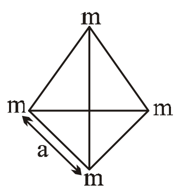
12. A small mass and a thin uniform rod each of mass ' m ' are positioned along the same straight line as shown. Find the force of gravitational attraction exerted by the rod on the small mass.



13. A point P lies on the axis of a fixed ring of mass M and radius a, at a distance a from its centre C. A small particle starts from P and reaches C under gravitational attraction only. Its speed at C will be _______.

14. An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:
(i) the initial speed of projection
(ii) the speed at half the maximum height.

15. Four masses (each of m)are placed at the vertices of a regular pyramid triangular base of side 'a'. Find the work done by the system while taking them apart so that they form the pyramid of side '2a'.

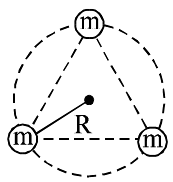


16. A thin spherical shell of total mass M and radius R is held fixed. There is a small hole in the shell. A mass m is released from rest a distance R from the hole along a line that passes through the hole and also through the centre of the shell. This mass subsequently moves under the gravitational force of the shell. How long does the mass take to travel from the hole to the point diametrically opposite.

17. A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5$J. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2 \times 10^5$J is equal to ________.

18. A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/ s. Then the time taken for the satellite to reach the earth is _______.

19. A hypothetical planet of mass M has three moons each of equal mass 'm' each revolving in the same circular orbit of radius R. The masses are equally spaced and thus form an equilateral triangle. Find :
(i) the total P.E. of the system
(ii) the orbital speed of each moon such that they maintain this configuration.



20. A remote sensing satellite is revolving in an orbit of radius x over the equator of earth. Find the area on earth surface in which satellite can not send message.

21. A pair of stars rotates about a common center of mass. One of the stars has a mass M which is twice as large as the mass m of the other. Their centres are a distance d apart, d being large compared to the size of either star. (a) Derive an expression for the period of rotation of the stars about their common centre of mass in terms of d,m, G. (b) Compare the angular momentum of the two stars about their common centre of mass by calculating the ratio L_m/L_M. (c) Compare the kinetic energies of the two stars by calculating the ratio K_m/K_M.

22. A small body is projected with a velocity just sufficient to make it reach from the surface of a planet (of radius 2R and mass 3M) to the surface of another planet (of radius R and mass M). The distance between the centers of the two spherical planets is 6R. the distance of the body from the center of bigger planet is 'x' at any moment. During the journey, find the distance x where the speed of the body is (a) maximum (b) minimum. Assume motion of body along the line joining centres of planets.

EXERCISE # 3

PART - I : IIT-JEE : QUESTIONS

* ***Marked Questions are having more than one correct option.***

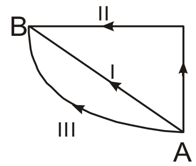
1. Distance between the centres of two stars is 10a. The masses of these stars are M and 16 M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? obtain the expression in terms of G. M and a. **[JEE - 1996, 5]**

2. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth : **[JEE (Scr) - 98, 2]**
(A) The acceleration of S is always directed towards the centre of the earth
(B) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(C) The total mechanical energy of S varies periodically with time
(D) The linear momentum of S remains constant in magnitude.

3. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is : **[JEE (Scr) - 2001, 1/35]**
(A) 1 (B) $\sqrt{2}$ (C) 4 (D) 2

4. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometers above the earth's surface (R_{Earth} = 6400 km) will approximately be : **[JEE(Scr) - 02, 3/84]**
(A) 1/2 hr (B) 1 hr (C) 2 hr (D) 4 hr

5. A particle of mass m is taken through the gravitational field produced by a source S, from A to B, along the three paths as shown in figure. If the work done along the paths I, II and III is W_I, W_{II} and W_{III} respectively, then : **[JEE (Scr.)-2003,3/84]**



(A) $W_I = W_{II} = W_{III}$ (B) $W_{II} > W_{III} = W_{II}$ (C) $W_{III} = W_{II} > W_I$ (D) $W_I > W_{II} > W_{III}$

6. A projectile is fired vertically up from the bottom of a crater (big hole) on the moon. The depth of the crater is R/100, where R is the radius of the moon. If the initial velocity of the projectile is the same as the escape velocity from the moon surface, determine in terms of R, the maximum height attained by the projectile above the lunar (moon) surface. **[JEE 2003(Main),4/60]**

