arride learning

MATHEMATICAL TOOLS

Contents

Syllabus

Differentiation, Integration & Vector

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MATHEMATICAL TOOLS

TRIGONOMETRY :

$$\theta = \frac{\text{Arc length}}{\text{Radius}} \quad \Rightarrow \quad \theta = \frac{\overset{\frown}{AB}}{r}$$

Angle Conversion formulas

1 degree = $\frac{\pi}{180}$ ($\approx$ 0.02) radian

Degrees to radians : multiply by $\frac{\pi}{180}$

1 radian $\approx$ 57 degrees

Radians to degrees : multiply by $\frac{180}{\pi}$

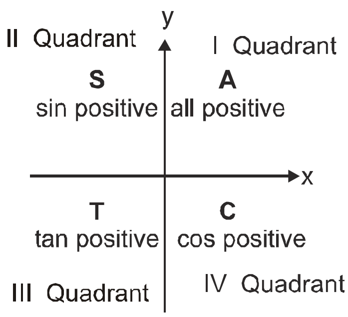
RULES FOR FINDING TRIGONOMETRIC RATIO OF ANGLES GREATER THAN 90° :

Step 1 → Identify the quadrant in which angle lies.

Step 2 → **(a)** If angle = $(n\pi \pm \theta)$ where n is an integer. Then trigonometric function of $(n\pi \pm \theta)$ = same trigonometric function of θ and sign will be decided by CAST Rule.

THE CAST RULE

A useful rule for remembering when the basic trigonometric functions are positive and negative is the CAST rule.
If you are not very enthusiastic about CAST. You can remember it as ASTC (After school to college)



(b) If angle = $\left[(2n+1)\frac{\pi}{2} \pm \theta\right]$ where n is an integer. Then

trigonometric function of $\left[(2n+1)\frac{\pi}{2} \pm \theta\right]$ = complimentary trigonometric function of θ

and sign will be decided by CAST Rule.

Values of sin θ, cos θ and tan θ for some standard angles.

Degree	0	30	37	45	53	60	90	120	135	180
Radians	0	$\pi/6$	$37\pi/180$	$\pi/4$	$53\pi/180$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$\sin\theta$	0	1/2	3/5	$1/\sqrt{2}$	4/5	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	0
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$1/\sqrt{2}$	3/5	1/2	0	$-1/2$	$-1/\sqrt{2}$	-1
$\tan\theta$	0	$1/\sqrt{3}$	3/4	1	4/3	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	0

GENERAL TRIGONOMETRIC FORMULAS :

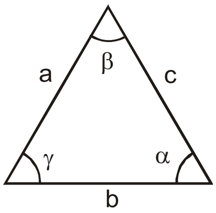
1.
$\cos^2\theta + \sin^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta.$
$1 + \cot^2\theta = \text{cosec}^2\theta.$

2.
$\cos(A + B) = \cos A \cos B - \sin A \sin B$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

3. $\sin 2\theta = 2 \sin\theta \cos\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

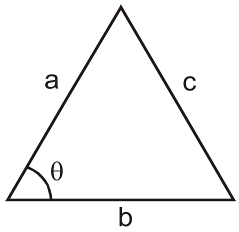
$\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

4. ***sine rule for triangles***



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

5. ***cosine rule for triangles***



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

RULES FOR DIFFERENTIATION :

$\frac{d}{dx}c = 0$; $\frac{d}{dx}x^n = nx^{n-1}$.

$\frac{d}{dx}(cu) = c\frac{du}{dx}$; $\frac{d}{dx}(u_1 + u_2 + ... + u_n) = \frac{du_1}{dx} + \frac{du_2}{dx} + ... + \frac{du_n}{dx}$.

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$; $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{d}{dx}(\sin x) = \cos x$; $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$; $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$; $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$; $\frac{d}{dx}(e^x) = e^x$; $\frac{dy}{dx} = f'[g(x)].g'(x)$

RULES FOR INTEGRATION :

$\int k f(x)dx = k\int f(x)dx$; $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

	<u>**Indefinite Integral**</u>	<u>**Reversed derivative formula**</u>
1.	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$, n rational	$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$
	$\int dx = \int 1dx = x + C$ (special case)	$\frac{d}{dx}(x) = 1$
2.	$\int \sin kx\,dx = -\frac{\cos kx}{k} + C$	$\frac{d}{dx}\left(-\frac{\cos kx}{k}\right) = \sin kx$
3.	$\int \cos kx\,dx = \frac{\sin kx}{k} + C$	$\frac{d}{dx}\left(\frac{\sin kx}{k}\right) = \cos kx$
4.	$\int \sec^2 x\,dx = \tan x + C$	$\frac{d}{dx}\tan x = \sec^2 x$

5. $\int \text{cosec}^2 x\, dx = -\cot x + C$ $\qquad \frac{d}{dx}(-\cot x) = \text{cosec}^2 x$

6. $\int \sec x \tan x\, dx = \sec x + C$ $\qquad \frac{d}{dx} \sec x = \sec x \tan x$

7. $\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + C$ $\qquad \frac{d}{dx}(-\text{cosec}\, x) = \text{cosec}\, x \cot x$

VECTOR :

DEFINITION OF VECTOR :

A physical quantity is called vector if in addition to magnitude -

(a) it has a specified direction

(b) it obeys the law of parallelogram of addition.

(c) It obeys commutative law of addition $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. If any of the above conditions is not satisfied the physical quantity cannot be a vector.

ADDITION OF VECTORS :

Addition of vectors is done by parallelogram law or its corollary, the triangle law :

(a) Parallelogram law of addition of vectors : If two vectors $\vec{A}$ and $\vec{B}$ are represented by two adjacent sides of a parallelogram both pointing outwards (and their tails coinciding) as shown. Then the diagonal drawn through the intersection of the two vectors represents the resultant

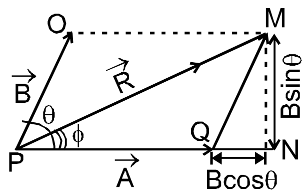
(i.e., vector sum of $\vec{A}$ and $\vec{B}$).

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

The direction of resultant vector $\vec{R}$ from $\vec{A}$ is given by

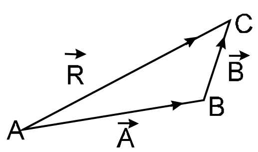
$$\tan\phi = \frac{MN}{PN} = \frac{MN}{PQ + QN} = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\phi = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

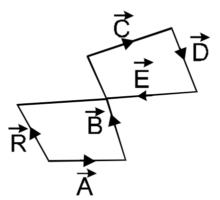


(b) Triangle law of addition of vectors : To add two vectors $\vec{A}$ and $\vec{B}$ shift any of the two vectors parallel to itself until the tail of $\vec{B}$ is at the head of $\vec{A}$. The sum $\vec{A} + \vec{B}$ is a vector $\vec{R}$ drawn from the tail of $\vec{A}$ to the head of $\vec{B}$, i.e., $\vec{A} + \vec{B} = \vec{R}$. As the figure formed is a triangle, this method is called ' triangle method' of addition of vectors.

If the 'triangle method' is extended to add any number of vectors in one operation as shown . Then the figure formed is a polygon and hence the name Polygon Law of addition of vectors is given to such type of addition.



$\vec{A} + \vec{B} = \vec{R}$



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{E}$

IMPORTANT POINTS :

- To a vector only a vector of same type can be added that represents the same physical quantity and the resultant is a vector of the same type.

- As R = $[A^2 + B^2 + 2AB \cos\theta]^{1/2}$ so R will be maximum when, $\cos\theta$ = max = 1, i.e., θ = 0º, i.e. vectors are like or parallel and R_{max} = A + B.

- The resultant will be minimum if, $\cos\theta$ = min = -1, i.e., θ = 180º , i.e. vectors are antiparallel and R_{min} = A – B.
- If the vectors A and B are orthogonal, i.e., θ = 90º, R = $\sqrt{A^2+B^2}$
- As previously mentioned that the resultant of two vectors can have any value from (A ~ B) to (A + B) depending on the angle between them and the magnitude of resultant decreases as θ increases 0º to 180º

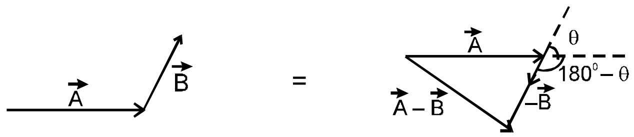
- Minimum number of unequal coplanar vectors whose sum can be zero is three.

- The resultant of three non-coplanar vectors can never be zero, or minimum number of non coplanar vectors whose sum can be zero is four.

- Subtraction of a vector from a vector is the addition of negative vector, i.e.,

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

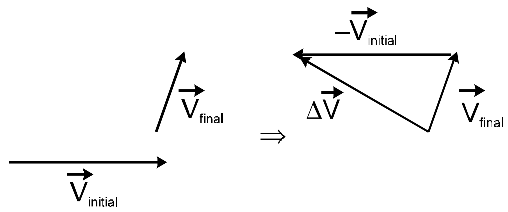
(a) From figure it is clear that $\vec{A}-\vec{B}$ is equal to addition of $\vec{A}$ with reverse of $\vec{B}$



$$|\vec{A}-\vec{B}| = [(A)^2 + (B^2) + 2AB\cos(180^\circ - \theta)]^{1/2}$$

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$$

(b) Change in a vector physical quantity means subtraction of initial vector from the final vector.