7. A double star system consists of two stars A and B which have time period T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option. **[JEE 2006, +3, –1/184]**

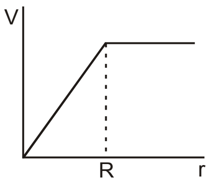
(A) If $T_A > T_B$ then $R_A > R_B$ (B) If $T_A > T_B$ then $M_A > M_B$

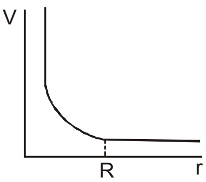
(C) $\left(\dfrac{T_A}{T_B}\right)^2 = \left(\dfrac{R_A}{R_B}\right)^3$ (D) $T_A = T_B$

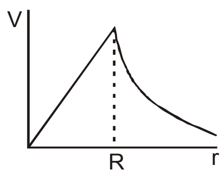
8. A spherically symmetric gravitational system of particles has a mass density **[JEE 2008, +3, –1/82]**

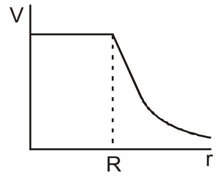
$$\rho = \begin{cases} \rho_0 & \text{for} \quad r \le R \\ 0 & \text{for} \quad r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A) 

(B) 

(C) 

(D) 

9. **STATEMENT -1** **[JEE 2008,+3, –1/82]**

An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT -2

An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall.

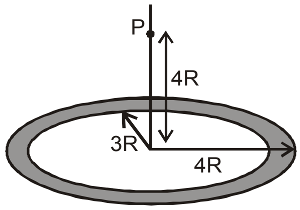
(A) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is a correct explanation for STATEMENT -1

(B) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is NOT a correct explanation for STATEMENT -1

(C) STATEMENT -1 is True, STATEMENT -2 is False

(D) STATEMENT -1 is False, STATEMENT -2 is True.

10. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is : **[JEE 2010, –1, 3/252]**



(A) $\dfrac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (B) $-\dfrac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (C) $\dfrac{GM}{4R}$ (D) $\dfrac{2GM}{5R}\left(\sqrt{2}-1\right)$

11. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S) where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is : **[JEE 2010, +3, 3/252]**

12. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be : **[JEE 2010, +3, 3/252]**

PART - II : AIEEE QUESTIONS

1. A satellite of the earth is revolving in a circular orbit with a uniform speed v. If the gravitational force suddenly disappears, the statellite will : **[AIEEE-2002, 4/300]**
(1) Continue to move with velocity v along the original orbit
(2) Move with a velocity v, tangentially to the original orbit
(3) Fall down with increasing velocity
(4) Ultimately come to rest somewhere on the original orbit

2. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period becomes **[AIEEE-2003, 4/300]**
(1) 10 hour (2) 80 hour (3) 40 hour (4) 20 hour

3. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45º with the vertical, the escape velocity will be : **[AIEEE-2003,4/300]**
(1) $11\sqrt{2}$ km/s (2) 22 km/s (3) 11 km/s (4) $11/\sqrt{2}$ m/s

4. A satellite of mass m revolves around earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is : **[AIEEE-2004, 4/300]**
(1) gx (2) $\frac{gR}{R-x}$ (3) $\frac{gR^2}{R+x}$ (4) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

5. The time period of an earth satellite in circular orbit is independent of : **[AIEEE-2004, 4/300]**
(1) the mass of the satellite
(2) radius of its orbit
(3) both the mass and radius of the orbit
(4) neither the mass of the satellite nor the radius of its orbit

6. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is : **[AIEEE-2004, 4/300]**
(1) 2mgR (2) $\frac{1}{2}$mgR (3) $\frac{1}{4}$mgR (4) mgR

7. The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then, which one of the following is correct ? **[AIEEE-2005, 4/300]**
(1) $d = \frac{h}{2}$ (2) $d = \frac{3h}{2}$ (3) d = 2h (4) d = h

8. A particle of mass 10 kg is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere (you may take G = 6.67×10^{-11} Nm^2/kg^2); **[AIEEE-2005, 4/300]**

(1) 13.34×10^{-10} J (2) 3.33×10^{-10} J (3) 6.67×10^{-9} J (4) 6.67×10^{-7} J

9. If g_E and g_m are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop expriment could be performed on the two surfaces, one will find the ratio

$\dfrac{\text{Electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be **[AIEEE-2007, 3/120]**

(1) 1 (2) 0 (3) g_E/g_M (4) g_M/g_E

10. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[AIEEE-2008, 3/105]**

(1) 11 km s^{-1} (2) 110 km s^{-1} (3) 0.11 km s^{-1} (4) 1.1 km s^{-1}

11. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is **[AIEEE-2009, 4/144]**

(1) $\frac{R}{\sqrt{2}}$ (2) $\frac{R}{2}$ (3) $\sqrt{2}$ R (4) 2R

12. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is : **[AIEEE 2011]**

(1) $-\frac{4Gm}{r}$ (2) $-\frac{6Gm}{r}$ (3) $-\frac{9Gm}{r}$ (4) zero

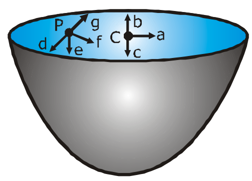
13. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be : **[AIEEE 2012]**

(1) 6.4×10^{11} J (2) 6.4×10^{8} J (3) 6.4×10^{9} J (4) 6.4×10^{10} J

EXERCISE # 4

NCERT QUESTIONS

1. (i) In the following two exercises, choose the correct answer from among the given ones :
The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see figure) (i) a, (ii) b, (iii) c, (iv) none.
(ii) For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.



2. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

3. A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = $6.4 \times 10^{23\,kg}$; radius of mars = 3395 kg; G = 6.67×10^{-11} N m^2 kg^{-2}.

4. A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is 1.50×10^8 km away from the sun?

5. A body weights 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

6. Assuming the earth to be a sphere of uniform mass density, how much would a body weight half way down to the centre of the earth if it weighted 250 N on the surface?

7. A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^6 m ; G = 6.67×10^{-11} N m^2 kg^{-2}.

8. The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

9. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expanded to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; G = 6.67×10^{-11} N m^2 kg^{-2}.

10. Two stars each of the one solar mass (= 2×10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

11. Two heavy spheres, each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

12. As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km.

13. A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun = 2×10^{30} kg).

14. A spaceship is stationed on Mars. How much energy must be expanded on the spaceship to launch it out to the solar system? Mass of the space ship = 1000 kg; mass of sun = 2×10^{30} kg ; mass of mars = 6.4×10^{23} kg ; radius of mars = 3395 kgm ; radius of the orbit of mars = 2.28×10^8 km; G = 6.67×10^{-11} N m^2 kg^{-2}.

ANSWERS

Exercise # 1

PART - I

A 1.	(A)	**A-2.**	(C)	**A-3**	(A)	**B-1.**	(B)	**B-2.**	(D)	**B-3.**	(D)	**B-4.**	(D)
B-5*.	(AD)	**B-6.**	(D)	**B-7.**	(B)	**B-8.**	(C)	**C-1.**	(C)				
C-2.	(a) (D) (b) (B) **(c)** (B) **(d)** (B)					**C-3.**	(D)	**D-1.**	(B)	**D-2.**	(D)	**D-3*.**	(AC)
D-4*.	(BC)	**D-5.**	(D)	**D-6*.**	(BC)	**D-7.**	(A)	**D-8.**	(A)	**D-9.**	(A)	**D-10.**	(B)
E-1.	(B)	**E-2.**	(A)	**E-3.**	(B)	**E-4.**	(A)	**E-5.**	(B)	**E-6.**	(B)	**E-7.**	(B)
E-8.	(A)												

PART - II

1.	(B)	**2.**	(A)	**3.**	(B)	**4.**	(C)	**5.**	(C)	**6.**	(A)	**7.**	(A)
8.	(B)	**9.**	(C)	**10.**	(A)	**11.**	(D)	**12.**	(B)	**13.**	(B)	**14.**	(C)
15.	(B)	**16.**	(D)	**17.**	(D)	**18.**	(A)						

19.