- $|\vec{A}|+|\vec{B}| \geq |\vec{A}+\vec{B}|$ (The triangle inequality)

MULTIPLICATION OF VECTORS :

THE SCALAR PRODUCT :

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}\cdot\vec{B}$ (read $\vec{A}$ dot $\vec{B}$) is defined as the product of their magnitude with cosine of angle

between them. Thus, $\vec{A}\cdot\vec{B}$ = AB cos θ {here θ is the angle between the vectors}

PROPERTIES :

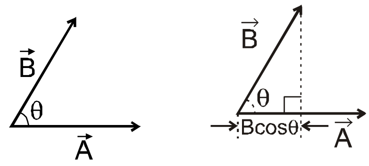
- It is always a scalar which is positive if angle between the vectors is acute (i.e. < 90º) and negative if angle between them is obtuse (i.e. 90º < $\theta \leq$ 180º)

- It is commutative, i.e., $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- It is distributive, i.e. $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

- As by definition $\vec{A}\cdot\vec{B}$ = AB cos θ. The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]$

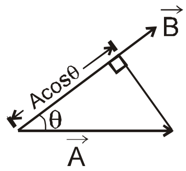
- $\vec{A}\cdot\vec{B}$ = A (B cos θ) = B (A cos θ)

  Geometrically, B cos θ is the projection of $\vec{B}$ onto $\vec{A}$ and A cos θ is the projection of $\vec{A}$ onto $\vec{B}$ as shown. So $\vec{A}\cdot\vec{B}$ is the product of the magnitude of $\vec{A}$ and the component of $\vec{B}$ along $\vec{A}$ and vice versa.



  Component of $\vec{B}$ along $\vec{A}$ = B cosθ = $\frac{\vec{A}\cdot\vec{B}}{A}$ = $\hat{A}\cdot\vec{B}$

  Component of $\vec{A}$ along $\vec{B}$ = A cosθ = $\frac{\vec{A}\cdot\vec{B}}{B}$ = $\vec{A}\cdot\hat{B}$



- Scalar product of two vectors will be maximum when cos θ = max = 1, i.e., θ = 0°, i.e., vectors are parallel ⇒ $(\vec{A}\cdot\vec{B})_{max}$ = AB
- If the scalar product of two nonzero vectors vanishes then the vectors are perpendicular.
- The scalar product of a vector by itself is termed as self dot product and is given by

  $(\vec{A})^2 = \vec{A}\cdot\vec{A}$ = AA cosθ = AAcos0° = A^2 ⇒ $A = \sqrt{\vec{A}\cdot\vec{A}}$
- In case of unit vector $\hat{n}$, $\hat{n}\cdot\hat{n}$ = 1 x 1 x cos 0° = 1 ⇒ $\hat{n}\cdot\hat{n} = \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
- In case of orthogonal unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$; $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- $\vec{A}\cdot\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)\cdot(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z) = [A_xB_x + A_yB_y + A_zB_z]$

VECTOR PRODUCT :

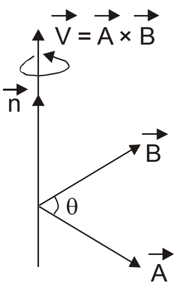
The vector product or cross product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}\times\vec{B}$ (read $\vec{A}$ cross $\vec{B}$) is defined as :

$$\vec{A}\times\vec{B} = AB \sin\theta\ \hat{n}$$

Here θ is the angle between the vectors and the direction $\hat{n}$ is given by the right-hand-thumb rule.

Right-Hand-Thumb Rule:

To find the direction of $\hat{n}$, draw the two vectors $\vec{A}$ and $\vec{B}$ with both the tails coinciding . Now place your stretched right palm perpendicular to the plane of $\vec{A}$ and $\vec{B}$ in such a way that the fingers are along the vector $\vec{A}$ and when the fingers are closed they go towards $\vec{B}$. The direction of the thumb gives the direction of $\hat{n}$.



PROPERTIES :

- Vector product of two vectors is always a vector perpendicular to the plane containing the two vectors i.e. orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

- Vector product of two vectors is not commutative i.e. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$.

  But $\left|\vec{A}\times\vec{B}\right| = \left|\vec{B}\times\vec{A}\right|$ = AB sin θ

- The vector product is distributive when the order of the vectors is strictly maintained i.e.

$$A\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}.$$

- The magnitude of vector product of two vectors will be maximum when sinθ = max = 1, i.e,, θ = 90º

$$|\vec{A}\times\vec{B}|_{max} = AB$$

  i.e., magnitude of vector product is maximum if the vectors are orthogonal.

- The magnitude of vector product of two non–zero vectors will be minimum when |sinθ| = minimum = 0,i.e., θ = 0º or 180º and $|\vec{A}\times\vec{B}|_{min} = 0$ i.e., if the vector product of two nonzero vectors vanishes, the vectors are collinear.

Note : When θ = 0º then vectors may be called as like vector or parallel vectors and when θ = 180º then vectors may be called as unlike vectors or antiparallel vectors.

- The self cross product i.e. product of a vector by itself vanishes i.e. is a null vector.

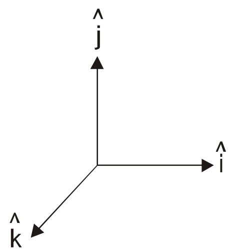
Note : Null vector or zero vector : A vector of zero magnitude is called zero vector. The direction of a zero vector is in determinate (unspecified).

$$\vec{A}\times\vec{A} = AA \sin 0^\circ\ \hat{n} = \vec{0}.$$

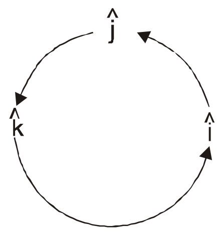
- In case of unit vector $\hat{n}$, $\hat{n}\times\hat{n} = \vec{0} \Rightarrow \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$

- In case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ in accordance with right-hand-thumb-rule,

  $\hat{i}\times\hat{j} = \hat{k}$ $\quad$ $\hat{j}\times\hat{k} = \hat{i}$ $\quad$ $\hat{k}\times\hat{i} = \hat{j}$



(A)



(B)

- In terms of components, $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$

$$\vec{A}\times\vec{B} = \hat{i}\,(A_y B_z - A_z B_y) + \hat{j}\,(A_z B_x - A_x B_z) + \hat{k}\,(A_x B_y - A_y B_x)$$

EXERCISE # 1

* ***Marked Questions are having more than one correct option.***

FUNCTION & DIFFERENTIATION

Section - (A) : Function

1. If $f(x) = 4x + 3$

Find $f(f(2))$

2. $f(x) = \log x^3$ and $g(x) = \log x$

Which of the following statement is / are true -

(A) $f(x) = gx$ (B) $3f(x) = g(x)$ (C) $f(x) = 3g(x)$ (D) $f(x) = (g(x))^3$

3. $f(x) = \cos x + \sin x$

Find $f(\pi/2)$

Section - (B) : Differentiation of Elementry Functions

Diffrentiate w.r.t. corresponding independent variable.

1. $s = 5t^3 - 3t^5$ **2.** $y = \tan x + \cot x$ **3.** $y = 5 \sin x$

4. $y = x^2 + x + 8$ **5.** $y = x^2 + \sin x$

Find the first derivative & second derivative of given functions w.r.t. corresponding independent variable.

6. $y = \ell nx + e^x$ **7.** $y = 6x^2 - 10x - 5x^{-2}$ **8.** $\omega = 3z^7 - 7z^3 + 21z^2$

9. $y = \sin x + \cos x$ **10.** $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$

Section - (C) : Differentiation by Product rule

Diffrentiate w.r.t. x

1. $y = (x^2 + 1)(x + 5 + \frac{1}{x})$ **2.** $y = \sin x \cos x$

3. $x \sin x$ **4.** $y = (x-1)(x^2 + x + 1)$

Section - (D) : Differentiation by Quotient rule

Find derivative of given functions w.r.t. the independent variable.

1. Suppose u and v are functions of x that are differentiable at x = 0 and that

$u(0) = 5$, $u'(0) = -3$, $v(0) = -1$ $v'(0) = 2$

Find the values of the following derivatives at x = 0.

(a) $\frac{d}{dx}(uv)$ (b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ (c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ (d) $\frac{d}{dx}(7v - 2u)$

2. $f(t) = \frac{t^2 - 1}{t^2 + t - 2}$, find f ' (t) ; Where : $t \neq 1$

3. $z = \frac{2x+1}{x^2-1}$, Find $\frac{dz}{dx}$ at x = 2,

4. $y = \frac{\sin x}{\cos x}$

5. $y = \frac{2x+5}{3x-2}$

Section - (E) : Differentiation by Chain rule

Find $\frac{dy}{dx}$ as a function of x

1. $y = (4 - 3x)^9$

2. $y = \left(1 - \frac{x}{7}\right)^{-7}$

3. $y = \left(\frac{x}{2} - 1\right)^{-10}$

4. $y = \sqrt{\sin \sqrt{x}}$

5. $y = \sin(x) + \ln(x^2) + e^{2x}$

6. $y = \sin 5x$

7. $y = 2 \sin(\omega x + \phi)$ where ω and ϕ constants

8. $y = (2x + 1)^5$

Section - (F) : Differentiation of Implicit functions

Find $\frac{dy}{dx}$

1. $x^2y + xy^2 = 6$

2. $(x + y)^2 = 4$

Section - (G) : Differentiation as a rate measurement

1. Suppose that the radius r and surface area $S = 4\pi r^2$ of a sphere are differentiable functions of t. Write an equation that relates $\frac{ds}{dt}$ to $\frac{dr}{dt}$.

2. Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t.Write an equation that relates dA / dt to dr / dt.

Section - (H) : Maxima & Minima

1. Find the values of function $2x^3 - 15x^2 + 36x + 11$ at the points of maximum and minimum

2. Particle's position as a function of time is given by $x = -t^2 + 4t + 4$ find the maximum value of position co-ordinate of particle.

Section - (I): Miscellaneous

Given y = f(u) and u = g(x) , find $\frac{dy}{dx}$

1. $y = \cos u$, $u = -\frac{x}{3}$

2. $y = \sin u$, $u = 3x + 1$

3. $y = 6u - 9$, $u = (1/2)x^4$

4. $y = 2u^3$, $u = 8x - 1$

INTEGRATION

Section - (A) : Integration of elementry functions

Find integrals of given functions

1. $2x$ **2.** x^2 **3.** $-3x^{-4}$ **4.** x^{-4} **5.** $\frac{1}{x^2}$

6. $\frac{5}{x^2}$ **7.** $\frac{3}{2}\sqrt{x}$ **8.** $\frac{3}{2\sqrt{x}}$ **9.** $\frac{4}{3}\sqrt[3]{x}$ **10.** $\frac{1}{3\sqrt[3]{x}}$

11. $\frac{1}{2}x^{-1/2}$ **12.** $-\frac{1}{2}x^{-3/2}$ **13.** $3\sin x$ **14.** $\sec^2 x$ **15.** $\csc^2 x$

16. $\sec x \tan x$ **17.** $\frac{1}{3x}$ **18.** $x^2 - 2x + 1$ **19.** $x^{-4} + 2x + 3$ **20.** $2 - \frac{5}{x^2}$

21. $\sqrt{x} + \frac{1}{\sqrt{x}}$ **22.** $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ **23.** $-\frac{3}{2}x^{-5/2}$ **24.** $(1 - x^2 - 3x^5)$

Section - (B) : Integration by substitution method

1. $\int \sec 2t \tan 2t \, dt$, (use , $u = 2t$)

2. $\int \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt$, (use , $u = 1-\cos\frac{t}{2}$)

3. $\int x^3 (x^4-1)^2 \, dx$, (use , $u = x^4 - 1$)

4. $\int \frac{9r^2}{\sqrt{1-r^3}}\, dr$, (use , $u = 1 - r^3$)

5. $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right) dx$, (use , $u = -\frac{1}{x}$)

6. $\int \sin 3x \, dx$, (use , $u = 3x$)

7. $\int x \sin(2x^2)\, dx$, (use , $u = 2x^2$)

Integrate by using a suitable substitution

8. $\int \frac{3}{(2-x)^2}\, dx$ **9.** $\int \frac{4y}{\sqrt{2y^2+1}}\, dy$ **10.** $\int \cos(3z+4)\, dz$

11. $\int \sin(8z-5)\, dz$ **12.** $\int \frac{1}{\sqrt{t}}\cos(\sqrt{t}+3)\, dt$ **13.** $\int (2x+1)^3\, dx$

Section - (C) : Definite integration

1. $\int_{-2}^{4} \left(\frac{x}{2}+3\right) dx$ **2.** $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$ **3.** $\int_{0}^{2\pi} \sin\theta\, d\theta$ **4.** $\int_{0}^{1} e^{x}\, dx$

Section - (D) : Calculation of area

Use a definite integral to find the area of the region between the given curve and the x–axis on the interval [0,b]

1. $y = 2x$ **2.** $y = \frac{x}{2} + 1$

Use a definite integral to find the area of the region between the given curve and the x–axis on the interval $[0, \pi]$

3. $y = \sin x$ **4.** $y = \sin^2 x$ **5.** $\int_{-2}^{1} 5\, dx$ **6.** $\int_{-4}^{-1} \frac{\pi}{2}\, d\theta$

VECTOR

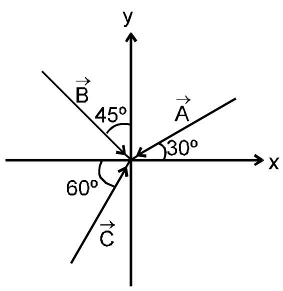
Section - (A) : Definition of vector & angle between vectors

1. Rain is falling vertically downwards with a speed 5 m/s. If unit vector along upward is defined as $\hat{j}$, represent velocity of rain in vector form.

2. The vector joining the points A (1, 1, –1) and B (2, –3, 4) & pointing from A to B is -

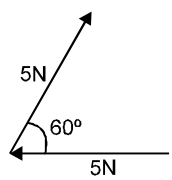
(A) $-\hat{i} + 4\hat{j} - 5\hat{k}$ (B) $\hat{i} + 4\hat{j} + 5\hat{k}$ (C) $\hat{i} - 4\hat{j} + 5\hat{k}$ (D) $-\hat{i} - 4\hat{j} - 5\hat{k}$.

3. Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are shown in figure. Find angle between



(i) $\vec{A}$ and $\vec{B}$, (ii) $\vec{A}$ and $\vec{C}$, (iii) $\vec{B}$ and $\vec{C}$.

4. The forces, each numerically equal to 5 N, are acting as shown in the Figure. Find the angle between forces?



Section - (B) : Addition of Vectors

1. Two force of $\vec{F}_1$ = 500 N due east and $\vec{F}_2$ = 250 N due north , Find $\vec{F}_2 - \vec{F}_1$?

2. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same

(A) magnitude (B) direction
(C) magnitude as well as direction (D) neither magnitude nor direction.

3. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors

(A) can be zero (B) cannot be zero
(C) lies in the plane of $\vec{A}$ & $\vec{B}$ (D) lies in the plane of $\vec{A}$ & $\vec{A}$ + $\vec{B}$

4. The vector sum of the forces of 10 N and 6 N can be

(A) 2 N (B) 8 N (C) 18 N (D) 20 N.

5. A set of vectors taken in a given order gives a closed polygon. Then the resultant of these vectors is a

(A) scalar quantity (B) pseudo vector (C) unit vector (D) null vector.

6. The vector sum of two force P and Q is minimum when the angle θ between their positive directions, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π.

7. The vector sum of two vectors $\vec{A}$ and $\vec{B}$ is maximum, then the angle θ between two vectors is -

(A) 0° (B) 30° (C) 45° (D) 60°

8. Given : $\vec{C}$ = $\vec{A}$ + $\vec{B}$. Also, the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively. The angle between $\vec{A}$ and $\vec{B}$ is

(A) 0° (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π.

9. If $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$ and θ is the angle between $\vec{P}$ and $\vec{Q}$, then

(A) θ = 0° (B) θ = 90° (C) P = 0 (D) Q = 0

10. The sum and difference of two perpendicular vectors of equal lengths are

(A) of equal lengths and have an acute angle between them
(B) of equal length and have an obtuse angle between them
(C) also perpendicular to each other and are of different lengths
(D) also perpendicular to each other and are of equal lengths.

11. A man walks 40 m North, then 30 m East and then 40 m South. Find the displacement from the starting point?

12. Two force $\vec{F}_1$ and $\vec{F}_2$ are acting at right angles to each other, find their resultant ?

13. A vector of magnitude 30 and direction eastwards is added with another vector of magnitude 40 and direction Northwards. Find the magnitude and direction of resultant with the east.

Section - (C) : Resolution of Vectors

1. One of the rectangular components of a velocity of 60 km h^{-1} is 30 km h^{-1}. Find other rectangular component?

2. If $0.5\hat{i} + 0.8\hat{j} + C\hat{k}$ is a unit vector. Find the value of C

3. The rectangular components of a vector are (2, 2). The corresponding rectangular components of another vector are (1, $\sqrt{3}$). Find the angle between the two vectors

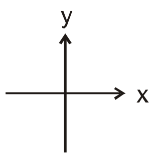
4. The x and y components of a force are 2 N and – 3 N. The force is

(A) $2\hat{i} - 3\hat{j}$ (B) $2\hat{i} + 3\hat{j}$ (C) $-2\hat{i} - 3\hat{j}$ (D) $3\hat{i} + 2\hat{j}$

5. Find the magnitude of $3\hat{i} + 2\hat{j} + \hat{k}$?

6. If $\vec{A} = 3\hat{i} + 4\hat{j}$ then find $\hat{A}$

7. What are the x and the y components of a 25 m displacement at an angle of 210º with the x-axis (anti clockwise)?



Section - (D) : Products of Vectors

1. Three non zero vectors $\vec{A}, \vec{B}$ & $\vec{C}$ satisfy the relation $\vec{A}.\vec{B} = 0$ & $\vec{A}.\vec{C} = 0$. Then $\vec{A}$ can be parallel to :

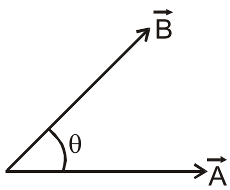
(A) $\vec{B}$ (B) $\vec{C}$ (C) $\vec{B}.\vec{C}$ (D) $\vec{B} \times \vec{C}$

2.* The magnitude of scalar product of two vectors is 8 and that of vector product is $8\sqrt{3}$. The angle between them is :