I	II	III
A	p	w
B	r	u
C	q	v
D	p	t

20. (A) p,r (B) p,r (C) q,r (D) p,r

Exercise # 2

PART - I

1.	(B)	**2.**	(A)	**3.**	(C)	**4.**	(D)	**5.**	(D)	**6.**	(C)	**7.**	(B)
8.	(B)	**9.**	(C)	**10.**	(A)	**11.**	(B)	**12.**	(C)	**13.**	(A)	**14.**	(A)
15.	(C)	**16.**	(ABCD)	**17.**	(ACD)	**18.**	(ABC)	**19.**	(B*C)	**20.**	(AD)	**21.**	(AD)
22.	(BCD)			**23.**	(AC)	**24.**	(AC)	**25.**	(BD)	**26.**	(A)	**27.**	(AC)
28.	(ABC)	**29.**	(AD)	**30.**	(ABD)								

PART - II

1. $\frac{4}{9}\pi^2\rho^2Gr^4$ **2.** $-20\hat{i}-40\hat{j}$, $|\vec{F}| = 5\sqrt{5}$ N, $\vec{F} = -5\hat{i}-10\hat{j}$ **3.** zero

4. $2\sqrt{\frac{G(M_A+M_B)}{d}}$ **5.** 1.5%

6. **(a)** $F = \frac{GM^2}{4R^2}$ **(b)** $\sqrt{\frac{GM}{4R}}$; $T = 4\pi\sqrt{\frac{R^3}{GM}}$ **(c)** $\sqrt{\frac{GM}{4R^3}}$ **(d)** $\frac{GM^2}{4R}$ **(e)** $\sqrt{\frac{4GM}{R}}$

7. $M_\alpha = \frac{4\pi^2[1.5\times10^{12}]^3}{3G[44.5\times365\times86400]^2} = 3.376\times10^{29}$ kg, $M_\beta = 2M_\alpha = 6.75\times10^{29}$ kg

8. (a) Since force is always acting towards centre of solid sphere. Hence it will strike at 'A'.

(b) $v = \sqrt{\frac{2\pi G\rho R^2}{3}}$

9. **(a)** $\sqrt{\frac{2GM_S}{R}}$ **(b)** $\left(\sqrt{2}-1\right)\sqrt{\frac{GM_S}{R}}$ **10.** $r_{min} = (GM_S/v_0^2)\,[\sqrt{1+(\ell v_0^2/GM_S)^2}-1]$

11. $T = 1 - \frac{1}{2}\frac{4\pi^2}{(86400)^2} \times 6400 \times \frac{10^3}{9.8} = 0.998$ s

12. $\frac{G\ m^2}{3\ L^2}$ **13.** $\sqrt{\frac{2GM}{a}\left(1-\frac{1}{\sqrt{2}}\right)}$ **14.** **(i)** $\frac{4}{3}\sqrt{\frac{Gm}{R}}$, **(ii)** $\frac{2}{3}\sqrt{\frac{2Gm}{5R}}$

15. $-\frac{3Gm^2}{a}$ **16.** $2 \times \sqrt{R^3 / GM}$ **17.** **1×10^5J** **18.** $t = \frac{GMm}{2C}\left(\frac{1}{R_e}-\frac{1}{r}\right)$

19. **(i)** $-\frac{3Gm}{R}\left(\frac{m}{\sqrt{3}}+M\right)$, **(ii)** $\sqrt{\frac{G}{R}\left(\frac{m}{\sqrt{3}}+M\right)}$ **20.** $\left(1-\frac{\sqrt{x^2-R^2}}{x}\right)4\pi R^2$

21. **(a)** $T=\frac{2\pi d^{3/2}}{\sqrt{3Gm}}$, **(b) 2,** **(c) 2** **22.** 2R, $3R[3-\sqrt{3}\,]$

Exercise # 3

PART - I

1. $V_{min} = \frac{3\sqrt{5}}{2}\sqrt{\frac{GM}{a}}$ **2.** (A) **3.** (D) **4.** (C) **5.** (A) **6.** 99.5R

7. (D) **8.** (C) **9.** (A) **10.** **(A)** **11.** **6** **12.** **3**

PART - II

1. (2) **2.** (3) **3.** (3) **4.** (4) **5.** (1) **6.** (2) **7.** (3)

8. (4) **9.** (1) **10.** (2) **11.** **(4)** **12.** (3) **13.** (4)

Exercise # 4

NCERT QUESTIONS

1. (i) (c), (ii) (e) **2.** 2.6×10^8 m **3.** 495 km **4.** 1.43×10^{12} m

5. 28 N **6.** 125 M **7.** 8.0×10^6 m from the earth's centre

8. 31.7 km/s **9.** 5.9×10^9 J **10.** 2.6×10^6 m/s

11. 0, 2.7×10^{-8} J/kg; an object placed at the mid point is in an unstable equilibrium.

12. -9.4×10^6 J/kg **13.** Yes **14.** 3×10^{11} J