(A) 30º (B) 60º (C) 120º (D) 150º

3. If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j}$ find (a) $\vec{A}.\vec{B}$ (b) $\vec{A} \times \vec{B}$

4. If $|\vec{A}| = 4$, $|\vec{B}| = 3$ and $\theta = 60°$ in the figure , Find (a) $\vec{A}.\vec{B}$ (b) $|\vec{A} \times \vec{B}|$



EXERCISE # 2

PART - I : OBJECTIVE QUESTIONS

Single Correct Answer Type

1. If the angle between two forces increases, the magnitude of their resultant
(A) decreases
(B) increases
(C) remains unchanged
(D) first decreases and then increases

2. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1} when it turns left through 90º. If the speed remains unchanged after turning, the change in the velocity of the car in the turning process is
(A) zero
(B) $50\sqrt{2}$ km h^{-1} S-W direction $50\sqrt{2}$ km h^{-1} S-W
(C) $50\sqrt{2}$ km h^{-1} N-W direction $50\sqrt{2}$ km h^{-1} N-W
(D) 50 km h^{-1} due west. 50 km h^{-1}

3. Which of the following sets of displacements might be capable of bringing a car to its returning point?
(A) 5, 10, 30 and 50 km
(B) 5, 9, 9 and 16 km
(C) 40, 40, 90 and 200 km
(D) 10, 20, 40 and 90 km

4. When two vector $\vec{a}$ and $\vec{b}$ are added, the magnitude of the resultant vector is always
(A) greater than (a + b)
(B) less than or equal to (a + b)
(C) less than (a + b)
(D) equal to (a + b)

5. Given : $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 5\hat{i} - 6\hat{j}$. The magnitude of $\vec{A}+\vec{B}$ is
(A) 4 units (B) 10 units (C) $\sqrt{58}$ units (D) $\sqrt{61}$ units.

6. Given : $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{B} = -\hat{i} - \hat{j} + \hat{k}$. The unit vector of $\vec{A} - \vec{B}$ is
(A) $\frac{3\hat{i}+\hat{k}}{\sqrt{10}}$ (B) $\frac{3\hat{i}}{\sqrt{10}}$ (C) $\frac{\hat{k}}{\sqrt{10}}$ (D) $\frac{-3\hat{i}-\hat{k}}{\sqrt{10}}$

7. Vector $\vec{A}$ is of length 2 cm and is 60º above the x-axis in the first quadrant. Vector $\vec{B}$ is of length 2 cm and 60º below the x-axis in the fourth quadrant. The sum $\vec{A} + \vec{B}$ is a vector of magnitude -
(A) 2 along + y-axis (B) 2 along + x-axis (C) 1 along – x axis (D) 2 along – x axis

8. Six forces, 9.81 N each, acting at a point are coplanar. If the angles between neighboring forces are equal, then the resultant is
(A) 0 N (B) 9.81 N (C) 2 × 9.81 N (D) 3 × 9.81 N.

9. A vector $\vec{A}$ points vertically downward & $\vec{B}$ points towards east, then the vector product $\vec{A} \times \vec{B}$ is
(A) along west (B) along east (C) zero (D) along south

10. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is
(A) 0º (B) 60º (C) 90º (D) 120º.

11. Given : $\vec{a}+\vec{b}+\vec{c} = 0$. Out of the three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ two are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. The angles between the vectors are:
(A) 90º, 135º,. 135º (B) 30º, 60º, 90º (C) 45º, 45º, 90º (D) 45º, 60º, 90º

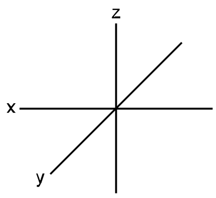
12. A hall has the dimensions 10 m × 12 m × 14 m. A fly starting at one corner ends up at a diametrically opposite corner. The magnitude of its displacement is nearly
(A) 16 m (B) 17 m (C) 18 m (D) 21 m.

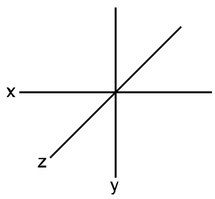
13. A vector is not changed if
(A) it is displaced parallel to itself
(B) it is rotated through an arbitrary angle
(C) it is cross-multiplied by a unit vector
(D) it is multiplied by an arbitrary scalar.

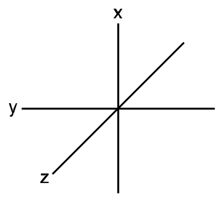
More than one choice type

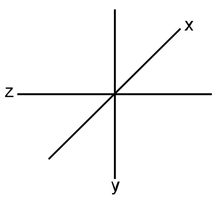
14. Which of the following is a true statement?
(A) A vector cannot be divided by another vector
(B) Angular displacement can either be a scalar or a vector.
(C) Since addition of vectors is commutative therefore vector subtraction is also commutative.
(D) The resultant of two equal forces of magnitude F acting at a point is F if the angle between the two forces is 120º.

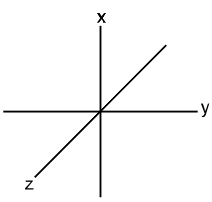
15. Which of the arrangement of axes in Fig. can be labelled "right-handed coordinate system"? As usual, each axis label indicates the positive side of the axis.

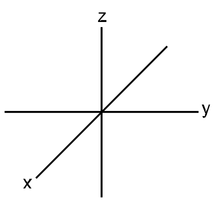
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) 

(A) (i), (ii) (B) (iii), (iv) (C) (vi) (D) (v)

PART - II : SUBJECTIVE QUESTIONS

FUNCTION & DIFFERENTIATION

1. $y = f(x) = \dfrac{2x-3}{3x-2}$

Find f(y)

2. If $f(x) = \begin{cases} x+2 & , \quad x<2 \\ 2x-1 & , \quad x \geq 2 \end{cases}$

Evaluate f(2), f(1) and f(3)

3. If $f(x) = \dfrac{x-1}{x+1}$ then find f {f(x)}

Find the first derivative and second derivative of given functions w.r.t. the independent variable x.

4. $y = \sqrt[7]{x} + \tan x$

5. $y = \ell n x^2 + \sin x$

Find derivative of given functions w.r.t. the corresponding independent variable.

6. $y = \left(x+\dfrac{1}{x}\right)\left(x-\dfrac{1}{x}+1\right)$

7. $y = x^2 \sin x + 2x \cos x - 2 \sin x$

8. $y = x^2 \cos x - 2x \sin x - 2\cos x$

9. $r = (1 + \sec \theta) \sin \theta$

10. $y = e^x \tan x$

11. $y = x^2 \sin^4 x + x \cos^{-2} x$

Find derivative of given functions w.r.t. the respective independent variable .

12. $p = \dfrac{\tan q}{1+\tan q}$

13. $y = \dfrac{\sin x+\cos x}{\cos x}$

14. $y = \dfrac{\cot x}{1+\cot x}$

Find $\dfrac{dy}{dx}$ as a function of x

15. $y = \left(\dfrac{x^2}{8}+x-\dfrac{1}{x}\right)^4$

16. $\sin^2 (x^2 + 1)$

17. $y = x (x^2+1)^{-1/2}$

18. $q = \sqrt{2r-r^2}$, find $\dfrac{dq}{dr}$

19. $y = \sin^3 x + \sin 3x$

20. $y = \sin \sin \sin x$

21. $y = e^{\sin x}$

Find $\frac{dy}{dx}$

22. $x^3 + y^3 = 18\,xy$

23. The radius r and height h of a circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$.
(a) If height is increasing at a rate of 5 m/s while radius is constant, Find rate of increase of volume of cylinder.
(b) If radius is increasing at a rate of 5 m/s while height is constant, Find rate of increase of volume of cylinder.
(c) If height is increasing at a rate of 5 m/s and radius is increasing at a rate of 5 m/s, Find rate of increase of volume of cylinder.

24. A sheet of area 40 m^2 in used to make an open tank with a square base, then find the dimensions of the base such that volume of this tank is maximum.

25. Find two positive numbers x & y such that x + y = 60 and xy is maximum -

26. y = cosu , u = sinx

27. y = sinu , u = x – cosx

28. Find y″ if (a) y = cosec x, (b) y = sec x.

INTEGRATION

Find integrals of given functions.

1. $\int (\sqrt{x} + \sqrt[3]{x})\, dx$

2. $\int x^{-3}(x+1)\, dx$

3. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\, dt$

4. $\int \frac{4 + \sqrt{t}}{t^3}\, dt$

5. $\int \cot^2 x\, dx$

6. $\int (1 - \cot^2 x)\, dx$

7. $\int \cos\theta(\tan\theta + \sec\theta)\, d\theta$

8. $\int (2x^3 - 5x + 7)\, dx$

9. $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right) dx$

Integrate by using the substitution suggested in bracket

10. $\int \sqrt{x}\,\sin^2(x^{3/2} - 1)\, dx$, (use , $u = x^{3/2} - 1$)

11. $\int \csc^2 2\theta \cot 2\theta\, d\theta$

(a) Using u = cot 2θ (b) Using u = csc 2θ

12. $\int \frac{dx}{\sqrt{5x+8}}$

(a) Using u = 5x + 8 (b) Using u = $\sqrt{5x+8}$

Integrate by using suitable substitution.

13. $\int \sqrt{3 - 2s}\, ds$

14. $\int \theta\, \sqrt[4]{(1 - \theta^2)}\, d\theta$

15. $\int 8\theta\, \sqrt[3]{\theta^2 - 1}\, d\theta$

16. $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2}\, dx$

17. $\int \frac{(1 + \sqrt{x})^3}{\sqrt{x}}\, dx$

18. $\int \sec^2(3x + 2)\, dx$

19. $\int \tan^2 x \sec^2 x \, dx$

20. $\int \sec\left(\upsilon + \frac{\pi}{2}\right)\tan\left(\upsilon + \frac{\pi}{2}\right) d\upsilon$

21. $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt$

22. $\int \frac{6\cos t}{(2+\sin t)^3} dt$

23. $\int 28(7x-2)^{-5} dx$, (use , $u = 7x - 2$)

24. $\int 12(y^4+4y^2+1)^2(y^3+2y) \, dy$, (use , $u = y^4 + 4y^2 + 1$)

25. $\int \csc\left(\frac{\upsilon-\pi}{2}\right)\cot\left(\frac{\upsilon-\pi}{2}\right) d\upsilon$

26. $\int_0^1 \frac{dx}{3x+2}$

27. $\int_0^{\sqrt[3]{7}} X^2 \, dx$

28. $\int_0^{\pi} \cos x \, dx$

29. $\int_0^{\sqrt{\pi}} x \sin x^2 \, dx$

30. $\int_{\pi}^{2\pi} \theta \, d\theta$

Use a definite integral to find the area of the region between the given curve and the x–axis on the interval [0,b],

31. $y = \sqrt{b^2 - x^2}$

32. $y = 3x^2$

VECTOR

33. The resultant of two vectors of magnitudes 2A and $\sqrt{2}$ A acting at an angle θ is $\sqrt{10}$ A. Find the value of θ ?

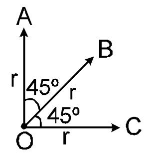
34. Vector $\vec{A}$ points N – E and its magnitude is 3 kg ms^{-1} it is multiplied by the scalar λ such that $\lambda = -4$ second. Find the direction and magnitude of the new vector quantity. Does it represent the same physical quantity or not ?

35. A force of 30 N is inclined at an angle θ to the horizontal . If its vertical component is 18 N, find the horizontal component & the value of θ .

36. Two vectors acting in the opposite directions have a resultant of 10 units . If they act at right angles to each other, then the resultant is 50 units . Calculate the magnitude of two vectors .

37. A vector $\vec{B}$ which has magnitude 8.0 is added to a vector $\vec{A}$ which lies along the x–axis. The sum of these two vectors is a 3rd vector which lies along the y–axis & has magnitude that is twice the magnitude of $\vec{A}$. Find the magnitude of $\vec{A}$.

38. Find the resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ each of magnitude r as shown in figure?



39. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j} + 2\hat{k}$ then find out unit vector along $\vec{A} + \vec{B}$

40. The x and y components of vector $\vec{A}$ are 4m and 6m respectively. The x,y components of vector $\vec{A} + \vec{B}$ are 10m and 9m respectively. Find the length of $\vec{B}$ and angle that $\vec{B}$ makes with the x axis. .

41. If $\vec{a} = x_1\hat{i} + y_1\hat{j}$ & $\vec{b} = x_2\hat{i} + y_2\hat{j}$. Find the condition that would make $\vec{a}$ & $\vec{b}$ parallel to each other.

42. The angle θ between directions of forces $\vec{A}$ and $\vec{B}$ is 90° where A = 8 dyne and B = 6 dyne. If the resultant $\vec{R}$ makes an angle α with $\vec{A}$ then find the value of 'α' ?

ADVANCE LEVEL PROBLEMS

DIFFERENTIATION

Find first derivative and second derivative of dependent variable.

1. $y = 4 - 2x - x^{-3}$

2. $y = -x^2 + 3$

3. $y = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{4}$

Find the derivative of functions using quotient rule.

4. Suppose u and v are differentiable functions of x and that
u (1) = 2, u'(1) = 0 v (1) = 5 v' (1) = – 1.
Find the values of the following derivatives at x = 1.

(a) $\frac{d}{dx}(uv)$ (b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ (c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ (d) $\frac{d}{dx}(7v - 2u)$.]

5. $g(x) = \frac{x^2 - 4}{x + 0.5}$

6. $y = \frac{3}{x} + 5 \sin x$

7. $y = \text{cosec } x - 4\sqrt{x} + 7$

8. $y = -10x + 3 \cos x$

Find $\frac{ds}{dt}$

9. $s = \frac{1 + \text{cosec } t}{1 - \text{cosec } t}$

10. $s = \frac{\sin t}{1 - \cos t}$

11. $s = \tan t - t$

12. $s = t^2 - \sec t + t$

Find $\frac{dp}{dq}$

13. $p = (1 + \text{cosec } q) \cos q$

14. $p = 5 + \frac{1}{\cot q}$

Find $\frac{dy}{dx}$ as a function of x.

15. $y = 5 \cos^{-4} x$.

16. $y = \sin^3 x$

Find the derivatives of the functions

17. $s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right)$

18. $r = (\text{cosec}\theta + \cot\theta)^{-1}$

19. $r = -(\sec\theta + \tan\theta)^{-1}$

20. $s = \frac{4}{3\pi} \sin 3t + \frac{4}{5\pi} \cos 5t$

INTEGRATION

Find an antiderivative for each function. Do as many as you can mentally.Check your answer by differentiation.

1. (a) $-\frac{2}{x^4}$ (b) $\frac{1}{2x^4}$ (c) $x^4 - \frac{1}{x^4}$

2. (a) $\frac{2}{3}x^{-1/3}$ (b) $\frac{1}{3}x^{-2/3}$ (c) $-\frac{1}{3}x^{-4/3}$

3. (a) $\pi \cos \pi x$ (b) $\frac{\pi}{2}\cos\frac{\pi x}{2}$ (c) $\cos\frac{\pi x}{2} + \pi \cos x$

4. (a) $\csc x \cot x$ (b) $-\csc 5x \cot 5x$ (c) $-\pi \csc\frac{\pi x}{2}\cot\frac{\pi x}{2}$

5. $(1 + 2\cos x)^2$

6. (a) $6x$ (b) x^7 (c) $x^7 - 6x + 8$

7. (a) $2x^{-4}$ (b) $\frac{x^{-4}}{2} + x^2$ (c) $-x^{-4} + x - 1$

Evaluating Integrals
Check your answers by differentiation.

8. $\int (5 - 6x)\, dx$

9. $\int \left(3t^2 + \frac{t}{2}\right) dt$

10. $\int \left(\frac{t^2}{2} + 4t^3\right) dt$

11. $\int x^{-1/3}\, dx$

12. $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$

13. $\int \left(8y - \frac{2}{y^{1/4}}\right) dy$

14. $\int 2x(1 - x^{-3})\, dx$

15. $\int (-2\cos t)\, dt$

16. $\int (-5\sin t)\, dt$

17. $\int 7\sin\frac{\theta}{3}\, d\theta$

18. $\int 3\cos 5\theta\, d\theta$

19. $\int (-3\csc^2 x)\, dx$

20. $\int \left(-\frac{\sec^2 x}{3}\right) dx$

21. $\int \frac{\csc\theta \cot\theta}{2}\, d\theta$

22. $\int \frac{2}{5}\sec\theta \tan\theta\, d\theta$

23. $\int (4\sec x \tan x - 2\sec^2 x)\, dx$

24. $\int \frac{1}{2}(\csc^2 x - \csc x \cot x)\, dx$

25. $\int (\sin 2x - \csc^2 x)\, dx$

26. $\int (2\cos 2x - 3\sin 3x)\, dx$

27. $\int 4\sin^2 y\, dy$

28. $\int \frac{\cos^2 y}{7}\, dy$

29. $\int \frac{1 + \cos 4t}{2}\, dt$

30. $\int \frac{1 - \cos 6t}{2}\, dt$

31. $\int (1 + \tan^2\theta)\, d\theta$

32. $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\, d\theta$

33. $\int (x + 1)\, dx$

Checking Integration Formulas

We will see where formulas like these come from.

34. $\int (3x-5)^{-2}\, dx = -\frac{(3x-5)^{-1}}{3} + C$

35. $\int \sec^2(5x-1)\, dx = \frac{1}{5}\tan(5x-1) + C$

36. $\int \csc^2\left(\frac{x-1}{3}\right) dx = -3\cot\left(\frac{x-1}{3}\right) + C$

37. $\int \frac{1}{(x+1)^2}\, dx = -\frac{1}{x+1} + C$

38. Right, or wrong ? Say which for each formula and give a brief reason for each answer.

(a) $\int (2x+1)\, dx = \sqrt{x^2+x+C}$ (b) $\int (2x+1)\, dx = \sqrt{x^2+x} + C$ (c) $\int (2x+1)\, dx = \frac{1}{3}(\sqrt{2x+1})^3 + C$

39. $\int (7x-2)^3\, dx = \frac{(7x-2)^4}{28} + C$

Evaluate Integrals by substitution method.

40. $\int 3y\sqrt{7-3y^2}\, dy$

41. $\int \sin^5 \frac{x}{3}\cos\frac{x}{3}\, dx.$

42. $\int \tan^7 \frac{x}{2}\sec^2\frac{x}{2}\, dx$

43. $\int r^2\left(\frac{r^3}{18}-1\right)^5 dr$

44. $\int r^4\left(7-\frac{r^5}{10}\right)^3 dr$

45. $\int x^{1/3}\sin(x^{4/3}-8)\, dx.$

46. $\int \csc\left(\frac{\upsilon-\pi}{2}\right)\cot\left(\frac{\upsilon-\pi}{2}\right) d\upsilon$

47. $\int \sqrt{\cot y}\,\csc^2 y\, dy$

48. $\int \frac{\sec z\tan z}{\sqrt{\sec z}}\, dz$

49. $\int \frac{1}{t^2}\cos\left(\frac{1}{t}-1\right) dt$

50. $\int \frac{1}{\sqrt{5s+4}}\, ds$

Find the definite integrals of following Functions

51. $\int_{-2}^{1} |x|\, dx$

52. $\int_{1/2}^{3/2} (-2x+4)\, dx$

Evaluate definite integrals of following Functions

53. $\int_{0}^{3b} x^2\, dx$

54. $\int_{0}^{\pi/2} \theta^2\, d\theta$

VECTOR

1. A sail boat sails 2km due East, 5km 37^0 South of East and finally has an unknown displacement. If the final displacement of the boat from the starting point is 6km due East, the third displacement is ________.

2. The resultant of two vectors u and v is perpendicular to the vector u and its magnitude is equal to half of the magnitude of vector v. Find the angle between u and v.

3. Let the resultant of three forces of magnitude 5N, 12N & 13N acting on a body be zero. If sin 23^0 = (5/13), find the angle between the 5N force & 13N force .

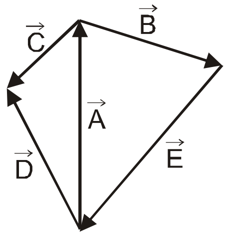
4. Two vectors $\vec{A}$ & $\vec{B}$ have the same magnitude . Under what circumstances does the vector $\vec{A} + \vec{B}$ have the same magnitude as $|\vec{A}|$ or $|\vec{B}|$. When does the vector difference $\vec{A} - \vec{B}$ have this magnitude .

5. The resultant of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled, resultant is doubled in magnitude, when $\vec{Q}$ is reversed, $\vec{R}$ is again doubled , find P : Q : R.

OBJECTIVES

Single choice type

15. In Figure, $\vec{E} + \vec{D} - \vec{C}$ equals



(A) $\vec{A}$ (B) $-\vec{A}$ (C) $\vec{B}$ (D) $-\vec{B}$

16. Two forces P and Q acting at a point are such that if P is reversed, the direction of the resultant is turned through 90º. Then
(A) P = Q (B) P = 2Q
(C) P = $\frac{Q}{2}$ (D) No relation between P and Q.

17. The sum of the magnitudes of two forces acting at a point is 16 N. The resultant of these force is perpendicular to the smaller force and has a magnitude of 8 N. If the smaller force is of magnitude x, then the value of x is
(A) 2 N (B) 4 N (C) 6 N (D) 7 N

18. Forces proportional to AB, BC and 2CA act along the sides of triangle ABC in order. Their resultant represented in magnitude and direction as
(A) CA (B) AC (C) BC (D) CB

19. A given force is resolved into components P & Q equally inclined to it . Then :
(A) P = 2 Q (B) 2 P = Q (C) P = Q (D) none of these

20. The resultant of two forces 3 P & 2 P is R, if first force is doubled, the resultant is also doubled . Then the angle between the forces is :
(A) 30º (B) 60º (C) 120º (D) 150º

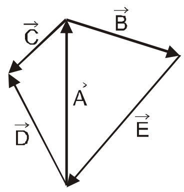
21. The resultant of two forces acting at an angle of 150º is 10 kg wt and is perpendicular to one of the forces . The other force is :
(A) $10\sqrt{3}$ kg wt (B) $20\sqrt{3}$ kg wt (C) 20 kg wt (D) $\frac{20}{\sqrt{3}}$ kg wt

22. The resultant of two equal forces is double of either of the forces . The angle between them is :
(A) 120º (B) 90º (C) 60º (D) 0º

23. A force of 6 kg wt. and another of 8 kg wt. can be applied together to produce the effect of a single force of:
(A) 1 kg wt. (B) 11 kg wt. (C) 15 kg wt. (D) 20 kg wt.

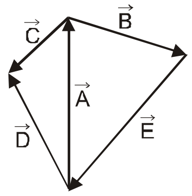
24. Let $\vec{u}$ be a constant vector and $\vec{v}$ be a vector of constant magnitude such that $|\vec{v}| = \frac{1}{2}|\vec{u}|$ and $|\vec{u}| \neq 0$.
Then the maximum possible angle between $\vec{u}$ and $\vec{u}+\vec{v}$ is :
(A) 30° (B) 60° (C) 120° (D) 150°

25. In Figure, $\vec{E}$ equals



(A) $\vec{A}$ (B) $\vec{B}$
(C) $\vec{A} + \vec{B}$ (D) $-(\vec{A} + \vec{B})$

26. In figure, $\vec{D} - \vec{C}$ equals



(A) $\vec{A}$

(B) $-\vec{A}$

(C) $\vec{B}$

(D) $-\vec{B}$

More than one choice type

27.* Which of the following is correct ?

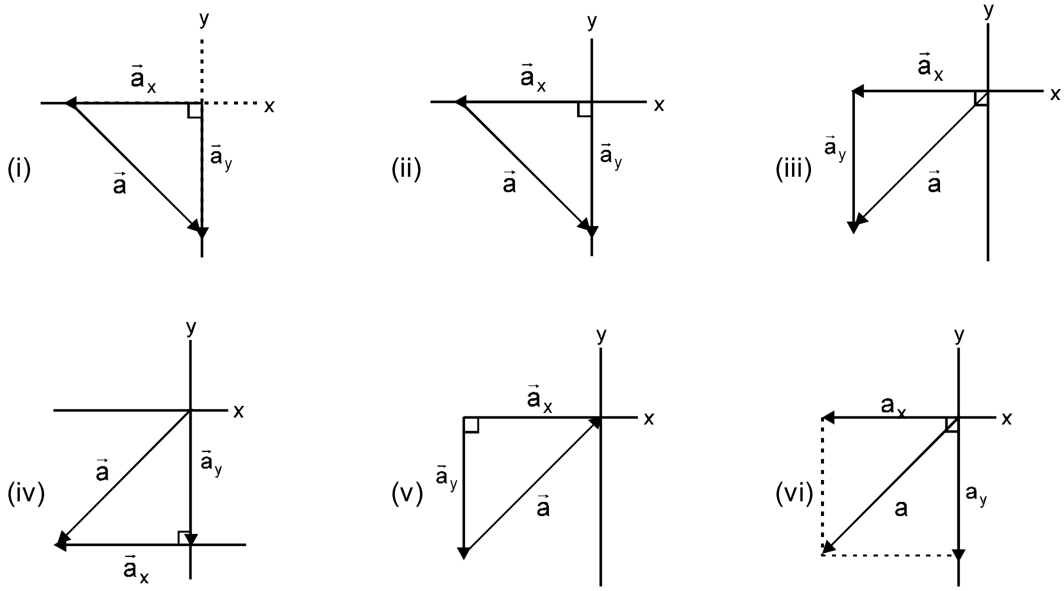
(A) The minimum number of vectors of unequal magnitude required to produce zero resultant is 3.

(B) When $\vec{A}$ is multiplied by '–3', the direction of $\vec{A}$ is reversed but magnitude becomes three times.

(C) The angle between $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ can vary between 0º and 180º.

(D) None of these.

28. In the Figure which of the ways indicated for combining the x and y components of vector a are proper to determine that vector?



(A) (iii)

(B) (iv)

(C) (vi)

(D) (i), (ii) and (v).

29*. Let $\vec{a}$ and $\vec{b}$ be two non-null vectors such that $|\vec{a}+\vec{b}| = |\vec{a}-2\vec{b}|$. Then the value of $\frac{|\vec{a}|}{|\vec{b}|}$ may be :

(A) $\frac{1}{4}$

(B) $\frac{1}{8}$

(C) 1

(D) 2

30.* Given : $\vec{A} + \vec{B} = \vec{C}$. If $\vec{B}$ is reversed, then the resultant becomes $\vec{D}$. Which of the following is incorrect ?

(A) C + D = 2(A + B)

(B) $C^2 + D^2 = 2(A^2 + B^2)$

(C) $C^2 + D^2 = A^2 + B^2$

(D) $C^2 - D^2 = 2(A^2 - B^2)$.

ANSWERS

Exercise # 1

FUNCTION & DIFFERENTIATION

Section - (A)

1. 47 **2.** (C) **3.** 1

Section - (B)

1. $\frac{ds}{dt} = 15t^2 - 15t^4$ **2.** $\sec^2 x - \text{cosec}^2 x$ **3.** $\frac{dy}{dx} = 5\cos x$

4. $\frac{dy}{dx} = 2x + 1$ **5.** $\frac{dy}{dx} = 2x + \cos x$ **6.** $\frac{dy}{dx} = \frac{1}{x} + e^x$, $\frac{d^2y}{dx^2} = -\frac{1}{x^2} + e^x$

7. $\frac{dy}{dx} = 12x - 10 + 10x^{-3}$, $\frac{d^2y}{dx^2} = 12 - 30x^{-4}$

8. $\frac{d\omega}{dz} = 21z^6 - 21z^2 + 42z \Rightarrow \frac{d^2\omega}{dz^2} = 126z^5 - 42z + 42$ **9.** $\frac{dy}{dx} = \cos x - \sin x$, $\frac{d^2y}{dx^2} = -\sin x - \cos x$

10. $\frac{dr}{d\theta} = -12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5} \Rightarrow \frac{d^2r}{d\theta^2} = 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

Section - (C)

1. $y' = 3x^2 + 10x + 2 - \frac{1}{x^2}$ **2.** $\cos^2 x - \sin^2 x$ **3.** $\sin x + x\cos x$ **4.** $\frac{dy}{dx} = 3x^2$

Section - (D)

1. (a) 13 (b) – 7 (c) $\frac{7}{25}$ (d) 20 **2.** $f'(t) = \frac{t^2 - 2t + 1}{(t^2 + t - 2)^2}$

3. $\frac{dz}{dx} = \frac{-2x^2 - 2x - 2}{(x^2 - 1)^2}$

$\left.\frac{dz}{dx}\right|_{\text{at } x = 2} = -\frac{14}{9}$

4. $\sec^2 x$ **5.** $y' = \frac{-19}{(3x - 2)^2}$

Section - (E)

1. $\frac{dy}{dx} = -27(4 - 3x)^8$ **2.** $\left(1 - \frac{x}{7}\right)^{-8}$ **3.** $\frac{dy}{dx} = -5\left(\frac{x}{2} - 1\right)^{-11}$ **4.** $\frac{\cos\sqrt{x}}{4\sqrt{x\sin\sqrt{x}}}$

5. $\frac{dy}{dx} = \cos(x) + \frac{1}{x^2}(2x) + e^{2x}.2 = \cos(x) + \frac{2}{x} + 2e^{2x}$ **Ans.**

6. $5\cos 5x$ **7.** $2\omega\cos(\omega x + \phi)$ **8.** $10(2x + 1)^4$

Section - (F)

1. $\frac{-2xy - y^2}{x^2 + 2xy}$ **2.** $\frac{dy}{dx} = -1$

Section - (G)

1. $\frac{ds}{dt} = 8\pi r \frac{dr}{dt}$ 2. $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$.

Section - (H)

1. 39, 38 2. 8

Section -(I)

1. $\frac{dy}{dx} = -\frac{1}{3}\sin\frac{x}{3}$ 2. 3 cos (3x + 1) 3. $12x^3$. 4. $\frac{dy}{dx} = 48(8x-1)^2$

INTEGRATION

Section - (A)

1. $x^2 + c$ 2. $\frac{x^3}{3} + c$ 3. $x^{-3} + c$ 4. $-\frac{1}{3}x^{-3} + c$

5. $-\frac{1}{x} + c$ 6. $-\frac{5}{x} + c$ 7. $\sqrt{x^3} + c$ 8. $3\sqrt{x} + c$

9. $x^{4/3} + c$ 10. $\frac{x^{2/3}}{2} + c$ 11. $x^{1/2} + c$ 12. $x^{-1/2} + c$

13. – 3 cos x + c 14. tan x + c 15. –cot x +c 16. sec x + c

17. $\frac{1}{3}\ell nx + c$ 18. $\frac{x^3}{3} - x^2 + x + c$ 19. $-\frac{1}{3}x^{-3} + x^2 + 3x + c$ 20. $2x + \frac{5}{x} + c$

21. $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c$ 22. $\frac{3x^{4/3}}{4} + \frac{3x^{2/3}}{2} + c$ 23. $x^{-3/2} + c$ 24. $x - \frac{x^3}{3} - \frac{x^6}{2} + C$

Section - (B)

1. $\frac{1}{2}\sec 2t + C$ 2. $\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C$ 3. $\frac{1}{12}(x^4-1)^3 + C$

4. $-6(1-r^3)^{1/2} + C$ 5. $-\frac{1}{2x} - \frac{1}{4}\sin\frac{2}{x} + C$ 6. $-\frac{1}{3}\cos 3x + C$

7. $-\frac{1}{4}\cos(2x^2) + C$ 8. $\frac{3}{2-x} + C$ 9. $2\sqrt{2y^2+1} + C$

10. $\frac{1}{3}\sin(3z+4) + C$ 11. $-\frac{\cos(8z-5)}{8} + C$ 12. $2\sin(\sqrt{t}+3) + C$

13. $\frac{(2x+1)^4}{8} + C$

Section - (C)

1. 21 2. 24 3. 0 4. e – 1

Section - (D) : Calculation of area

1. Area = $\int_0^b 2x\,dx = b^2$ units 2. $\frac{b^2}{4} + b = \frac{b(4+b)}{4}$ units

3. 2 units 4. $\pi/2$ units

5. 15 6. $\frac{3\pi}{2}$

VECTOR

Section - (A)

1. $\vec{V}_R = -5\hat{j}$ 2. (C) 3. (i) 105º , (ii) 150º , (iii) 105º. 4. 120º

Section - (B)

1. $250\sqrt{5}$ N, $\tan^{-1}(2)$ W of N **2.** (A) **3.** (B) **4.** (B)

5. (D) **6.** (D) **7.** (A) **8.** (C) **9.** (D)

10. (D) **11.** 30 m East **12.** $\sqrt{F_1^2 + F_2^2}$ **13.** 50, 53° with East

Section - (C)

1. $30\sqrt{3}$ km h^{-1}. **2.** $\pm\frac{\sqrt{11}}{10}$ **3.** 15° **4.** (A) **5.** $\sqrt{14}$

6. $\frac{3\hat{i}+4\hat{j}}{5}$ **7.** – 25 cos 30° and –25 sin 30°

Section - (D)

1. **(D)** **2.*** **(BC)** **3.** (a) 3 (b) $-\hat{i} + 2\hat{j} - \hat{k}$ **4.** (a) 6 (b) $6\sqrt{3}$

Exercise # 2

PART-I

Single choice type

1. (A) **2.** (B) **3.** (B) **4.** (B) **5.** (C)

6. (A) **7.** (B) **8.** (A) **9.** (D) **10.** (D)

11. (A) **12.** (D) **13.** (A)

More than one choice type

1. (ABD) **2.** (ABC)

PART-II

FUNCTION & DIFFERENTIATION

1. x **2.** f(2) = 3, f(1) = 3, f(3) = 5 **3.** $-\frac{1}{x}$

Section- (B)

4. $\frac{dy}{dx} = \frac{x^{-\frac{6}{7}}}{7} + \sec^2 x$, $\frac{d^2y}{dx^2} = \frac{-6}{49} x^{\frac{-13}{7}} + 2\tan x \sec^2 x$

5. $\frac{dy}{dx} = \frac{2}{x} + \cos x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$

Section - (C)

6. $\frac{dy}{dx} = 1 + 2x + \frac{2}{x^3} - \frac{1}{x^2}$ **7.** $x^2 \cos x$

8. $\frac{dy}{dx} = -x^2 \sin x$ **9.** $\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$

10. $e^x(\tan x + \sec^2 x)$ **11.** $2x \sin^4 x + 4x^2 \sin^3 x \cos x + \cos^{-2} x + 2x \cos^{-3} x \sin x$

Section - (D)

12. $\frac{\sec^2 q}{(1+\tan q)^2}$ **13.** $\frac{dy}{dx} = \sec^2 x$ **14.** $\frac{-\csc^2 x}{(1+\cot x)^2}$

Section - (E)

15. With $u = \left(\frac{x^2}{8}\right) + x - \left(\frac{1}{x}\right)$, $y = u^4$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 4u^3.\left(\frac{x}{4}+1+\frac{1}{x^2}\right) = 4\left(\frac{x^2}{8}+x-\frac{1}{x}\right)^3\left(\frac{x}{4}+1+\frac{1}{x^2}\right)$

16. $4x \sin(x^2+1)\cos(x^2+1)$ **17.** $\frac{1}{(x^2+1)^{3/2}}$ **18.** $\frac{1-r}{\sqrt{2r-r^2}}$

19. $3\sin^2 x \cos x + 3\cos 3x$ **20.** $\cos[\sin\{\sin x\}]\cos\{\sin x\}\cos x$ **21.** $\cos x\, e^{\sin x}$

Section - (F)

22. $\frac{dy}{dx} = \frac{18y-3x^2}{3y^2-18x}$

Section - (G)

23. (a) $\frac{dV}{dt} = \pi r^2\frac{dh}{dt} = 5\pi r^2$ (b) $\frac{dV}{dt} = 2\pi hr\frac{dr}{dt} = 10\pi rh$ (c) $\frac{dV}{dt} = \pi r^2\frac{dh}{dt} + 2\pi hr\frac{dr}{dt} = 5\pi r^2 + 10\pi rh$

Section - (H)

24. $\sqrt{\frac{40}{3}}$ m **25.** $x = 30$ & $y = 30$

Section - (I)

26. $-\sin(\sin x)\cos x$. **27.** $\frac{dy}{dx} = \cos(x-\cos x)(1+\sin x)$

28. (a) $2\,\text{cosec}^3 x - \text{cosec}\, x$ (b) $2\sec^3 x - \sec x$

INTEGRATION

Section (A): Integration of elementry functions

1. $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$ **2.** $-\frac{1}{x} - \frac{1}{2x^2} + C$ **3.** $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

4. $-2t^{-2} - \frac{2}{3}t^{-3/2} + C$ **5.** $-\cot x - x + C$ **6.** $2x + \cot x + C$

7. $-\cos\theta + \theta + C$ **8.** $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$ **9.** $\frac{x}{5} + \frac{1}{x^2} + x^2 + C$

Section - (B)

10. $\frac{1}{3}(x^{3/2}-1) - \frac{1}{6}\sin(2x^{3/2}-2) + C$

11. (a) $-\frac{1}{4}(\cot^2 2\theta) + C$ (b) $-\frac{1}{4}(\csc^2 2\theta) + C$

12. $\left[\frac{2}{5}\sqrt{5x+8}\right] + C$ **13.** $-\frac{1}{3}(3-2s)^{3/2} + C$ **14.** $-\frac{2}{5}(1-\theta^2)^{5/4} + C$

15. $3(\theta^2-1)^{4/3} + C$ **16.** $\frac{(-2)}{(1+\sqrt{x})} + C$ **17.** $\frac{(1+\sqrt{x})^4}{2} + C$

18. $\frac{1}{3}\tan(3x + 2) + C$

19. $\frac{\tan^3 x}{3} + C$

20. $\sec\left(\upsilon + \frac{\pi}{2}\right) + C$

21. $\frac{1}{2\cos(2t+1)} + C$

22. $\frac{-3}{(2+\sin t)^2} + C$

23. $-(7x-2)^{-4} + C$

24. $(y^4 + 4y^2 + 1)^3 + C$

25. $-2\csc\left(\frac{\upsilon-\pi}{2}\right) + C$

Section - (C)

26. $\frac{1}{3}\ell n\frac{5}{2} = \ell n\left(\frac{5}{2}\right)^{\frac{1}{3}}$

27. $\frac{7}{3}$

28. 0

29. 1

30. $\frac{3\pi^2}{2}$

Section - (D)

31. $\frac{\pi b^2}{4}$

32. Using n subintervals of length $\Delta x = \frac{b}{n}$ and right–end point values :

$$\text{Area} = \int_0^b 3x^2\,dx = b^3$$

VECTOR

1. 45^o

2. $\vec{B} = \lambda\vec{A} = -4 \times 3$ N-E
$= 12$ S-W
No it does not represent the same physical quantity.

3. 24 N ; 37^0 approx
$= \sqrt{576} = 24$N

4. P = 40 ; Q = 30

5. $\frac{8}{\sqrt{5}}$

6. $r(1 + \sqrt{2})$

7. $\frac{4\hat{i}+5\hat{j}+2\hat{k}}{\sqrt{45}}$

8. $3\sqrt{5}$, $\tan^{-1}\frac{1}{2}$

9. $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

10. 37^o

ADVANCE LEVEL PROBLEMS

PART-I

DIFFERENTIATION

1. $\frac{dy}{dx} = -2 + 3x^{-4}$

$\frac{d^2y}{dx^2} = -12x^{-5}$

2. $\frac{dy}{dx} = -2x$, $\frac{d^2y}{dx^2} = -2$

3. $\frac{dy}{dx} = x^2 + x + \frac{1}{4}$; $\frac{d^2y}{dx^2} = 2x + 1$

4. (a) –2 (b) $\frac{2}{25}$ (c) $-\frac{1}{2}$ (d) –7

5. $g'(x) = \frac{x^2+x+4}{(x+0.5)^2}$

6. $\frac{dy}{dx} = -\frac{3}{x^2} + 5\cos x$

7. $-\csc x \cot x - \frac{2}{\sqrt{x}}$

8. $-10 - 3\sin x$

9. $\frac{-2\,\text{cosec}\,t\cot t}{(1-\text{cosec}\,t)^2}$

10. $\frac{ds}{dt} = \frac{1}{\cos t - 1}$

11. $\sec^2 t - 1$

12. $\frac{ds}{dt} = 2t - \sec t\tan t + 1$

13. $\frac{dp}{dq} = -\text{sing}\ q - \text{cosec}^2 q$

14. $\sec^2 q$

15. $\frac{dy}{dx} = 20\sin x\cos^{-5} x$

16. **With** $u = \sin x$, $y = u^3$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 3u^2\cos x = 3\sin^2 x(\cos x)$

17. $\frac{3\pi}{2}\left[\cos\left(\frac{3\pi t}{2}\right) - \sin\left(\frac{3\pi t}{2}\right)\right]$

18. $\frac{\text{cosec}\,\theta}{\cot\theta + \text{cosec}\,\theta}$

19. $\frac{\sec\theta}{\sec\theta + \tan\theta}$

20. $\frac{4}{\pi}(\cos 3t - \sin 5t)$

INTEGRATION

1. (a) $\frac{2}{3x^3} + C$ (b) $\frac{-x^{-3}}{6} + C$ (c) $\frac{x^5}{5} + \frac{1}{3x^3} + C$

2. (a) $x^{2/3} + C$ (b) $x^{1/3} + C$ (c) $x^{-1/3} + C$

3. (a) $\sin \pi x + C$ (b) $\sin\frac{\pi x}{2} + C$ (c) $\frac{2}{\pi}\sin\frac{\pi x}{2} + \pi\sin x + C$

4. (a) $-\csc x + C$ (b) $\frac{1}{5}\csc(5x) + C$ (c) $2\csc\left(\frac{\pi x}{2}\right) + C$

5. $3x + \sin 2x + 4\sin x + C$

6. (a) $3x^2 + C$ (b) $\frac{x^8}{8} + C$ (c) $\frac{x^8}{8} - 3x^2 + 8x + C$

7. (a) $\frac{-2}{3x^3} + C$ (b) $\frac{-x^{-3}}{6} + \frac{x^3}{3} + C$ (c) $\frac{x^{-3}}{3} + \frac{x^2}{2} - x + C$

8. $5x - 3x^2 + C$

9. $t^3 + \frac{t^2}{4} + C$

10. $\frac{t^3}{6} + t^4 + C$

11. $\frac{3}{2}x^{2/3} + C$

12. $\frac{x^{\frac{3}{2}}}{3} + 4x^{1/2} + C$

13. $4y^2 - \frac{8}{3}y^{3/4} + C$

14. $x^2 + \frac{2}{x} + C$

15. $-2\sin t + C$

16. $5\cos t + C$

17. $-21\cos\frac{\theta}{3} + C$

18. $\frac{3}{5}\sin 5\theta + C$

19. $3\cot x + C$

20. $\frac{-\tan x}{3} + C$ **21.** $-\frac{1}{2}\csc\theta + C$ **22.** $\frac{2}{5}\sec\theta + C$

23. $4\sec x - 2\tan x + C$ **24.** $-\frac{1}{2}\cot x + \frac{1}{2}\csc x + C$ **25.** $-\frac{1}{2}\cos 2x + \cot x + C$

26. $\sin 2x + \cos 3x + C$ **27.** $2y - \sin 2y + C$ **28.** $\frac{y}{14} + \frac{\sin 2y}{28} + C$

29. $\frac{t}{2} + \frac{\sin 4t}{8} + C$ **30.** $\frac{t}{2} - \frac{\sin 6t}{12} + C$ **31.** $\tan\theta + C$

32. $\tan\theta + C$ **33.** $\frac{x^2}{2} + x + C$ **34.** Right

35. Right **36.** Right **37.** Right

38. (a) Wrong : $\frac{d\sqrt{x^2+x+C}}{dx} = \frac{2x+1}{2\sqrt{x^2+x+C}}$

(b) Wrong : $\frac{d\left(\sqrt{x^2+x}+C\right)}{dx}$

$= \frac{1}{2\sqrt{x^2+x}}(2x+1) = \frac{2x+1}{2\sqrt{x^2+1}}$

(c) Wrong : $\frac{d\left(\left(\frac{1}{3}\sqrt{2x+1}\right)^3 + C\right)}{dx}$

$= \frac{3}{2}\cdot\frac{1}{3}(2x+1)^{1/2}\cdot 2 = \sqrt{(2x+1)}$

39. Right **40.** $-\frac{1}{3}(7-3y^2)^{3/2} + C$ **41.** $\frac{1}{2}\sin^6\left(\frac{x}{3}\right) + C$

42. $\frac{1}{4}\tan^8\frac{x}{2} + C$ **43.** $\left(\frac{r^3}{18}-1\right)^6 + C$ **44.** $-\frac{1}{2}\left(7-\frac{r^5}{10}\right)^4 + C$

45. $-\frac{3}{4}\cos(x^{4/3}-8) + C$ **46.** $-2\csc\left(\frac{\upsilon-\pi}{2}\right) + C$ **47.** $-\frac{2}{3}(\cot^3 y)^{1/2} + C$

48. $2\sqrt{\sec z} + C$ **49.** $-\sin\left(\frac{1}{t}-1\right) + C$ **50.** $\frac{2}{5}(5s+4)^{1/2} + C$

51. Area = 2.5 square units **52.** Area = 2 square units **53.** $9b^3$ **54.** $\frac{\pi^3}{24}$

VECTOR

1. $\overrightarrow{BC} = 3\hat{j}$ = 3 km in north **2.** 150^0 **3.** 113^0

4. when the angle between A & B is 120^0 ; when it is 60^0

5. Hence P : Q : R = $\sqrt{2} : \sqrt{3} : \sqrt{2}$

PART-II

OBJECTIVES

Single choice type

15. (D) **16.** (A) **17.** (C) **18.** (A) **19.** (C)

20. (C) **21.** (C) **22.** (D) **23.** (B) **24.** (A)

25. (D) **26.** (A)

More than one choice type

27.* (ABC) **28.** (ABC) **29*.** (CD) **30.*** (ACD)